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Application of B-Spline to Numerical Solution of Free Damped Vibration Equation with Boundary Values Conditions

-سبلا ين عددياً مع مسائل القيم الحدوديةBحل معادلة الاهتزاز الحر بطريقة

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Abstract :

In this paper were recruit B -spline method to Solve one types of ordinary differential equations Free Damped Vibration where a solution by B – spline method Further confirmation of the results by comparing with exact solution, the comparison also a way to resolve numerical approximations other generally accepted called finite Deference method . Proved that the way B–spline is the best in terms of error rate through compliance with the exact solution.

الخلاصة:

في هذا البحث تم حل احد أنواع المعادلات التفاضلية الاعتيادية وهي معادلة الاهتزاز الحر بوجود معامل التثبيط.علاوةً على تأكيد النتائج عن طريق المقارنة بالحل الدقيق، فقد تمت المقارنة أيضا بطريقه حل عدديه تقريبية أخرى متعارف عليها تدعى الفرو قات المحددة.وأثبتت إن طريقة الحل بـشريحة– B هي الأفضل من حيث نسبة الخطأ من خلال مطابقتها مع الحل الدقيق.

1. INTRODUCTION

Many of the mathematical models of physics and engineering problems are expressed in terms of partial differential equations. Two of the most popular techniques for solving partial differential equations are the Finite Difference Method and the Finite Element Method. In the Finite Difference Method, a solution is derived at a finite number of points by approximating the derivatives at each of them. The accuracy of this method is based on the refinement level of the grid points where the solution is being evaluated.

In recent years (2007–2012) and several authors have published their works on the application of B-spline Collocation Method in the solution of boundary value problems (BVP) in the form of differential equations. For example, some of the cases studied including the one-dimensional hyperbolic telegraph equation **[10, 12]**, damped wave equation **[12]**, one-dimensional heat equation**[6,8,11]**, the boundary value problems for a system of singularly perturbed second order ordinary differential equations**[9]**, and advection-diffusion equations **[12]**. The B-spline Collocation Method procedure is simple and easy to apply to many problems involving differential equations. This is an advantage over the Finite Element Method.

Boundary value problems for ordinary differential equations (ODE) arise in various fields of application such as thermodynamics, elasticity, classical and quantum mechanics. When numerical aspects are considered, the theory of spline functions has been very active in the field of approximation theory, boundary value problems and partial differential equations. As a piecewise polynomial, B-spline can become a fundamental tool for numerical methods to get the solution of the differential equations. This is because it allows the application of the basis functions of B-spline **[3, 1,2]** which can be accurately and efficiently computed.

In this paper, the case study is investigated where B-spline collocation scheme is applied. This is the numerical solution of spring oscillation equation (**Free Damped Vibration**).

The collocation method together with B-spline approximations only requires the evaluation of the unknown parameters at grid points. As known, the success of the B-spline collocation method is dependent on the choice of B-spline basis functions (***U***-vector) **[9],[13]**. Several experiments are used to build up the approximation solutions for some equations so that the best B-spline basis functions that give most accurate solution can be established.

Collocation method is a method for the numerical solution of ordinary differential equations, partial differential equations and integral equations. It is the basic method used in finite element techniques. It involves satisfying a differential equation to some tolerance at a finite number of points, called collocation points. The idea is to choose a finite-dimensional space of candidate solutions (in our case a B-spline basis) and a number of points in the domain (collocation points), and to select that solution which satisfies the given equation at the collocation points and at the given boundary conditions. The method of evaluating these points is explained in due course of the solutions.

The basic theory behind the method is briefly explained below:

Consider the equation $u''(t) = f(t, u, u')$. $a < t < b$, with the boundary conditions :

$u(a) = \alpha \qquad u(b) = \beta$

The approximate solution would be of the following format:

$$u(t) \sim v(t,B) = \sum_{i=1}^{n} B_i \, \varphi_i(t) \qquad (1.1)$$

where φ_i are basis functions defined on [a, b], and B_i's are parameters to be determine. The choice of φ_i's can vary depending on the problem. In our case φ_i's are the B-spline basis functions. **[13]**

2. B-spline Basis Function and Curve

The following equation is known as the Cox-de Boor recursion formula. **[3]**

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \qquad (*)$$

Where p is the degree of basis function

And knot vector U is defined as set of $m+1$ non-decreasing numbers $u_0 \le u_1 \le \ldots \le u_m$ .. These u_i's are called knots, and the half-open interval $[u_i, u_{i+1})$ is the i^{th} knot span.

The B-spline curve is defined as:

$$C(u) = \sum_{i=0}^{p-1} d_i N_{i,p}(u) \qquad (**)$$

the curve is determined by the control points $d_{l-n}, d_{l-n+1}, \ldots, d_l$, and the other control points d_i have no influence.

2.1 ONE DIMENSIONAL B-SPLINE COLLOCATION

The B-spline collocation method involves the determination of knot vectors and control points such that the differential equation, or boundary value problem, is satisfied to some tolerance at a finite set of collocation points. These collocation points reside on the Cartesian abscissa axis. In this paper, a nodal type, called the Greville Abscissa approach **[2-4]**, has been adopted and the collocation points are located at the Greville Abscissa points along the Cartesian abscissa axis. Other methods are also available in the literatures which will not be considered here.

Once the collocation method has been selected, the solution is achieved by following these procedure steps:

1. Select the appropriate knot vector
2. Calculate all required basis functions, by equation (*)
3. Calculate abscissa coordinates for the required control points, using the Greville Abscissa equation, by (2.1)
   $x_i = (t_i + t_{i+1} + \ldots + t_{i+n-1})/n$ For: $i=0, 1,\ldots,m-n$ (2.1)
   where *'n'* is the degree of the basis functions, or ***p*** and '***m***' is the total number of knots in the knot vector.
4. Calculate the B-spline curve equations and the required B-spline curve derivatives.
5. Use the boundary conditions to solve for end ordinate values of control points.
6. Substitute B-spline curves and the derivatives into the differential equation or boundary value problem.
7. Calculate the remaining interior ordinates of the control points by evaluating the differential equation at the corresponding abscissa of the collocation points.

In order to quantify the approximation results, the error has been calculated at each collocation point using the relative error formula:

$err\% = [1-|Bspline/Exact|]*100$ (2.2)

The Root Mean Square errors is a technique used in statistics for tolerance analysis:

$$RMS = \{(1/N) \sum_{i=1}^{N} (err\%)_i^2\}^{1/2} \quad (2.3)$$

A reasonable baseline value for a successful approximation has been set at RMS ≤ 0.005.

Free Damped Vibration
As Example:

A 16 lb object stretches a spring (8/9) ft by itself. There is no damping and no external forces acting on the system. Attached, a damper to the spring that exerts a force (F_d) of 5 lbs when the velocity is 2 lb/sec. Find the displacement at any time ***t, P(t).*** (See the diagram below) **[7]**.
The mass (***m*** = 1/2) and spring constant (***k*** = 18).
The damping coefficient (γ) is derived from :
$F_d = \gamma P'$ gives $\gamma = 5/2 = 2.5$

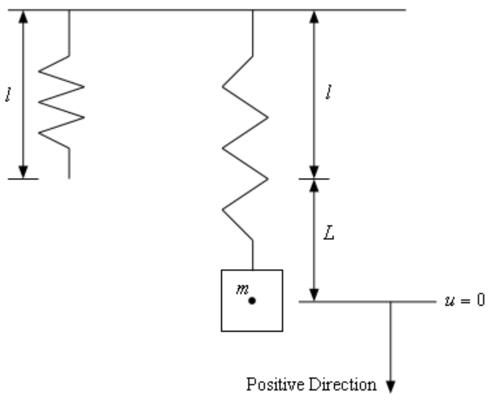


Figure 1 Spring Oscillation

The *BVP* for this example is then,

$\frac{1}{2} P'' + (5/2) P' + 18 P = 0$ for $0 < t < 1$ (2.4)

With the boundary conditions: $P(0) = 0$ and $P(1) = 1$.

The exact solution is computed using a Matlab program solver of second order ordinary differential equation with the specified boundary conditions:

dsolve (' D2y + 5*Dy +36* y = 0 ',' y(0)=y0 ',' y(1)=y1 ') (2.5)

Which gives the following exact solution:

$$y = 1/\sin(119^{1/2}/2)/(\cosh(5/2)-\sinh(5/2)).*\exp(5t/2).*\sin(119^{1/2}\, t/2) \quad (2.6)$$

Computation time =3.38 seconds.

1- Solution A

The first step is to select the appropriate knot vector that provides the resolution required for the solution. Since the ODE is of second order, the least U-vector to choose is of degree 2.

U = [0 0 0 1 1 1].

The required basis functions are:

$N_{0,2}(t) = (1-t)^2 \qquad N_{1,2}(t) = 2t(1-t)^2 \qquad N_{2,2} = t^2 \qquad (2.7)$

The basis functions for both intervals are shown in Figure 2. Below:

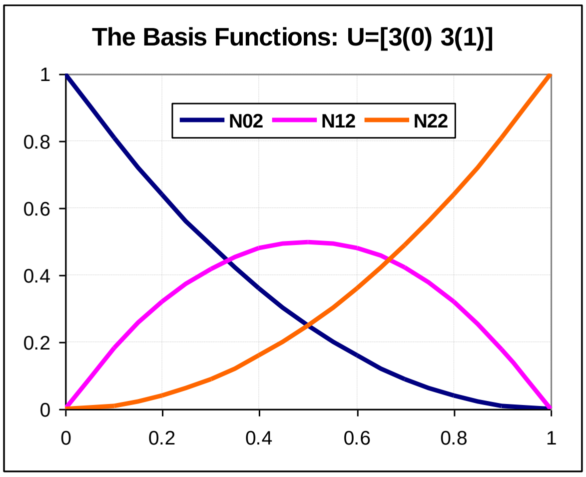


Figure2. Basis function: U= [0(3) 1(3)]

The abscissa coordinates for the control points are: (0 ½ 1)

The B-spline curve and its first and second derivatives are given by (as shown in chapter 3):

$$P(t) = \sum_{i=0}^{p} B_i N_{i.p}(t) \qquad (2.8)$$

$$P'(t) = \sum_{i=0}^{p} B_i N'_{i.p}(t) \qquad (2.9)$$

$$P''(t) = \sum_{i=0}^{p} B_i N''_{i.p}(t) \qquad (2.10)$$

$P(t) = B_0 N_{0,2}(t) + B_1 N_{1,2}(t) + B_2 N_{2,2}(t)$

$P(t) = B_0[(1-t)^2] + B_1[2t(1-t)^2] + B_2[t^2]$

$P'(t) = B_0[-2(1-t)] + B_1[(2-4t)] + B_2[2t]$

$P''(t) = B_0[2] + B_1[-4] + B_2[2]$

By applying the boundary conditions to equation P(t):

$P(0) = 0$ gives $B_0 = 0 = y_1$

$P(1) = 1$ gives $B_2 = 1 = y_3$

Substitute the values of B_0 and B_2 in the differential equation and solve at internal ordinate coordinate x_2=1/2

$(½)(B_0[2] + B_1[-4] + B_2[2]\} + (5/2)\{B_0[-2(1-t)] + B_1[(2-4t)] B_2[2t]\} + 18\{B_0[(1-t)^2] + B_1[2t(1-t)^2] + B_2[t^2]\} = 0$

The remaining internal ordinate coordinate is then calculated to be:

$B_1 = y_2 = -1.5$

Now that all of the coordinates for the control points have been calculated, they can be substituted into the B-spline curve equation:

P(t) = $[-3t(1-t)^2] + [t^2]$
Plot results along with the exact solution and compute the RMS

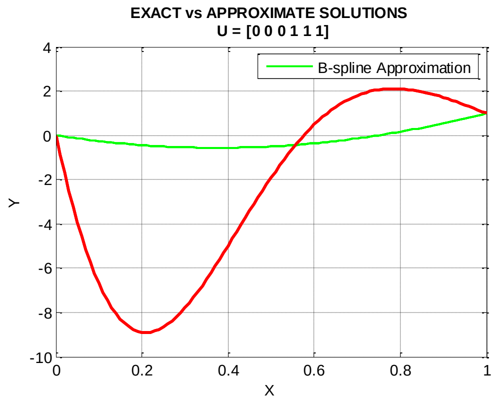


Figure 3. Exact vs. approximate solution at U= [0(3) 1(3)]

Collocation Points Evaluation				
y	X	Approx	Exact	%RelErr
0.0	0.0	0.0	0.0	0.0
-1.5	0.5	-0.5	-1.91	73.78
1.00	1.00	1.00	1.00	0.00

RMS	Time
0.426	3.53 sec

By plotting the two functions for the entire range [0, 1], as seen in Figure 3. The areas where refinement is necessary can be determined. One method of refinement is to increase the order of the B-spline Basis Functions. Another method is to add a collocation point within the range where the highest error percentage is. By a visual inspection of Figure 2. , it can be seen that the difference between the two functions is within the whole range of $0 < x < 1$. Since the RMS is very high, in the next solution (B) it shows what will happen when one additional collocation points is added, and at the same time the degree of the basis functions increased to 3. In order to add one collocation points, one intermediate point is added to the knot vector.

2- *Solution B*

The knot vector ***U*** = [0 0 0 0 ½ 1 1 1 1].
By following the same procedure in the above example, the function P(t) is computed for all ranges The basis functions for both intervals are shown in Figure 4. Below:

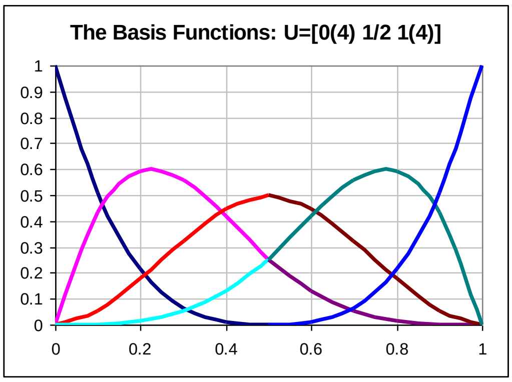


Figure 4. Basis function: U = [0(3) ½ 1(3)]

The remaining internal ordinate coordinate is then calculated by solving the three simultaneous linear equations. The final result is
$B_1 = y_2 = -12.7344$, $B_2 = y_3 = 2.7344$, $B_3 = y_4 = 3.5156$
Plot results Figure 5. along with the exact solution and compute the RMS

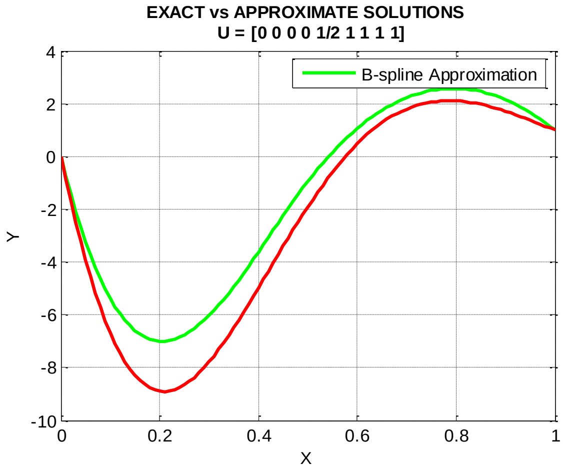


Figure 5. Exact vs. approximate solution at ***U***= [0(3) ½ 1(3)]

Collocation Points Evaluation				
y	X	Approx	Exact	%RelErr
0	0.00	0.00	0.00	0.00
-12.734	0.17	-6.81	-8.59	20.82
2.7344	0.5	-0.94	-1.91	50.83
3.5156	0.83	2.52	2.03	24.12
1.00	1.00	1.00	1.00	0.00

RMS	Time
0.268	4.55 sec

The RMS is still very high and not acceptable. By plotting the two functions for the entire range [0,1], as seen in Figure 5.the areas where refinement is necessary is still not confined to a specific range.

In the next solution (***C***) one more additional collocation points are added, and at the same time the degree of the basis functions kept at 3. In order to add two collocation points, two more intermediate points are added to the knot vector.

3- *Solution C*

The knot vector ***U*** = [0 0 0 0 1/3 2/3 1 1 1 1].

By following the same procedure in the above examples, the function P(t) is computed for all ranges.

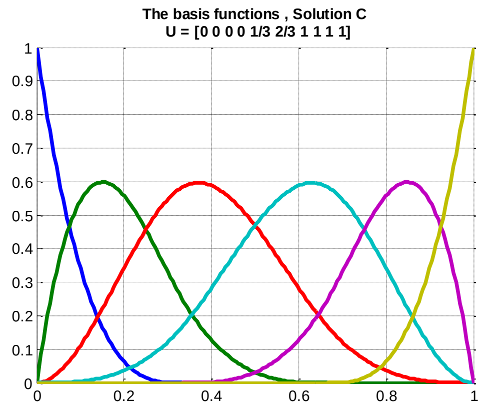


Figure 6. The basis functions

Plot results Figure 7. along with the exact solution and compute the RMS

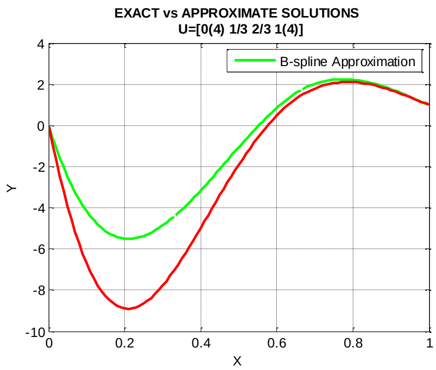


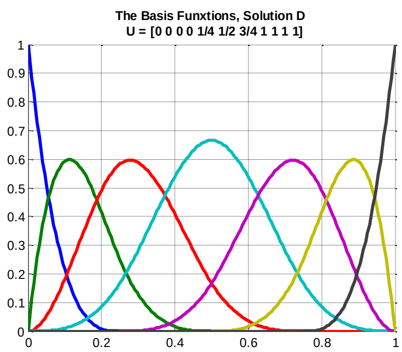
Figure7. Exact vs approximate solution at
U= [0(4) 1/3 2/3 1(4)]

Collocation Points Evaluation						
y	X	Approx	Exact	%RelErr		
0	0.00	0.00	0.00	0.00		
-6.5302	0.11	-4.43	-7.13	37.82		
-5.7957	0.33	-4.37	-6.96	37.21		
3.8447	0.67	1.75	1.48	17.98		
1.8860	0.89	1.84	1.77	3.47	RMS	Time
1.00	1.00	1.00	1.00	0.00	0.229	5.02 sec

The RMS is still very high and not acceptable. By plotting the two functions for the entire range [0,1] , as seen in Figure 7 , the areas where largest error is within the range of 0 < x < 0.6 .
It can be concluded that inserting another internal point improved the approximation at the end of the range (i.e t > 0.5). Therefore, inserting more internal points would bring the solution closer to exact solution, yet is not improving on the whole range. The next example confirms this argument.
In the next solution (***D***) three collocation points are inserted, hence three intermediate points are added to the knot vector.

4- *Solution D*

The knot vector ***U*** = [0 0 0 0 1/4 1/2 3/4 1 1 1 1].
By following the same procedure in the above examples, the function P(t) is computed for all ranges.
The results along with the exact solution are shown in Figure 9.



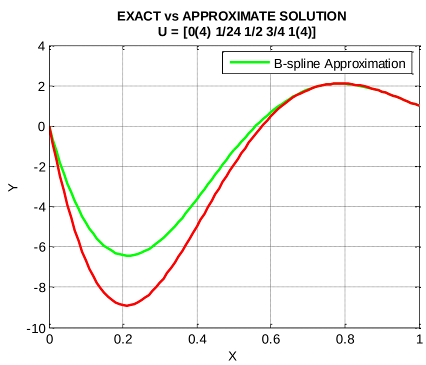


Figure 8. Basis function: Figure 9. Exact vs approximate
U = [0(4) ¼ ½ 3/4 1(4)]

Collocation Points Evaluation				
Y	X	Approx	Exact	%RelErr
0	0.00	0.00	0.00	0.00
-5.5621	0.08	-4.24	-5.89	28.02
-8.2567	0.25	-6.29	-8.66	27.37
-0.4837	0.50	-1.20	-1.91	36.92
2.9754	0.75	2.06	2.06	0.15
1.6086	0.92	1.60	1.60	0.34
1.00	1.00	1.00	1.00	0.00

RMS	Time
0.2034	5.72 sec

From the results of the above experiments, it can be concluded that the best improvement occurred when the ***degree*** of the basis function increased from ***two*** to ***three***. Inserting more points had less impact on the approximation by comparison. Therefore, increasing the degree further is expected to have a vast improvement on the solution. This is shown in the next solution (***E***) where the degree is ***n*** = 4.

5- *Solution E*

The knot vector U = [0 0 0 0 0 1/4 1/2 3/4 1 1 1 1 1].

By following the same procedure in the above examples, the function ***P(t)*** is computed for all ranges. The basis functions are shown in Figure 10.

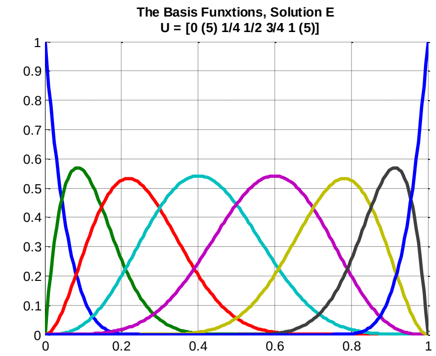


Figure 10. Basis function: ***U*** = [0(5) ¼ ½ 3/4 1(5)]

The solution results along with the exact solution are shown in Figure 11.

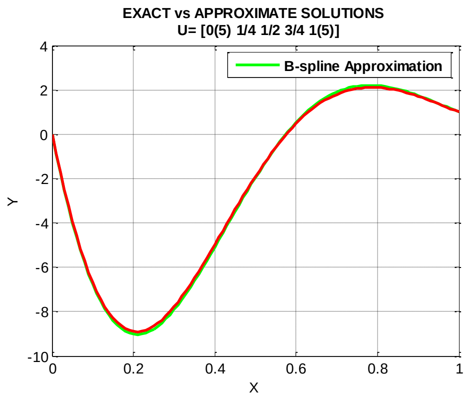


Figure11. Exact vs. approximate solution at ***U*** = [0(5) ¼ ½ 3/4 1(5)

Collocation Points Evaluation						
Y	X	Approx	Exact	%RelErr		
0	0.00	0.00	0.00	0.00		
-5.7500	0.06	-4.78881	-4.72614	1.33E+00		
-12.096	0.19	-8.95165	-8.82715	1.41E+00		
-6.1971	0.38	-5.8736	-5.75667	2.03E+00		
2.9912	0.63	0.99011	0.91526	8.18E+00		
2.5008	0.81	2.16299	2.08297	3.84E+00		
1.4875	0.94	1.48224	1.46016	1.51E+00	RMS	Time
1.00	1.00	1.00	1.00	0.00	0.0338	8.8 sec

The results of this experiment confirmed our expectations. The results can further improved by having yet higher B-spline basis degree. One last experiment will certainly be worthwhile in order to improve on the accuracy. The next solution will be of degree *5*.

6- *Solution F*

The knot vector ***U*** = [0 (6) 1/4 1/2 3/4 1 (6)].

By following the same procedure in the above examples, the function ***P***(***t***) is computed for all ranges. The basis functions are shown in Figure 12.

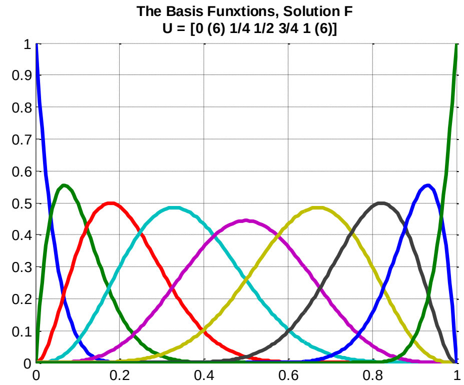


Figure 12. Basis function: ***U*** = [0(6) ¼ ½ 3/4 1(6)]

The results along with the exact solution are shown in Figure 13.

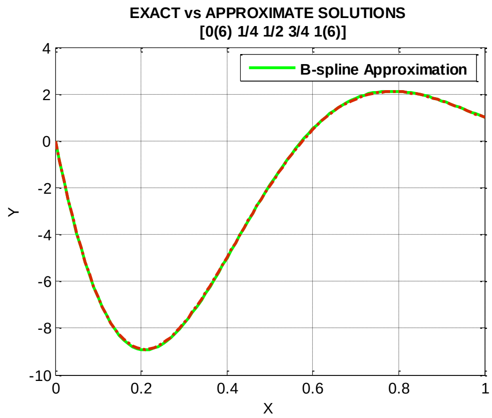


Figure13. Solution F: Exact vs approximate solution at
U= [0(6) ¼ ½ 3/4 1(6)]

Collocation Points Evaluation						
Y	X	Approx	Exact	%RelErr		
0	0.00	0.00	0.00	0.00		
-4.5162	0.05	-3.94	-3.93	0.24		
-10.739	0.15	-8.31	-8.29	0.23		
-9.9748	0.30	-7.82	-7.79	0.33		
0.1938	0.50	-1.90	-1.91	0.56		
2.9441	0.70	1.82	1.80	1.40		
2.1422	0.85	1.98	1.97	0.60		
1.3788	0.95	1.37	1.37	0.26	RMS	Time
1.00	1.00	1.00	1.00	0.00	0.0057	18.25 sec

In the next experiment, one internal point is removed and the degree increased another notch to ***n*** = 6**.** The computation time is also monitored.

7- *Solution G*

The knot vector ***U*** = [0 (7) 1/3 2/3 1 (7)].

By following the same procedure in the above examples, the function P(t) is computed for all ranges**.** The basis functions are shown in Figure 14.

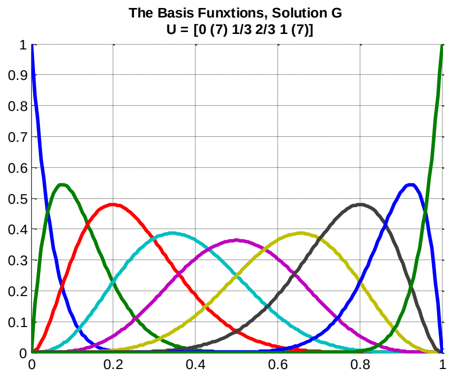


Figure 14. Basis function: ***U*** = [0(7) 1/3 2/3 1(7)]

The results along with the exact solution are shown in Figure 15.

Collocation Points Evaluation						
Y	X	Approx	Exact	%RelErr		
0	0.00	0.00	0.00	0.00		
-4.9783	0.06	-4.27	-4.29	0.46		
-11.673	0.17	-8.56	-8.60	0.46		
-9.5860	0.33	-6.94	-6.96	0.41		
1.1118	0.50	-1.90	-1.91	0.39		
2.9390	0.67	1.47	1.48	0.60		
2.2887	0.83	2.02	2.03	0.34		
1.4075	0.94	1.41	1.41	0.17	RMS	Time
1.00	1.00	1.00	1.00	0.00	0.0037	27.86 sec

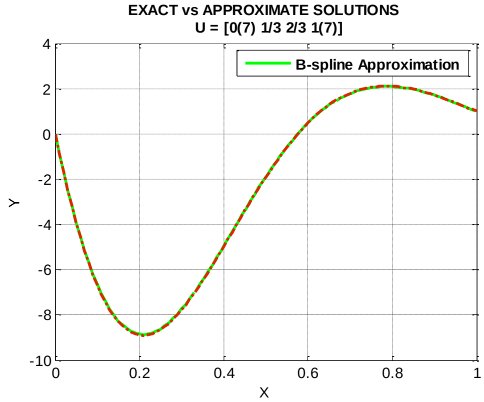


Figure 15. Exact vs approximate solution at
U= [0(7) 1/3 2/3 1(7)]

The conclusion here is that the computation time is higher but the errors are less. A trade off for higher accuracy is worth accepting.

In the next experiment, two internal points are removed and the degree increased another notch. This will complete the series of experiments.

8- *Solution H*

The knot vector ***U*** = [0 (8) 1/2 1 (8)].

By following the same procedure in the above examples, the function P(t) is computed for all ranges. The basis functions are shown in Figure 16.

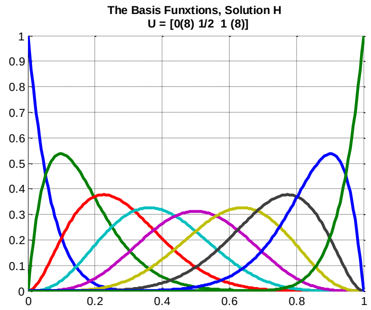


Figure 16.Basis function: ***U*** = [0(8) ½ 1(8)]

The results along with the exact solution are shown in Figure 17.

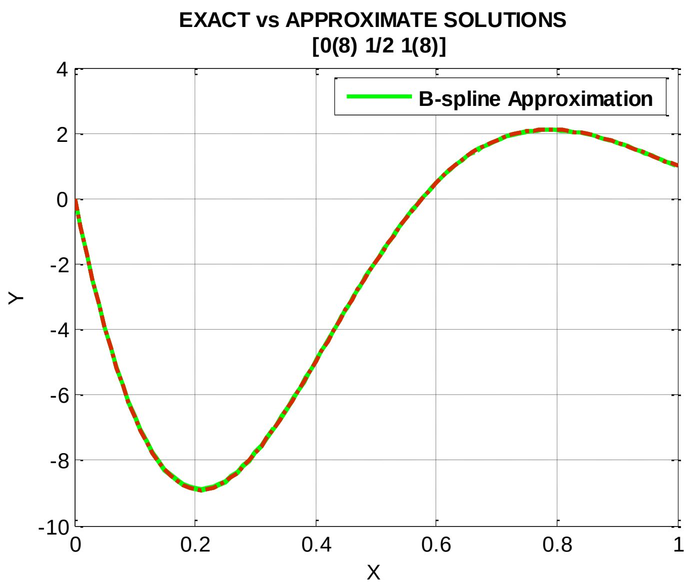


Figure17. Exact vs approximate solution at ***U*** = [0(8) ½ 1(8)]

Collocation Points Evaluation				
Y	X	Approx	Exact	%RelErr
0	0.00	0.00	0.00	0.00
-6.4259	0.07	-5.25	-5.25	0.05
-14.055	0.21	-8.91	-8.90	0.06
-8.0347	0.36	-6.29	-6.29	0.04
1.3504	0.50	-1.91	-1.91	0.00
2.9764	0.64	1.18	1.18	0.09
2.6800	0.79	2.11	2.11	0.08
1.5232	0.93	1.52	1.52	0.09
1.00	1.00	1.00	1.00	0.00

RMS	Time
0.00058	44.9 sec

This last experiment concludes that a higher degree B-spline basis functions gives best approximation and longer computation time.

3. VERIFICATION OF THE B-SPLINE COLLOCATION METHOD

In this section, the finite difference method (FDM) is chosen to solve the vibration problems. The purpose of this exercise is to cross correlate the solutions of the B-spline method with another numerical method. The FDM is commonly used to solve ordinary differential equations that have conditions imposed on the boundary rather than at the initial point (BVP) **[5]**. The equations take the following form.

$y''(x) = f(x,y,y')$

$y'(x) = f(x,y) \qquad a \le x \le b \qquad (3.1)$

with boundary conditions

$y(a) = y_a$ and $y(b)=y_b$

Approximating the derivatives dy/dx and d^2y/dx^2 at a node by the central divided difference approximation, Figure 24.gives.

$d^2y/dx^2|_i \sim (y_{i+1} - 2y_i + y_{i-1})/(\Delta x)^2 \qquad (3.2)$

$dy/dx|_i \sim (y_{i+1} - y_i)/(\Delta x) \qquad (3.3)$

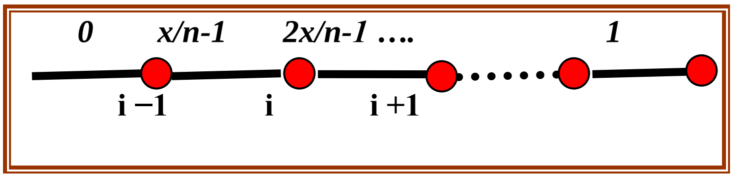


Figure17. Illustration of finite difference nodes method.
Using central divided difference method

3.1 Free Damped Vibration

As Exemple:

*('D2y + 5*Dy +36* y = 0','y(0)=y0','y(1)=y1')*

Similar to the procedure used in the B-spline method, the solution results are also compared with the exact solution in order to refine its accuracy.

We can rewrite the differential equation as:

$(y_{i+1} - 2y_i + y_{i-1})/(\Delta x)^2 + 5\,(y_{i+1} - y_i)/(\Delta x) + 36\,y_i = 0 \qquad (3.4)$

Solution 1: four nodes (***n***=4)

For ***n=4***

$x_0 = 0, \; x_1 = x/n\text{-}1 = 1/3, \qquad x_2 = 2/3, \; x_3=1$

From the boundary condition at x = 0 , we obtain

Node 1: At x =0, $y_1 = 0$

Node 4: At x =1, $y_4 = 1$

For the rest of the nodes, the following equation applies :

$$24\, y_{i+1} + 3y_i + 9y_{i-1} = 0 \qquad (3.5)$$

And the solution can be found by the following constructed matrix of linear equations:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 9 & 3 & 24 & 0 \\ 0 & 9 & 3 & 24 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (3.6)$$

which gives the values of y_i at each node.The results are depicted in Figure 25. The ***RMS*** = 0.4864

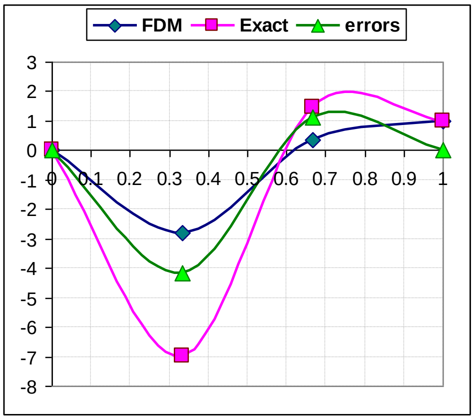


Figure 25. FDM solution at ***n***=4, Free Damped Vibration

In order to improve on the accuracy of the solution, higher number of nodes is needed.

Solution 2: ten nodes (***n***=10)

$x_i = x/n\text{-}i$

From the boundary condition at x = 0 , we obtain

Node 1: At $x = 0$, $y_1 = 0$

Node 10: At $x = 1$, $y_{10} = 1$

For the rest of the nodes, the following equation applies:

$$81\, y_{i-1} - 171 y_i + 126 y_{i+1} = 0 \qquad (3.7)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & . & . & . & \\ 81 & -171 & 126 & 0 & 0 & . & . & . \\ 0 & 81 & -171 & 126 & 0 & 0 & . & . \\ 0 & 0 & 81 & -171 & 126 & 0 & . & . \\ 0 & 0 & . & . & . & . & . & . \\ 0 & 0 & . & . & . & . & . & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_{10} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ 1 \end{bmatrix} \qquad (3.8)$$

Solving the set of equations (3.6), gives the solution at each node. The results are depicted in Figure 26. The RMS = 0.4731

From the above two solutions, it can be concluded that in order to achieve an acceptable results, a very large number of nodes is necessary. For this purpose the computation procedure was programmed, using Matlab, and used for the solution.

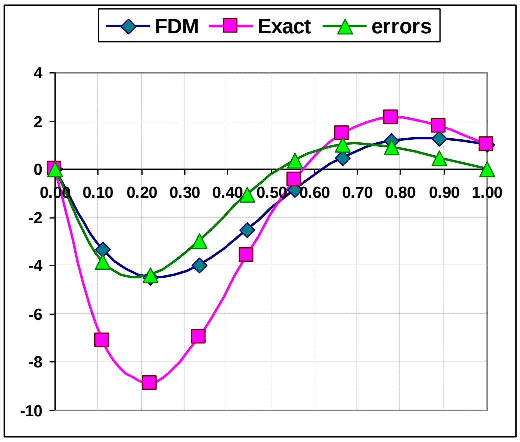


Figure 26. FDM solution at ***n***=10, Free Damped Vibration

1.5 Solutions with a large number of nodes:

Some of the results are depicted in graphical format as shown in Figures 27. A table of the RMS errors corresponding to solutions with the specified number of nodes is also given.

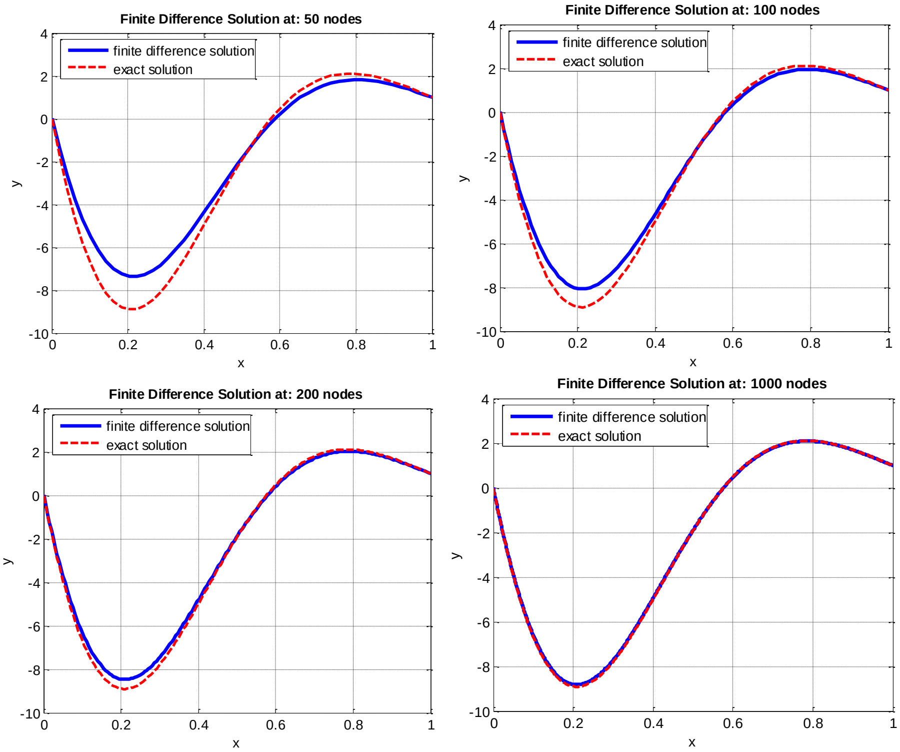


Figure 27. FDM solution at larger ***n***, Free Damped Vibration

Number of Nodes	4	10	12	21	50	100	200	500	1000
RMS	0.4864	0.4731	0.407	0.3559	0.3292	0.2043	0.1352	0.0825	0.0591
Time sec							4.48	5.17	(13sec)

CONCLUSIONS

1. B-spline can become a fundamental tool for numerical methods to get the solution of differential equations. The solution can be accurately and efficiently computed.
2. When solving BVP, the refinement of the B-spline approximation is best done at a higher degree. (As fig 16.)

3. The most efficient way of approximation is to find the right balance between the approximation order, number of internal knot points, i.e. the U-vector structure, and computation time.
4. The B-spline method was further verified by comparing it with the numerical Finite Difference method. The targeted RMS of 0.005 was not easily achievable by FDM despite the large number of nodes used (up to 1000 nodes) in the case of free damped vibration problem. Another proof to the superiority of the B-spline method. (As fig 27.)

The B-spline Collocation Method has a few distinct advantages over the Finite Difference Methods. B-spline provides a piecewise-continuous, closed form solution. Its accuracy can be improved by simply inserting extra knot exactly where refinement is needed

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