Numerical Modeling of Wire and Tube Condenser Used in Domestic Refrigerators

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Abstract

This paper presents modeling of wire and tube condensers that are commonly used in a vapor compression cycle based domestic refrigeration. A simulation model was developed by using the finite element with a combination of thermodynamic correlations. The application of this model used in the design of condenser in vapor compression cycle with new refrigerant.

Keywords: Condenser; Refrigerator; Alternative refrigerant

الخلاصة

تم في هذا البحث نمذجة المكثف السلكي المستخدم عادة من منظومات التنقيح الانظغاطية المنزلية. تم بناء الأنموذج الحسابي بالاعتماد على طريقة العناصر المحددة، استخدام هذا الأنموذج في تصميم المكثفات المستخدمة في منظومات التنقيح الانظغاطية المنزلية عند استخدام المواد البديلة.

1. Introduction
One of the commonly used condensers in domestic refrigerators is the wire-and-tube condenser. The wire-and-tube condenser is usually a natural convection heat exchanger. It consists of a single copper or steel tube and solid steel wires that serve as extended surfaces. The tube which carries the refrigerant, is bent into a single-passage serpentine shape with wires symmetrically spot-welded to both sides in a direction normal to the tubes as shown in Fig. 1a. It had been used and widely studied since 1957. Witzell and Fantaine [1] studied condenser and get correction for condenser, and concluded that any additional of wire on the external surface of condenser did not increase in the rate of heat transfer from condenser. Collicott et al. [2] works experimental study to calculate heat transfer factor by free convection and shape factor for wire-and-tube condenser. They find that the diameter of tube or wire proportional to space between the tube or wire. Ellison et al. [3] developed a computer model for an air-cooled condenser, the model relies on a tube-by-tube computational approach calculating the thermal and fluid flow performance of each tube in the heat exchanger individually, using local temperatures and heat transfer coefficients. Hoke et al. [4] and Lee and Lee [5] developed the air side empirical heat transfer correlations for forced-convection for wire-and-tube condensers, while Tagliafico and Tanda [6] for natural-convection type condensers. Hussein [7] devised a computational model for a vapour compression cycle of simple refrigeration system that could yield accurate prediction using (CFC) refrigerant and hydrocarbon (HC) refrigerants. However, the aim of this works is to present the development and application of the design and optimization of a wire-and-tube condenser in the domestic refrigerators. The model is developed by using a combination of finite element along with thermodynamic correlations. Thermodynamic and transport properties of refrigerants used from ASHRAE [8].

Fig.1. (a) Schematics of a wire-and-tube condenser. (b) Elemental unit of the condenser, and (c) parameters of the wire-and-tube condenser.

2. Modeling of the Condensers
2-1 Heat Transfer Analysis

The condenser model is developed by using finite element shown in (fig. 2) where the rate of heat transfer from an element of tube length, \( \Delta z \) can be expressed as:

\[
Q_{\text{ele}} = U_A \Delta \theta \quad \text{..........................(1)}
\]

Heat is transferred by convection from the condensing fluid inside the tube to the tube wall. The heat is then transferred by conduction through the tube wall to the outside tube surface. Heat flow to the surrounding air takes place by several processes, natural convection from the exposed tube wall, and convection from the tube wall. Heat transferred by Radiation from both tube surfaces and wire surfaces. The wire and tube conductance (UAele) applied to each element is expressed as:

\[
\frac{1}{U_A} = R_t + R_w + \frac{1}{R_{oc}} + \frac{1}{R_{or}} = \left( \frac{1}{h_A} + \ln \left( \frac{r_i}{r_o} \right) \right) + \frac{1}{2 \pi K \Delta z} \left( A_s (h_{oc} + h_{or}) \right) \quad \text{..........................(2)}
\]

The elemental tube length is equal to the pitch of wire, \( \Delta z = p_w \) (see figs. 1a and 2) and the elemental outer area of heat transfer, \( A_o \) is given as:

\[
A_o = A_t + A_w = \pi d_{t,o} p_w + 2\pi d_w p_t \quad \text{..........................(3)}
\]

Fig. 2. Finite element model

The design parameters for the current condenser are given in (table 1) [7]

<table>
<thead>
<tr>
<th>Tube material</th>
<th>Steel</th>
</tr>
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</table>

Table 1. Design parameters and geometrical data of current wire-and-tube condenser
2-2 Forced convection heat transfer inside tube

Heat transfer process inside the condenser tube can be divided into three main phases, which are:

a- Desuperheating
b- Two-phase forced convection with condensation
c- Sub-cooling

a- Desuper Heating

Desuperheating zone occupies a relatively small portion of the condenser, due to the large temperature differential between the superheated refrigerant vapour and the ambient air. In this zone refrigerant vapour is cooled to refrigerant condensing temperature corresponding to condenser pressure. The non-dimensional heat transfer parameter describing these phenomena, Nusselt number, is related to the non-dimensional Reynolds and Prandtl numbers in the following form [9]:

\[ Nu = 0.023 \, Re^{0.8} \, Pr^{0.3} \]

Where

\[ Nu = \frac{h_i \, d_{t,i}}{k} \quad ; \quad Re = \frac{G \, d_{t,i}}{\mu} \quad ; \quad Pr = \frac{\mu \, c_p}{k} \]

b- Two-Phase Forced Convection with Condensation

Condensing takes place in predominantly most of the condenser length at a substantially constant temperature. At the end of the condensing region the refrigerant is mainly in the saturated liquid phase. The predominant flow pattern during condensation in refrigerator condenser is the annular flow with liquid refrigerant flowing on the pipe wall and vapor refrigerant flowing in the core [3]. Traviss et al. [10] proposed correlation to calculate the local heat transfer coefficient as a function of quality is:

Where
\[ \text{Re}_i = \frac{G(1-x)d_{ij}}{\mu_i} \quad ; \quad \text{Pr}_i = \frac{\mu_i c_p i}{k_i} \] .................................(5)

\( b = 1 \quad \text{for} \quad F_i \leq 1, b = 1.15 \quad \text{for} \quad F_i > 1 \)

F1 and F2 in equation (5) are dimensionless parameters expressed as follows:

\[ F_1 = 0.15 \left( \frac{1}{X_n} + \frac{2.85}{X_n^{0.478}} \right) \] ...........................(6)

\[ F_2 = 0.707 \quad \text{Pr}_i \quad \text{Re}_i^{0.5} \quad \text{for} \quad \text{Re}_i < 50 \] ............ (7)

\[ F_2 = 5 \quad \text{Pr}_i + 5 \ln \left[ 1 + \text{Pr}_i \left( 0.09636 \quad \text{Re}_i^{0.585} - 1 \right) \right] \quad \text{for} \quad 50 < \text{Re}_i < 1125 \] ........................(8)

\[ F_2 = 5 \quad \text{Pr}_i + 5 \ln \left( 1 + 5 \text{Pr}_i \right) + 2.5 \ln \left( 0.00313 \quad \text{Re}_i^{0.812} \right) \quad \text{for} \quad \text{Re}_i > 1125 \] ........................(9)

Parameter \( X_n \), formulated by Lockhart – Martinelli \[^{11} \] has the following form:

\[ X_n = \left( \frac{1-x}{x} \right)^{0.9} \left( \frac{V_i}{V_g} \right)^{0.5} \left( \frac{\mu_i}{\mu_g} \right)^{0.1} \] ........................(10)

Equation (5) is applicable where conditions for annular condensation in a tube exist. Such condition may be assumed to exist for flow qualities ranging for \((0.1 \text{ to } 0.9)\) \[^{12} \]. At qualities larger than \((0.9)\) the whole tube inner surface is not covered by a liquid film and part of heat transfer is just that of single-phase convection. At qualities less than \((0.1)\), the flow was observed to be slug flow regime \[^{12} \]. It is assumed that in the quality range \((0.0 \text{ to } 0.1)\) and \((0.9 \text{ to } 1.0)\), the heat transfer coefficient changes linearly from a two-phase flow value to a single phase flow value and is calculated using linear interpolation between values obtained from equation (4) and (5) \[^{12} \].

**c-Sub-cooling**

Sub-cooling mainly depends on the temperature differential between the condensing temperature and the ambient temperature. Heat transfer coefficient is calculated using equation (4).

**2-3 Heat transfer from outside surfaces**
There are two modes of heat transfer from outside surfaces:

a. Natural convection heat transfer.
b. Radiation heat transfer.

**a- Natural convection heat transfer**

The computation of the convection heat transfer involves two sections of the heat exchanger, namely the (X-Y) and (Y-Z) sections see (fig. 1a) the convection heat transfer coefficients for the vertical section (X-Y) was computed using Mc Adam correlation\(^{[13]}\) given as:

\[
h_{oc} = 0.27 \left( \frac{\Delta T}{d_{t,o}} \right)^{0.25} \text{ .......... ..........} \quad \ldots (11)
\]

For the (Y-Z) section, which is the main frame of the wire-and-tube condenser, the convection heat transfer is assumed the same for the tube and wire was computed using Tagliafico and Tanda\(^{[6]}\) correlation expressed as:

\[
h_{oc} = \frac{Nu \times k_o}{H} \text{ .................}(12)
\]

Where

\[
Nu = 0.66 \left( \frac{Ra \times H}{d_{t,o}} \right)^{0.25} \times \left\{ 1 - \left[ 1 - 0.45 \left( \frac{d_{t,o}}{H} \right)^{0.25} \right] \times \exp \left( -\frac{s_w}{\psi} \right) \right\} \text{ ..........}(13)
\]

\[
Ra = \left( \frac{\beta \rho^2 cp}{\mu k} \right)_{\omega} \left( T_{t,o} - T_{\infty} \right) g H^3 \text{ .................}(14)
\]

\[
\psi = \left( \frac{28.2}{H} \right)^{0.4} s_t^{0.9} s_t^{-1} + \left( \frac{28.2}{H} \right)^{0.8} \left( \frac{264}{T_{t,o} - T_{\infty}} \right)^{0.5} s_t^{-1.5} s_t^{-0.5} \text{ ..........}(15)
\]

\[
s_t = \frac{(p_1 - d_{t,o})}{d_{t,o}} \text{; } s_w = \frac{(p_w - d_w)}{d_w} \text{ .........................}(16)
\]

\[
\beta = \frac{1}{T_{film}} \text{; } T_{film} = \frac{T_{t,o} + T_{\infty}}{2} \text{ .........................}(17)
\]

**(a) Radiation heat transfer**

The radiation heat transfer coefficient if given by\(^{[9]}\) as:
\[ h_{or} = e \sigma \frac{(T_{ex}^4 - T_{w}^4)}{(T_{ex}^4 - T_{\infty}^4)} \]  \hspace{1cm} \text{(18)}

Where

\( e \) is the average surface emissivity equal (0.95), and \( \sigma \) is the Stefan Boltzmann constant equal \((5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)\).

\( T_{ex} \) is the mean surface temperature of the heat exchanger. This can be expressed as a function of the surface temperature of the tube, \( T_{t,o} \), and the mean surface temperature of the wires, \( T_{w} \), as:

\[ T_{ex} = \frac{(A_t T_{t,o} + A_w T_{w})}{A_o} \]  \hspace{1cm} \text{(19)}

Where \( T_{t,o} \) is the outer tube temperature, initially is assumed to be \((0.5 \degree C)\) lower than the refrigerant temperature. This value is iterated after the elemental heat transfer has been computed, and \( T_{w} \) is the wire temperature can be found by definition of fin efficiency \([9]\) as:

\[ \eta_{w} = \frac{(T_{w} - T_{\infty})}{(T_{t,o} - T_{\infty})} \hspace{1cm} \text{or} \hspace{1cm} T_{w} = \eta_{w} (T_{t,o} - T_{\infty}) + T_{\infty} \]  \hspace{1cm} \text{(20)}

Fin efficiency of wire, \( \eta_{w} \) for cylindrical bar with one-dimensional heat flow can be expressed by the equation \([9]\).

\[ \eta_{w} = \frac{\tanh \left( \frac{m p_i}{2} \right)}{\left( \frac{m p_i}{2} \right)} \hspace{1cm} \text{where} \hspace{1cm} m = \sqrt{\frac{4 h_{w}}{k_{w} d_{w}}} \]  \hspace{1cm} \text{(21)}

Substituting eqs. (3) and (20) into eq. (6), then \( T_{ex} \) can be written as:

\[ T_{ex} = \frac{T_{t,o} + \frac{A_w}{A_t} (\eta_{w} (T_{t,o} - T_{\infty}) + T_{\infty})}{\left( 1 + \frac{A_w}{A_t} \right)} \]  \hspace{1cm} \text{(22)}

\[ \frac{A_w}{A_t} = 2 \left( \frac{p_i}{d_{t,o}} \right) \left( \frac{d_{w}}{p_w} \right) \]  \hspace{1cm} \text{(23)}

Now found the inner and outer heat transfer coefficients, the elemental heat transfer, \( Q_{el} \) was computed from Eqs. (1) and (2). As the heat transfer from the refrigerant to the tube must be
equal to the heat transfer from the tube to the ambient (in the steady state), the outer tube temperature, $T_{t,o}$ can be obtained from the following.

$$T_{t,o} = T_{ref} - Q_{ele}^* \left( \frac{1}{h_i A_i} + \frac{\ln \left( r_e / r_i \right)}{2 \pi k l} \right)_{ele} \quad \ldots \ldots \ldots (24)$$

This value is then compared against the initial value of $T_{t,o}$. If the error is larger than $0.05^\circ C$, a new $T_{t,o}$ is substituted into Eq. (22) and the computations are repeated until convergence is achieved. This yields the actual heat transfer. The enthalpy of the refrigerant at the outlet of an element can then be given by:

$$h_{o,ele} = Q_{ele}^* m^* + h_{t,ele} \quad \ldots \ldots (25)$$

**Pressure loss correlation:**

Total pressure drop for a fluid flowing inside tube can be expressed by equation.

$$\Delta P_{tot} = \Delta P_f + \Delta P_a + \Delta P_g \quad \ldots \ldots (26)$$

Pressure drop due to gravity effect is very small and may be neglected. Only pressure drop due to friction and due to momentum change will be considered with distinction between single and two-phase flow.

**Single-phase pressure drop:**

Frictional pressure drop for a vapour region in a tube can be calculated by fanning equation [12].

$$\Delta p_f = f \frac{G^2}{2 \rho_m} \frac{\Delta z}{d_{i,f}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (27)$$

$$f = 0.184 \Re^{-0.2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (28)$$

$$\frac{1}{\rho_m} = 0.5 \left( \frac{1}{\rho_i} + \frac{1}{\rho_o} \right) = \frac{(v_i + v_o)}{2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (29)$$

Pressure drop due to momentum change can be calculated by the following equation.

$$\Delta P_a = \frac{G^2}{2} \left( V_o - V_i \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (30)$$
Total pressure drop in return bend is expressed by \(^{(14)}\):

\[
\Delta P_{rb} = n K \frac{G^2}{2 \rho_m} \quad \text{..........(31)}
\]

Where, \(n\) = no. of turns, and \(K = 1.4\)

The expression of the liquid phase pressure drop in the normal incompressible flow relation \(^{(8)}\):

\[
\Delta P_f = f \frac{G^2}{2 \rho_i d_{t,i}} \quad \text{..........(32)}
\]

**Two-phase pressure drop:**

The two-phase frictional pressure drop generally constitutes the largest part of the total pressure drop. By using the definitions developed by Lochart & Martinelli \(^{(11)}\) as given below:

\[
\Delta P_f = \Delta P_l \phi_l^2 \quad \text{..........(33)}
\]

\[
\Delta P_l = f_t \frac{G_l^2 \Delta z}{2 \rho_l d_{t,i}} \quad \text{..........(34)}
\]

\[
G_l = G(1 - x)
\]

\[
f_t = 0.0316 \text{Re}_t^{-0.25}
\]

\[
\text{Re}_t = \frac{G_l d_{t,i}}{\mu_l}
\]

The liquid phase multiplier, \(\phi_l\) a correction factor for the frictional pressure drop of the single phase and can be defined as a function of the Martinelli parameter \(X_a\) \(^{(11)}\) as :

\[
\phi_l^2 = 1 + 8 X_a^{-1} + X_a^{-2} \quad \text{..........(35)}
\]

The two-phase momentum pressure drop can be used equation given from \(^{(15)}\) as:

\[
\Delta p_a = -\frac{G^2}{\rho_g} \left\{ \left[ 1 + \left( \frac{\rho_g}{\rho_l} \right)^{\frac{1}{3}} - \left( \frac{\rho_g}{\rho_l} \right)^{\frac{2}{3}} \right] \left[ x_o^2 - x_i^2 \right] \right. \\
\left. - \left[ 2 \left( \frac{\rho_g}{\rho_l} \right)^{\frac{1}{3}} - \left( \frac{\rho_g}{\rho_l} \right)^{\frac{2}{3}} \left( x_o - x_i \right) \right] \right\} \quad \text{..........(36)}
\]
The pressure drop in the return bend assuming constant quality through the bend is defined by Pierre [14] as:

\[ \Delta p_f = K_f \frac{G^2 v_m}{2} \] \hspace{1cm} (37)

\[ \Delta p_a = K_m \frac{G^2 v_m}{2} \] \hspace{1cm} (38)

\[ v_m = x v_g + (1 - x) v_l \] \hspace{1cm} (39)

\[ K_m = 0.9; K_f = 0.47 \]

Where \( x \): quality at bend inlet.

Then pressure drop for each element is:

\[ \Delta p_{ele} = p_{o,ele} - p_{i,ele} = \Delta p_f + \Delta p_a \] \hspace{1cm} (40)

The total pressure loss and heat transfer rate are simply the sum of all elemental pressure losses and heat transfer.

\[ \Delta p_{tot} = \sum \Delta p_{ele} \] \hspace{1cm} (41)

\[ Q_{tot} = \sum Q_{ele} \] \hspace{1cm} (42)

The computation process is repeated for subsequent elements where the inlet condition of the current element is equal to the outlet conditions of the previous element, namely:

\[ (T_i; p_i; \rho_i; q_{ref,i}; h_i)_i = (T_o; p_o; \rho_o; q_{ref,o}; h_o)_{i-1} \]

This yields the computation of the condenser capacity, pressure drop and other performance characteristics of the condenser.

### 3. Results and discussion:

The solution of heat transfer and pressure drop equation for every element along the condenser give rise to the distribution of all the studied variables such as quality, mass flow rate, refrigerant temperature, etc. Figures (1, 2, 3, 4) show the numerical result obtained from the model under operating conditions (\( T_{in} = 85 \) °C, \( P_{in} = 10 \) bar, \( T_{amb} = 35 \) °C). These figures show the effect of mass flow rate on the quality, refrigerant temperature, inside heat transfer coefficient and heat transfer for each element with length of tube, when increasing mass flow rate there is an increase in the quality at the end of condenser and increase value of inside heat transfer coefficient. From the above when increasing mass flow rate, degree of sub cooling decrease in the end of condenser tube and this is reduce capacity of it. Figures (5, 6) show the effect of condensing pressure on the inside heat transfer coefficient and heat transfer for each element with the length of tube. Figures (7, 8) show the effect of the refrigerant temperature at inlet of condenser on the inside heat transfer coefficient and heat transfer for each element with
the tube length. Figures (9, 10, 11) show the effect of the ambient temperature on the inside heat transfer coefficient, refrigerant temperature and heat transfer for each element with tube length.

Fig. (7)

Fig. (8)

Fig. (9)

Fig. (10)
4. Conclusions:

The modeling results show the effect of mass flow rate, pressure, refrigerant temperature and ambient temperature on the performance of condenser. The model can be used for the design the wire-and-tube condenser used in the refrigeration system work with alternative refrigerants.

Nomenclature

A area (m²)
cp constant pressure specific heat (J/kg K)
d diameter (m)
f friction factor (–)
g acceleration due to gravity (m/s)
G  flow rate per unit area or mass flux (kg/m²s)

h  heat transfer coefficient (W/m²K)

h  refrigerant enthalpy (kJ/kg)

H  heat exchanger height (m)

k  thermal conductivity (W/mK)

l  length (m)

m  fin property parameter (–)

m•  mass flow rate (kg/s)

Nu  Nusselt number (–)

p  pitch (m)

P  Pressure (kPa)

Pr  Prandtl number (–)

Q•  heat flow rate (W)

r  radius (m)

R  thermal resistance (K/W)

Ra  Rayleigh number (–)

Re  Reynolds number (–)

T  temperature (K)

U  over all heat transfer coefficient (W/m²K)

V  specific volume (m³/kg)

x  Refrigerant quality (–)

Xtt  Martinelli parameter (–)

Δz  Elemental length (m)

ΔP  pressure drop (pa)

ΔT  temperature difference (K)

Greek letters

π  pie, π = 3.1416

β  thermal expansion coefficient (1/K)

ε  thermal emittance (–)

η  efficiency (–)

μ  dynamic viscosity (kg/m s)

ρ  Density (kg/m³)

σ  Stefan–Bolzmann constant, σ = 5.67×10⁻⁸ (W/K^4 m²)

ϕ  liquid phase multiplier

Subscripts

a  air, acceleration

c  convective

ele  elemental

ex  heat exchanger

f  frictional

g  gravitational
5. References


