On supra compactness in supratopologicol spaces

Taha H. Jassim
Dep. of mathematics, College of computer sciences& Math. , University of Tikrit , Tikrit , Iraq

(Received 6 / 8 / 2008 , Accepted 29 / 10 / 2008)

Abstract:

In this paper we investigate and study some properties of compactness in supratopologicol space and show that if f is S^*-continuous function from (X,τ_X^*) to (Y,τ_Y^*), and E is S -compact in X then $f(E)$ is S -compact in (Y,τ_Y^*), also if X,Y are a two S -compact supratopologicol space then $X\times Y$ is S -compact.

Keywords: Supratopologicol space, supra continuous, supra relative space, supra compact, supra open cover, supra open sub cover.

1-Introduction*:*

Let X,Y be any topological spaces. sub class $\tau^*\subseteq p(X)$ is called supratopology on X if $X\in\tau^*$ and τ^* is closed under arbitrary union .(X,τ^*) is called asupratopological space, The members of τ^* are called supra open sets and it's Complement is an supra closed sets[1]. Let (X,τ) be a topological space and τ^* be a supratopology on X, we called τ^* asupratopology associated with τ if $\tau\subseteq\tau^*$. Let $(X,\tau_x{}^*)and(Y,\tau_y{}^*)$ be supratopological spaces. Afunction $f:X\to Y$ is an S^*-continuous function if the inverse image of each supra open set in Y is a supra open set in X [3].let E be asubset of X, then the classes τ_E^* of all intersections of E with τ^*-supra open subsetsof X belong to τ^* is a topology on E it is called relative supratopology,We say that a collection $\{u_\alpha^*\}_{\alpha\in\Lambda}$ of asupra open sub set of X be an supra open cover of E if and only if $E\subseteq\bigcup_{\alpha\in\Lambda}u_\alpha^*$ then (X,τ^*) is supra compact (S -compact for short) if and only if every supra open cover of E has finite supra sub cover. For example let $X=\{a,b,c,e\}$, τ^* is supratoplogical of with empty set and also $u_1{}^*=\{a\},u_2{}^*=\{b,c,e\}$ hence $X\subseteq\bigcup_{i=1}^{2}u_i^*$ then (X,τ^*) is S -compact space. The product of supratopology is $\tau_{XXY}^*=\{\ \bigcup_{i,j}u_i^*\times v_j^*,u_i^*\in\tau_X^*,v_j^*\in\tau_Y^*\}$[2].For xample let $X=\{a,b,e,f\},Y=\{1,2\}$, $\tau_X^*=\{X,\{a\},\{b\},\{a,b\}\}$, $\tau_Y^*=\{Y,\{1\},\{2\}\}$ then

$\tau_{X\times Y}^*=\{X\times Y,X\times\{1\},X\times\{2\},\{a\}\times Y,\{a\}\times\{1\},$
$\{a\}\times\{2\},\{b\}\times Y,\{b\}\times\{1\},\{b\}\times\{2\},\{a,b\}\times Y,$
$\{a,b\}\times\{1\},\{a,b\}\times\{2\}\}$.=
$\{X\times Y,\{(a,1),(b,1),(c,1),(d,1)\},\{(a,2),$
$(b,2),(c,2),(d,2)\},\{(a,1),(a,2)\},\{(a,1)\},$
$\{(a,2)\},\{(b,1),(b,2)\},\{(b,1)\},\{(b,2)\},\{(a,1),$
$(a,2),(b,1),(b,2)\},\{(a,1),(b,1)\},\{(a,2),(b,2)\}\}$

2- S - Compactness in supratopologicol spaces

Theorem 2-1: Any supra closed sub set of S -compact is S -compact

Proof.

Let (X,τ^*) be a S *-compact* space, let E be any supra closed sub set of X and $\{E_\alpha:\alpha\in I\}$ supra open cover of aset E, to show there exist finite Supra open cover of E, since E is supra closed set then E^c supra open set, $E^c\bigcup\{E_\alpha:\alpha\in I\}$ supra open cover in (X,τ^*) by hypothesis there exist supra open cover has finite supra open sub cover $\{E_\alpha:\alpha\in I\}$containing sub cover such that if E^c not supra open cover any part E (because $E\bigcap E^C=\phi$) such that $E\subset\bigcup_{i=1}^{n}E_{\alpha i}$ is finite supra open sub cover on E, then E is S -compact.

Remark 2-2: Every finite supratopologicol space is S -compact.

Let $X=\{x_1,x_2,...,x_n\}$, let $x_1\in u_1^*,x_2\in u_2^*....x_n\in u_n^*$ then $x\in u_1^*\bigcup u_2^*\bigcup....\bigcup u_n^*$ then $x\subseteq\bigcup_{i=1}^{n}u_i^*$

Theorem 2-3: let (X,τ^*) be a supratopologicol space and let (E,τ_E^*)be a subspace of (X,τ^*) then (E,τ_E^*) is S -compact iff every supra open cover of E is consist of supra open set contained in X has finite supra open cover of E.

Proof.

Suppose that E is S -compact, $\{E_\alpha:\alpha\in I\}$ supra open cover of E the family $\{E_\alpha\bigcap E:\alpha\in I\}$ supra open cover in (E,τ_E^*).since E is S -compact there exist finite supra open sub cover $\{E_{\alpha_1}\bigcap E....E_{\alpha n}\bigcap E\}$ cover of E then $\{E_{\alpha1},.....E_{in}\}$supra open cover of E.

Necessity. To prove E is S -compact. Let $\{E_\alpha^c\}=E^c$ supra open cover on E, $\forall\alpha$ such that $E_\alpha^c=E_\alpha\bigcap E$, the family $\{E_\alpha\}$supra open cover set on E, $E\subseteq X$ by hypothesis $\exists$ finite supra open cover on E then $\{E_{\alpha1}^c,...E_{\alpha n}^c\}$ finite supra open sub cover on E^c cover set on E.

Theorem 2-4: Let E be a sub set of a supratopologicol space (X,τ^*) then E is S -compact in (X,τ^*) if and only if E is S -compact in (E,τ_E^*).

Proof.

Sufficiency. Suppose that $\{u_i^*\}$ be a τ_E^* -supra open cover of E, there exist $H_i \in \tau^*$ such that $u_i^* = E\bigcap H_i, u_i^* \subseteq Hi$ hence $E \subseteq \bigcup_i u_i^*, E \subset \bigcup_i H_i^* and \{H_i\}$ is a $\tau^* -$ supra open cover of E .since E is S -compact on τ^* so $\{H_i^*\}$ contains a finite supra sub open cover we say $E \subseteq H_{i1}\bigcup H_{i2}\bigcup ...\bigcup H_{in}$, $H_{ik} \in \{H_i\}$ But $E = u_{i1}^*\bigcup ....\bigcup u_{im}^*$ thus H_{im}^* contains a finite supra open sub cover then (E,τ_E^*) is S -compact space.

Necessity.

let $\{H_i\}$ be a $\tau^* -$ supra open cover of E $u_i^* = E\bigcap H_i, E \subseteq \bigcup_i H_i$ thus $u_i^* = \bigcup_i (E\bigcap H_i) = \bigcup u_i^*$ but $u_i^* \in \tau_E^*$ so $\{H_i\}$ is a $\tau_E^* -$ supra open cover of hypothesis $\{u_i^*\} = E\bigcap (H_i\bigcup ...\bigcup H_{im}) \subseteq H_{i1}\bigcup ....\bigcup H_{im}$ then E is S - compact space.

Theorem 2-5: Let (X,τ^*) be a supratopologicol space, Let (E,τ_E^*) be a sub space of X ,if $Y \subseteq E$,then Y is S -compact set in τ^* iff Y is S -compact set in τ_E^* .

Proof.

Sufficiency. Suppose that y is S -compact set in τ_E^* .to show y is S -compact set in τ^* , let $\{u_\alpha^*\}$ supra open cover set y on τ^* such that

$y = y\bigcap E \subset \left(\bigcup_\alpha u_\alpha^*\right)\bigcap E = \bigcup_\alpha (u_\alpha^*\bigcap E) = \bigcup_\alpha u_\alpha^{**}$ Thus $\{u_\alpha^{**}\}$ supra open cover set y on τ^* ,but y is S -compact set in τ_E^* , there exist finite supra open cover $y \subseteq \bigcup_{i=1}^n u_i^{**}, y \subseteq \bigcup_{i=1}^n (u_\alpha^*\bigcap E) \subseteq \bigcup_{i=1}^n u_\alpha^*$ is finite supra open cover then y is S - compact set in τ^* .

Conversely.

Let y is S -compact set in τ^* , $\{u_\alpha^{**}\}$ supra open cover set y on τ_E^* , $y \subseteq \bigcup u_\alpha^{**}$,by definition relatively $u_\alpha^{**} = u_\alpha^* \bigcap E, \forall \alpha$ such that

$y \subseteq \bigcup_\alpha u_\alpha^* = \bigcup_\alpha (u_\alpha^*\bigcap E) = \left(\bigcup_\alpha u_\alpha\right)\bigcap E \subseteq \bigcup_\alpha u_\alpha$ since y is S - compact set in τ^* , $\exists$ finite supra open sub cover, $y \subseteq \bigcup_{i=1}^n u_{\alpha i}$,

Thus $y = y\bigcap E \subseteq \left(\bigcup_{i=1}^n u_{\alpha i}^*\right)\bigcap E = \bigcup_{i=1}^n (u_\alpha^*\bigcap E) = \bigcup_{i=1}^n u_{\alpha i}^*$ is finite supra open cover on y sets then y is S - compact set in τ_E^*

Theorem 2-6: Asupratopologicol space (X,τ^*) is S - compact if and only if For every class $\{F_i\}$ of supra closed sub sets of X , $\bigcap F_i = \phi$ impels $\{F_i\}$ contains a finite sub class $\{F_{i1}, F_{i2}, ...F_{im}\}$ with $F_{i1}, F_{i2}, ..F_{im} = \phi$

Proof.

Suppose $\bigcap F_i = \phi$, then by De Morgan's low. $X = \bigcup F_i^c$ So $\{F_i^c\}$ is an supra open cover of X ,Since each F_i is closed but X is S -compact hence there exist $F_{i1}^c, ...F_{im}^c \in \{F_i^c\}$ thus $\phi = X^C =$

$(F_{i1}^c\bigcup ....\bigcup F_{im}^c)^c = F_{i1}^{cc}\bigcap ....\bigcap F_{im}^{cc}$

$= F_{i1}\bigcap ....\bigcap F_{im}$

Necessity.

let be $\{G_I\}$ be an supra open cover of $X = \bigcup_i G_i$ by De Morgan's low $\phi = X^C = \left(\bigcup_i G_i\right)^c = \bigcap_i G_i^c$, since each G_i is open. $\{G_i^c\}$ Is class of supra closed sets and has a empty intersection Hence there exist $G_{i1}^c, ...G_{im}^c \in \{G_i^c\}$ such that $G_{i1}^c \bigcap ...\bigcap G_{im}^c = \phi$ Thus $X = \phi^c = G_{i1}\bigcup ....\bigcup G_{im}$ then X is S -compact

3- S^* - continuous of supratopologicol spaces

Theorem 3-1:

Let $f : (X,\tau_X^*) \to (Y,\tau_Y^*)$ be a S^* - continuous, and let E be a S -Compact sub set of X then it's image $f(E)$ is S - compact sub set of Y .

Proof.

Suppose $\{G_I\}$ is an supra open cover of $f(E)$ hence $\{f^{-1}(G_i)\}$ is an supra open cover of E . Since S^* - continuous and each G_i is an supra open set, so $\{f^{-1}(G_i)\}$ is an supra open cover.

Theorem 3-2: each of (X,τ_x^*) and (Y,τ_y^*) be a S - compact spaces, Let: $f : (X,\tau_x^*) \to (Y,\tau_y^*)$ be a S^* - continuous then $f(X)$ is S -compact.

Proof.

Suppose $(G_i)_{i\in I}$ be supra open cover to a set $f(X)$ on Y , the family $\{(f^{-1}(A_i) : A \in G)\}_{i\in I}$ is supra open cover on X ,since is S^* - continuous and A is supra open set in Y then $f^{-1}(A)$ is supra open set in X . since X is

S -compact then the there exist finite sets $f^{-1}(A_1),\dots f^{-1}(A_n)$ supra open cover to X the set $A_1,\dots A_n$ in $(G_i)_{i\in I}$ cover to $f(X)$ then $f(X)$ is S -compact.

Theorem 3-3: Let $(X,\tau_x{}^*)$ and $(Y,\tau_y{}^*)$ are a S -compact spaces then the product of supratopology XxY is a S -compact space

Proof.

Since $(X,\tau_x{}^*)$ is S -compact there exist finite supra open sub cover $\left\{\bigcup_i u_i^*\right\}$ cover of X then $\{u_{i1},\dots\dots u_{in}\}$ supra open sub cover of X, and Since $(Y,\tau_y{}^*)$ is S -compact there exist finite supra open sub cover $\left\{\bigcup_i v_i^*\right\}$ cover of Y then $\{v_{i1},\dots\dots v_{in}\}$ supra open sub cover of Y. By definition The product of supratopolog is $\tau^*_{XXY}=\{\ \bigcup_{i,j} u_i^*\times v_j^*, u_i^*\in\tau_X^*, v_j^*\in\tau_Y^*\}$ Thus $\left\{\bigcup_{i,j} u_{i1}^*\times v_{i1}^*,\dots,\bigcup_{i,j} u_{in}^*\times v_{jm}^*, u_i^*\in\tau_x^*, v_j^*\in\tau_y^*\right\}$ hence $\exists$ finite supra open sub cover on XxY then $\left(X\times Y,\tau^*_{x\times y}\right)$ is S -compact .

References:

1 .A.S. Mashhor, A.A. Allam , F.S. Mahmoud and F.H. Khedr ,on supratopological spaces , Indain J.pure Appl. Math 14(1983),502-510

2. J. Dugundji , topology , library of congress catalog card number :66-1094 printed in the United States of America

3. W.K. Min , H. S.chang , on M-continuity , Kangweon – Kyungki. Jour.6 , no 2(1998),32

حول السبرا- تراص في الفضاءات السبراتبولوجية(التبولوجية الفوقية)

طه حميد جاسم الدوري

قسم الرياضيات ، كلية علوم الحاسبات والرياضيات ، جامعة تكريت ، تكريت ، العراق

(تاريخ الاستلام: ٦ / ٨ /٢٠٠٨ ، تاريخ القبول: ٢٩ / ١٠ / ٢٠٠٨)

الملخص

في هذا البحث تحرينا ودرسنا بعض خواص التراص في الفضاءات السبراتبولوجية(التبولوجية الفوقية)ووجدنا انه إذا كانت f دالةمستمرةمن النمط S^*- من (X,τ_X^*) إلى (Y,τ_Y^*)، وكانت E مرصوصة من النمط- S في X فان $f(E)$ هي مرصوصة من النمط- S في (Y,τ_Y^*) .وكذلك ووجدنا انه اذاكانت كل من X,Y هما فضاءئين تبولوجين فوقيين ومرصوصين من النمط- S فان $X\times Y$ هو فضاء مرصوص من النمط- S .