

FUZZY DUAL GRAPH

***Nuha Abdul-Jabbar, Jehan H. Naoom and, Eman H. Ouda**
Department of Applied Sciences, University of Technology.
***E-mail :Nuha1900@yahoo.com.**

Abstract

The definition of fuzzy dual graphs are considered with the following properties are obtained, which are the dual of the dual of fuzzy graph is the fuzzy graph itself, and the dual of fuzzy bipartite graph is Eulerian fuzzy graph.

Key word: fuzzy graph, dual graph, euler graph.

Introduction

Researches on the theory of fuzzy sets has been witnessing an exponential growth both within mathematics and in its applications, this ranges from traditional mathematical subjects like logic, topology, algebra, analysis, etc. to pattern recognition, information theory, artificial intelligence, operation research, neural networks and planning, etc. Consequently fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest [5,7].

Rosenfeld in 1975 considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs, and then some basic fuzzy graph theoretic concepts and applications have been indicated, many authors found deeper results, and fuzzy analogues of many other graph theoretic concepts, this include fuzzy trees, fuzzy line graphs, operations on fuzzy graphs, automorphism of fuzzy graph, fuzzy interval graphs, cycles and cocycles of fuzzy graphs, bipartite fuzzy graph and metric aspects in fuzzy graph [3,4,5].

In this paper the definition of dual fuzzy graph is modified and we get some properties of it. Which are the dual of dual fuzzy graph is the fuzzy graph itself, and the dual of bipartite fuzzy graph is an Eulerian fuzzy graph.

Basic Concept

In this section we explain some basic definitions of the graph G with n vertices and m edges consisting of a vertex set $V(G) = \{v_1, \dots, v_n\}$ and an edge set $E(G) = \{e_1, \dots, e_m\}$ where each edge consists of two vertices called its endpoints.

Definition (1) [6, p.16]

Suppose that the vertex set of a graph G can be split into two disjoint sets V_1 and V_2 , in such a way that every edge of G joins a vertex of V_1 to a vertex in V_2 : G is then said to be a *bipartite* graph, which we denoted by $G(V_1, V_2)$ as shown in Fig. (1).

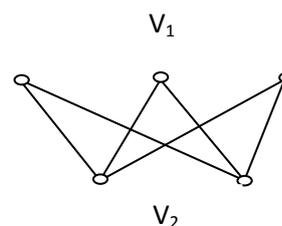


Fig. (1) : Bipartite graph.

Definition (2) [6, p.30]

A connected graph G is called an *Eulerian* graph if there exists a circuit which includes every edge of G ; such a circuit is called an Eulerian circuit, (See Fig. (2)).

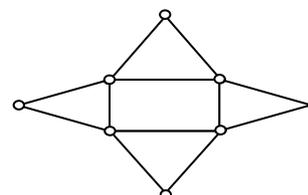


Fig. (2) : Eulerian graph.

Theorem(1) [2,p.86], [6,p.32]

A given connected graph G is an Eulerian Graph if and only if all vertices of G are of even degree.

Definition (3) [2, p.247], [6,p.64]

A planar graph is a graph drawn in the plane in such a way that no two edges intersect geometrically except at a vertex to which they are both incident, each closed region in a planar graph called face.

An example of a planar graph is given in Fig. (3).

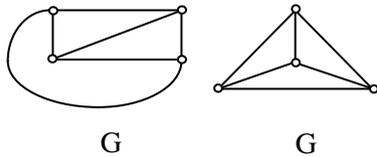


Fig. (3) : Planar graphs.

Corollary [6, p.67]

If G is a connected simple planar graphs with $n(\geq 3)$ vertices and m edges, then $m \leq (3n-6)$.

Remark [2, p.256]

A simple planar graph G with $(3n-6)$ edges, and every face is a triangle called a maximal planar graph.

Definition (4) [2, p.250]

Suppose G is a planar graph. The dual graph G^* of G is a planar graph having a vertex for each face in G . The edges of G^* correspond to the edge of G as follow: if e is an edge of G that has face F_i on one side and face F_j on the other side, then the corresponding dual edge $e \in E(G^*)$ is an edge joining the vertices x, y of G^* that corresponding to the faces F_i, F_j of G .

Below we have drawn a plane graph G with solid edge, and its dual G^* with dashed edges (see Fig.(4)).

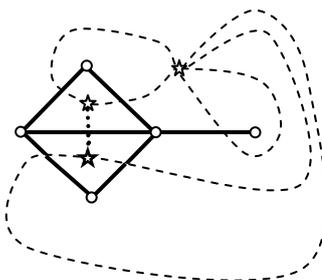


Fig. (4) :Dual graph G^* .

Lemma [6, p.73]

Let G be a planar connected graph with n vertices, m edges and f faces, and let its geometric dual G^* have n^* vertices, m^* edges and f^* faces. Then $n^*=f, m^*=m,$ and $f^*=n$.

Theorem (2) [6, p.74]

Let G be a planar connected graph. Then G^{**} is isomorphic to G .

Theorem (3) [2, p.235]

Edges in a planar graph G form a cycle in G if and only if corresponding dual edges form a bond (cut edge) in G^* .

Definition (5)[1]

Planar graph is said to be dual Eulerian if it has an Eulerian circuit e_1, \dots, e_k such that e_1^*, \dots, e_k^* form an Eulerian circuit in G^* .

Fuzzy Dual Graph

Let $G(V,E,F)$ be a graph with V the set of vertices, E the set of edges, and F the set of faces .

A fuzzy subsets of a set F is a mapping $\lambda:F \rightarrow [0,1]$ for any $f \in F, \lambda(f)$ is called the degree of membership of f in \tilde{G} (the fuzzy graph), a fuzzy subset of a set V is a mapping $\sigma:V \rightarrow [0,1]$ for any $v \in V, \sigma(v)$ is called the degree of membership of V in \tilde{G} . If λ and σ is into $[0,1]$ then they are called the characteristic functions of a subset F and V respectively , and they are known as ‘‘crisp’’.

A fuzzy relation on V is a fuzzy subset of $V \times V$, a fuzzy relation μ on V is a fuzzy relation on \bar{b} which is $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

For all u, v in V , where \wedge stands for minimum.

In this paper we define a fuzzy planar graph which is defined by the triple functions $\tilde{G}(\sigma, \mu, \lambda)$ where \bar{b} is fuzzy subset of V, μ is symmetric fuzzy relation on \bar{b} , and λ is a fuzzy subset of F .

Note that the crisp graph (V, E, F) is a special case of a fuzzy graph with each membership equal one.

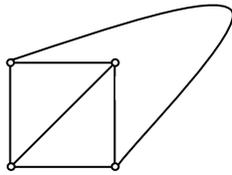


Fig. (5): Fuzzy planar graph.

Definition (6)

The maximal fuzzy planar graph is the fuzzy planar graph with maximum number of edges such that, when we add any other edge, the graph be fuzzy non planar graph.

Note that the maximal fuzzy bipartite planar graph having even cycles of length four, see Fig.(6).

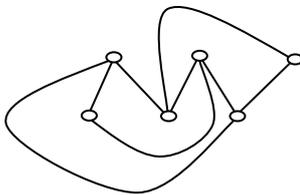


Fig. (6) : Fuzzy bipartite planar graph.

Definition (7)

Given a fuzzy planar graph \tilde{G} , we shall construct another fuzzy graph \tilde{G}^* called dual fuzzy graph of \tilde{G} .

The construction is in three stages:

- 1- Inside each face \tilde{F}_i of \tilde{G} we choose a point \tilde{v}_i^* these points are the vertices of \tilde{G}^* with the degree of membership $\sigma(v_i^*) = \lambda(f_i)$
- 2- Corresponding to each edge \tilde{e} of \tilde{G} we draw a line \tilde{e}^* which crosses \tilde{e} but not other edge of \tilde{G} , and joins the vertices \tilde{v}_i^* which lie in the faces \tilde{f}_i adjoining \tilde{e} these line are the edges of \tilde{G}^* with degree of membership $\mu(e_i^*) = \mu(e_i)$; and μ still satisfy the condition $\mu(e_i^*) \leq \min(\sigma(v_i^*), \sigma(v_{i+1}^*))$ where $\tilde{v}_i^*, \tilde{v}_{i+1}^*$ is the end of \tilde{e}_i^* .
- 3- Every faces of \tilde{G}^* has a membership λ such that $\lambda(f_i^*) = \sigma(v_i)$.

The fuzzy dual of the planar graph in Fig.(5) is shown in Fig.(7).

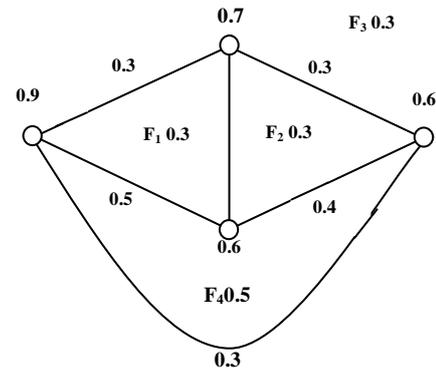


Fig. (7) : Fuzzy planar graph.

Proposition (1)

The dual of the dual fuzzy graph is the fuzzy graph itself $\tilde{G}^{**} = \tilde{G}$

Proof:

By using the definition of fuzzy dual graph the dual of fuzzy graph \tilde{G} having the degree of membership as follow:

$$\sigma(v_i^*) = \lambda(f_i)$$

$$\mu(e^*) = \mu(e)$$

$$\lambda(f^*) = \sigma(v)$$

and the dual of the dual fuzzy graph has the degree of membership as follow:

$$\sigma(v^{**}) = \lambda(f^*) = \sigma(v)$$

$$\Rightarrow \sigma(v^{**}) = \sigma(v)$$

$$\mu(e^{**}) = \mu(e^*) = \mu(e)$$

$$\Rightarrow \mu(e^{**}) = \mu(e)$$

$$\lambda(f^{**}) = \sigma(v^*) = \lambda(f)$$

$$\Rightarrow \lambda(f^{**}) = \sigma(v^*)$$

Such that the membership of the dual of the dual fuzzy graph is the membership of the fuzzy graph itself such that $\tilde{G}^{**} = \tilde{G}$

Proposition (2)

The dual of fuzzy bipartite graph is an Eulerian fuzzy graph.

Proof:

In this prove we consider the maximal fuzzy bipartite graph with the degree of membership $\sigma(\tilde{v}), \mu(e)$ and $\lambda(f)$ for the vertices, edges, and faces respectively.

Since it is maximal fuzzy bipartite planar graph then by definition (6) each cycles of

الخلاصة

even length, such that every face \tilde{f} in \tilde{G} have even lengths.

So the dual of these faces f_i for all $i=1, \dots, k$ obtained a vertices v_i^* of even degree such that by theorem (1) the dual graph is Eulerian fuzzy graph, with the degree of membership $\sigma(v^*) = \lambda(f)$, $\mu(e^*) = \mu(e)$ and $\lambda(f^*) = \sigma(v)$.

في هذا البحث تم تعريف البيان الثنائي الضبابي ودراسة بعض خصائصه وكما يلي الثنائي الضبابي للبيان الثنائي الضبابي يبقى ثنائيا ضبابيا، والثنائي الضبابي للبيان الثنائي التجزئة الضبابي يكون بيانا اويلريا ضبابيا.

References

- [1] B. Servatius and H. Servatius, "Dual Eulerian Graphs", *Obzornik Za Matematiko in Fiziko*, Vol.50, No.1, 2003, pp.14-20.
- [2] D.B.West, "Introduction to Graph Theory", Prentice-Hall Inc., 2001.
- [3] L.S.Bershtein and T.A. Dziouba, "Allocation of Maximal Bipartite Part from Fuzzy Graph" <http://citeseerix.ist.psu.doc>, 2000, pp.207-211.
- [4] M.R Berthold and k.huber, "Constructing Fuzzy Graphs from Examples", *Intelligent data Analysis*, Vol. 3, 1999, pp 37-53.
- [5] M.S.Sunitha, "Complement of a fuzzy Graph" *Indian J. pure appl. math.*, Vol.33No.9.2002,pp. 1451-1464.
- [6] R.J. Wilson, "Introduction to Graph Theory" Third Edition, Longman Inc, 1985.
- [7] W.B. kandasamy, and F.Smarandaohe, "Basic Neutrosophic Algebraic Structures and their Application to Fuzzy Models" American Reseach Press, 2004.