Fuzzy almost pairwise semi-pre continuous mappings and fuzzy almost semi-pre open (semi-pre closed) mappings

Mohammed Jassim Mohammed
Department of Mathematics, College of Education
University of Thi-Qar , thiqaruni.org

Abstract

The purpose of this paper to introduce and study the concepts of fuzzy semi-pre open sets in fuzzy bitoplogical spaces also we introduce and study the concepts of fuzzy almost pairwise semi-pre continuous mappings and fuzzy almost pairwise semi-pre open (semi-pre closed) mappings. Their properties have been investigated.

Introduction:

The notion of bitoplogical spaces was initially studied by Kelly [3], the concepts of fuzzy sets was introduced by Zadchin [12]/ chang [2] first introduced the fuzzy topological spaces. Kandil [4] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological space. Thakar [11] introduced and studied fuzzy semi-pre open sets and fuzzy semi-continuity in fuzzy topology.

Kumar [7,8] defined the (T_i,T_j)-fuzzy semiopen (semiclosed) sets, (T_i,T_j)-fuzzy preopen (preclosed) sets, also Kamar [7,8] defined the fuzzy semi-continuous mapping, fuzzy pairwise pre-continuous mappings.

Krsteska [6] introduce and studied (T_i,T_j)-strongly semiopen sets and almost pairwise strongly semi-continuous mappings.

In this paper we introduced and studied semi-pre open sets of fuzzy bitopological space and also we introduce the fuzzy almost pairwise semi-pre continuous mappings, fuzzy almost pairwise semi-pre open (semi-pre closed) mappings.

1. Preliminaries

Through this paper, fbts X denotes a fuzzy bitopological space (X,τ_1,τ_2) and the indices i,j takes value {1,2} and $i \neq j$, For a fuzzy set A of a fbts (X,τ_1,τ_2), τ_i-intA and τ_j-clA means, respectirely, the interior and closure of A with respect to fuzzy topologies τi and τ_j.

Definition (1-1)[6,7,8]:

Let A be a fuzzy set of fbts X. Then A is called:
1) a (τ_i, τ_j)-fuzzy semiopen set if and only if $A \leq \tau_i\text{-cl}(\tau_j\text{-int}A)$.
2) a (τ_i, τ_j)-fuzzy pre-open set if and only if $A \leq \tau_i\text{-int}(\tau_j\text{-cl}A)$.
3) a (τ_i, τ_j)-fuzzy α-open set if and only if $A \leq \tau_i\text{-int}(\tau_j\text{-cl}(\tau_i\text{-int}A))$.
4) a (τ_i, τ_j)-fuzzy semi-pre open set if and only if $A \leq \tau_i\text{-cl}(\tau_j\text{-int}(\tau_i\text{-cl}A))$.
5) a (τ_i, τ_j)-fuzzy regular open set if and only if $A=\tau_i\text{-int}(\tau_j\text{-cl}A)$.
The family of all (τ_i, τ_j)-fuzzy semiopen sets, (τ_i, τ_j)-fuzzy pre-open sets, (τ_i, τ_j)-fuzzy α-open sets, (τ_i, τ_j)-fuzzy semi-pre open sets and (τ_i, τ_j)-fuzzy regular open sets of a fbts (X,τ_1,τ_2) will be denoted by (τ_i, τ_j)-FSO(X), (τ_i, τ_j)-FPO(X), (τ_i, τ_j)-FαO(X), (τ_i, τ_j)-FSPO(X) and (τ_i, τ_j)-FRO(X) respectively.

Definition (1-2) [7,12]:

Let A be a fuzzy set of a fbts X. Then A is called:
1) a (τ_i, τ_j)-fuzzy semiclosed set if and only if A^c is a (τ_i, τ_j)-fuzzy semiopen set.
2) a (τ_i, τ_j)-fuzzy preclosed set if and only if A^c is a (τ_i, τ_j)-fuzzy preopen set.
3) a (τ_i, τ_j)-fuzzy α-closed set if and only if A^c is a (τ_i, τ_j)-fuzzy α-open set.
4) a (τ_i, τ_j)-fuzzy semi-pre closed set if and only if A^c is a (τ_i, τ_j)-fuzzy semi-pre open set.
5) a (τ_i, τ_j)-fuzzy regular closed set if and only if A^c is a (τ_i, τ_j)-fuzzy regular open set.
The family of all (τ_i, τ_j)-fuzzy semiclosed sets, (τ_i, τ_j)-fuzzy pre-closed sets, (τ_i, τ_j)-fuzzy α-closed sets, (τ_i, τ_j)-fuzzy semi-pre closed sets and (τ_i, τ_j)-fuzzy regular closed sets will be denoted by (τ_i, τ_j)-FSC(X), (τ_i, τ_j)-FPC(X), (τ_i, τ_j)-FαC(X), (τ_i, τ_j)-FSPC(X) and (τ_i, τ_j)-FRC(X) respectively.

Definition (1-3) [6,7,8]:

A mapping f: $(X,\tau_1,\tau_2) \to (Y,\delta_1,\delta_2)$ from a fbts X into a fbts Y is called:
1) a fuzzy pairwise semicontinuous if $f^{-1}(A)$ is a (T_i,T_j)-fuzzy semiopen set of X for each δ_i-fuzzy open set A of Y.
2) a fuzzy pairwise precontinuous if $f^{-1}(A)$ is a (T_i,T_j)-fuzzy preopen set of X for each δ_i-fuzzy open set A of Y.
3) a fuzzy pairwise α-continuous if $f^{-1}(A)$ is a (T_i,T_j)-fuzzy α-open set of X for each δ_i-fuzzy open set A of Y.
4) a fuzzy pairwise semi-pre continuous if $f^{-1}(A)$ is a (T_i,T_j)-fuzzy semi-pre open set of X for each δ_i-fuzzy open set A of Y.
5) a fuzzy pairwise regular continuous if $f^{-1}(A)$ is a (T_i,T_j)-fuzzy regular open set of X for each δ_i-fuzzy open set A of Y.

2. Fuzzy almost pairwise semi-pre continuous mappings

Definition 2-1:

A mapping $f:(X,\tau_1,\tau_2) \to (Y,\delta_1,\delta_2)$ from a fbts X into a fbts Y is called fuzzy almost pairwise semi-pre continuous if $f^{-1}(A)$ is a (τ_i, τ_j)-fuzzy semi-pre open set of X for each (δ_i,δ_j)-fuzzy regular open set A of Y.

Remark 2-2:

Let $f:X \to Y$ be a mapping from a fbts X into fbts Y. If f is fuzzy pairwise semi-pre continuous, then f is a fuzzy almost pairwise continuous, the converse of the statements need not to be true, as in the following example.

Example 2-3:

Let X={2,3,6} and μ,η,λ be fuzzy sets of X defined as follows:
$\mu(2)=0.3$ $\mu(3)=0.2$ $\mu(6)=0.4$

$\eta(2)=0.2$ $\eta(3)=0.4$ $\eta(6)=0.2$
$\lambda(2)=0.4$ $\lambda(3)=0.6$ $\lambda(6)=0.5$

If we put $\tau_1=\tau_2\{0,\mu,\mu\vee\lambda,1\}$, $\delta_1=\delta_2=\{0,\mu,\lambda,\mu\wedge\lambda, \mu\vee\lambda,1\}$, let $f:(X,\tau_1,\tau_2)\to (Y,\delta_1,\delta_2)$ identity mapping f is fuzzy almost pairwise semi-pre continuous but f is not fuzzy pairwise semi-pre continuous mapping.

Theorem 2-3:

Let $f:(X,\tau_1,\tau_2)\to(Y,\delta_1,\delta_2)$ be a mapping from a fbts X into a fbts Y. Then the following statements are equivalent:

i. f is a fuzzy almost pairwise semi-pre continuous mapping.
ii. $f^{-1}(A)$ is a (τ_i, τ_j)-fuzzy semi-pre closed set of X for each (δ_i,δ_j)-fuzzy regular closed set A of Y.
iii. (τ_i, τ_j)-spcl $f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}A))\leq f^{-1}(A)$ for each δ_i-fuzzy closed set A of Y.
iv. $f^{-1}(A)\leq (\tau_i, \tau_j)$-spint $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$, for each (δ_i,δ_j)-fuzzy open set A of Y.
v. $f^{-1}(A)\leq (\tau_i, \tau_j)$-spint $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$, for each (δ_i,δ_j)-fuzzy pre open set A of Y.
vi. (τ_i, τ_j)-spcl $f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}A)) \leq f^{-1}(A)$, foe each (δ_i,δ_j)-fuzzy pre closed set A of Y.
vii. (τ_i, τ_j)-spcl $f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}(\delta_i\text{-cl}A)))\leq f^{-1}(A)$, foe each (δ_i,δ_j)-fuzzy α-closed set A of Y.
viii. $f(A)\leq (\tau_i, \tau_j)$-spint $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$, for each (δ_i,δ_j)-fuzzy α-open set A of Y.

Proof:

(i)$\to$(ii)

Let A be any (δ_i,δ_j)-fuzzy regular closed set of fbts Y so A^c is a (δ_i,δ_j)-fuzzy regular open set of fbts Y.

Since s is a fuzzy almost pairwise semi-pre continuous then $f^{-1}(A^c)$ is a (τ_i, τ_j)-fuzzy semi-pre open set in fbts X.

Since $f^{-1}(A^c)=(f^{-1}(A))^c$

Therefore $f^{-1}(A)$ is a (τ_i, τ_j)-fuzzy semi-pre closed set in fbts X.

(ii) $\to$(iii)

Let A be δ_i-fuzzy closed set of Y.

$\Rightarrow\delta_i\text{-cl}(\delta_j\text{-int}A)\leq A$

$\Rightarrow f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}A))\leq f^{-1}(A)$

Since $\delta_i\text{-cl}(\delta_j\text{-int}A)$ is a (δ_i,δ_j)-fuzzy regular closed set then $f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}A))$ is a (τ_i, τ_j)-fuzzy semi-pre closed set in fbts X.

$\Rightarrow f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}A))= (\tau_i, \tau_j)\text{-spcl}f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}A))$

Therefore $(\tau_i, \tau_j)\text{-spcl}f^{-1}(\delta_i\text{-cl}(\delta_j\text{-int}A))\leq f^{-1}(A)$

(iii) $\to$(iv)

It can be prove by using complement.

(iv) $\to$(v)

Let A is (δ_i,δ_j)-fuzzy pre-open set of fbts Y

$\Rightarrow A\leq \delta_i\text{-int}(\delta_j\text{-cl}A)$

$\Rightarrow f^{-1}(A)\leq f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$

Since $\delta_i\text{-int}(\delta_j\text{-cl}A)$ is δ_i-fuzzy open set.

Then $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))\leq (\tau_i, \tau_j)\text{-soint } f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}(\delta_i\text{-cl}(\delta_j\text{-cl}A)))= (\tau_i, \tau_j)\text{-soint } f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$

Therefore $f^{-1}(A) \leq(\tau_i, \tau_j)\text{-soint } f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$

(v)$\to$(vi)

It can be prove by using complement.

(vi)) $\rightarrow$ (vii)

Let A be (δ_i, δ_j)-fuzzy α-closed set of fbts Y

So δ_i-cl(δ_j-int(δ_i-cl(δ_j-clA))) $\leq$ A

$\Rightarrow$ f^{-1}(δi-cl(δ_j-int(δ_i-cl(δ_j-clA)))) $\leq$ f^{-1}(A)

Since δ_i-cl(δ_j-int(δ_i-cl(δ_j-clA))) is a (δ_i, δ_j)-fuzzy pre closed set

$\Rightarrow$(τ_i, τ_j)-spcl f^{-1}(δ_i-cl(δ_j-int(δ_j-cl(δ_i-cl(δ_j-int(δ_i-clA)))) $\leq$ f^{-1}(δ_i-cl(δ_j-int(δ_i-clA)))

Therefore (τ_i, τ_j)-spcl f^{-1}(δ_i-cl(δ_j-int(δ_i-clA))) $\leq$ f^{-1}(A)

(vii) $\rightarrow$ (viii)

It can be prove by using complement.

(viii) $\rightarrow$ (i)

Let A be (τ_i, τ_j)-regular open set of fbts Y

$\Rightarrow$ A= δ_i-int(δ_j-clA))

So A is a (δ_i, δ_j)-α-open set

Then f^{-1}(A) $\leq$ (τ_i, τ_j)-spint f^{-1}(δ_i-int(δ_j-cl(δ_i-intA))) $\leq$ (τ_i, τ_j)-spint f^{-1}(δ_i-int(δ_j-clA))= (τ_i, τ_j)-spint f^{-1}(A)

$\Rightarrow$ f^{-1}(A) $\leq$ (τ_i, τ_j)-spint f^{-1}(A)

So A is a (τ_i, τ_j)-semi-pre open set

Therefore f is a fuzzy almost pairwise semi-pre conyinuous.

Theorem 2-4:

Let f:(X,τ_1,τ_2) $\rightarrow$ (Y,δ_1,δ_2) be a mapping from a fbts X into a fbts Y. Then the following statements are equivalent:

i. **f is a fuzzy almost pairwise semi-pre continuous mapping.**
ii. **(τ_i,τ_j)-spcl f^{-1}(A) $\leq$ f^{-1}(δ_i-clA) for each (δ_i, δ_j)-fuzzy semi-pre open set A of Y.**
iii. **f^{-1}(δ_i-clA) $\leq$ (τ_i,τ_j)-spint f^{-1}(A) for each (δ_i, δ_j)-fuzzy semi-pre closed set A of Y.**
iv. **f^{-1}(δ_i-clA) $\leq$ (τ_i,τ_j)-spint f^{-1}(A) for each (δ_i, δ_j)-fuzzy semi closed set A of Y.**
v. **(τ_i,τ_j)-spcl f^{-1}(A) $\leq$ f^{-1}(δ_i-clA) for each (δ_i, δ_j)-fuzzy semi open set A of Y.**

Proof:

(i) $\rightarrow$ (ii)

Let A be any (δ_i, δ_j)-semi-pre open set of fbts Y

$\Rightarrow$A $\leq$ δ_i-cl(δ_j-int(δ_i-clA))

So f^{-1}(A) $\leq$ f^{-1}(δ_i-cl(δ_j-int(δ_i-clA)))

Since δ_i-cl(δ_j-int(δ_i-clA)) is (δ_i, δ_j)-fuzzy regular closed

So f^{-1}(δ_i-cl(δ_j-int(δ_i-clA))) is (τ_i,τ_j)-fuzzy semi-pre closed set in X

Then (τ_i,τ_j)-spcl f^{-1}(δ_i-int(δ_j-clA))= f^{-1}(δ_i-int(δ_j-clA))

Therefore (τ_i,τ_j)-spcl f^{-1}(A) $\leq$ f^{-1}(δ_i-cl(δ_j-int(δ_i-clA))) $\leq$ f^{-1}(δ_i-clA)

(ii) $\rightarrow$ (iii)

It can be prove by using complement.

(iii) $\rightarrow$ (iv)

Let A a (δ_i, δ_j)-fuzzy semi-closed set of fbts Y

$\Rightarrow$ δ_i-int(δ_j-clA) $\leq$ A

$\Rightarrow$ δ_i-int(δ_j-cl(int-A)) $\leq$ A

Since A is a (τ_i,τ_j)-fuzzy semi-pre closed set in fbts Y

f^{-1}(int-A) $\leq$ (τ_i,τ_j)-spint f^{-1}(A)

(iv) $\rightarrow$ (v)

It can be prove by using complement.

(v) $\rightarrow$ (i)

Let A be a (δ_i, δ_j)-fuzzy regular closed set of fbts Y
$\Rightarrow$ A= δ_i-cl(δ_j-intA))
So A is semi-open set
Then (τ_i,τ_j)-spcl $f^{-1}(A) \leq f^{-1}(\delta_i\text{-cl}A)$
$\Rightarrow (\tau_i,\tau_j)$-spcl $f^{-1}(A) \leq f^{-1}(A)$
Therefore $f^{-1}(A)$ is a (τ_i,τ_j)-semi-pre closed set
$\Rightarrow$ f is almost pairwise semi-pre closed set

Corollary 2-5:

Let f:$(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ be a fuzzy almost pairwise semi-pre continuous mapping. Then the following statements holds:

1. **(τ_i,τ_j)-spcl $f^{-1}(A) \leq f^{-1}(\delta_i\text{-cl}A)$, for each δ_i-fuzzy open set A of Y.**
2. **$f^{-1}(\delta_i\text{-int}A) \leq (\tau_i,\tau_j)$-spint $f^{-1}(A)$, for each δ_i-fuzzy open set A of Y.**

Theorem 2-6:

Let f:$(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ be a mapping from a fbts X into a fbts Y. Then the following statements are equivalent:

- **i. f is a fuzzy almost pairwise semi-pre continuous.**
- **ii. for each fuzzy singleton x_t of X and δ_i-fuzzy open set A contain $f(x_t)$. there exists fuzzy semi-pre open set B of X containing x_t such that $f(B) \leq \delta_i$-int(δ_j-clA).**
- **iii. for each fuzzy singleton x_t of X and (δ_i, δ_j)-fuzzy regular open set A containing $f(x_t)$. there exists (τ_i,τ_j)-fuzzy semi-pre open set B of X containing x_t such that $f(B) \leq$ B.**

Proof:

(i)$\rightarrow$(ii)

Let x_t be fuzzy singleton and A is δ_i-fuzzy open set of a fbts Y containing $f(x_t)$
$f(x_t) \leq A \Rightarrow x_t \leq f^{-1}(A)$
Since A is δ_i-fuzzy open set
So $A \leq \delta_i$-int(δ_j-clA)
$\Rightarrow x_t \leq f^{-1}(A) \leq f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$
Since δ_i-int(δ_j-clA) is a (δ_i, δ_j)-fuzzy regular open and f is a fuzzy almost pairwise semi-pre continuous.
Then $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$ is a (τ_i,τ_j)-fuzzy semi-pre open
$\Rightarrow f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))= (\tau_i,\tau_j)$-spint $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$
$x_t \leq f^{-1}(A) \leq (\tau_i,\tau_j)$-spint $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$
Let B=(τ_i,τ_j)-spint $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))$
$\Rightarrow$ B is (τ_i,τ_j)-fuzzy semi-pre open and $x_t \leq B$
f(B)=f((τ_i,τ_j)-spint $f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))) \leq f(f^{-1}(\delta_i\text{-int}(\delta_j\text{-cl}A))) \leq \delta_i\text{-int}(\delta_j\text{-cl}A)$
Therefore $f(B) \leq \delta_i$-int(δ_j-clA).

(ii)$\rightarrow$(iii)

Let x_t be a fuzzy singleton in X and A is a (δ_i, δ_j)-fuzzy regular open set of Y containing $f(x_t)$
Since A=δ_i-int(δ_j-clA)
$\Rightarrow$ A is δ_i-fuzzy open set
be (ii) we get that there exists (τ_i,τ_j)-fuzzy semi-pre open set B of a fbts X containing x_t such that
$f(B) \leq \delta_i$-int(δ_j-clA)

Since A is (δ_i, δ_j)-fuzzy regular open set

$\Rightarrow$ A= δ_i-int(δ_j-clA)

Therefore f(B) $\leq$ A

(iii) $\rightarrow$ (i)

Let A be (δ_i, δ_j)-fuzzy regular open set and x_t be a fuzzy singleton such that $x_t \leq f^{-1}(A)$

$\Rightarrow f(x_t) \leq A$

by (iii) we get that there exists (τ_i,τ_j)-fuzzy semi-pre open set B of a fbts X containing x_t and f(B) $\leq$ A

$\Rightarrow B \leq f^{-1}(A)$

So $x_t \leq B \leq f^{-1}(A)$

Since B is (τ_i,τ_j)-fuzzy semi-pre open set

$\Rightarrow$ B=(τ_i,τ_j)-spint B $\leq$ (τ_i,τ_j)-spint $f^{-1}(A)$

Since x_t arbitrary and $f^{-1}(A)$ is the union of all fuzzy singleton of $f^{-1}(A)$

$f^{-1}(A) \leq (\tau_i,\tau_j)$-spint $f^{-1}(A)$

$\Rightarrow$ $f^{-1}(A)$ is (τ_i,τ_j)-fuzzy semi-pre open set

Therefor f is a fuzzy semi-pre continuous.

3. Fuzzy almost pairwise semi-pre open (closed) mapping:

Definition 3-1:

A mapping f:$(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ from a fbts X into a fbts Y is called:

1. Fuzzy pairwise semi-pre open mapping if f(A) is (δ_i, δ_j)-fuzzy semi-pre open set of Y for any τ_i-fuzzy open set A of X.
2. Fuzzy pairwise semi-pre closed mapping if f(A) is (δ_i, δ_j)-fuzzy semi-pre closed set of Y for any τ_i-fuzzy closed set A of X.
3. Fuzzy almost pairwise semi-pre open mapping if f(A) is a (δ_i, δ_j)-fuzzy semi-pre open set of Y for any (τ_i,τ_j)-fuzzy regular open set A of X.
4. Fuzzy almost pairwise semi-pre closed mapping if f(A) is a (δ_i, δ_j)-fuzzy semi-pre closed set of Y for any (τ_i,τ_j)-fuzzy regular closed set A of X.

Remark 3-2:

Every fuzzy pairwise semi-pre open (resp. fuzzy pairwise semi-pre closed) mapping is fuzzy almost pairwise semi-pre open (resp. fuzzy almost pairwise semi-pre closed) mapping. But the converse may not be true as in the following example.

Example 3-3:

Let X={a,b} and μ,λ be fuzzy sets defined as follows:

$\mu(a)$=0.4 $\mu(b)$=0.5

$\lambda(a)$=0.6 $\lambda(b)$=0.6

If we put $\tau_1=\tau_2=\{0,1,\mu,\lambda, \mu\vee\lambda\}$, $\delta_1=\delta_2=\{0,1, \mu\}$, and let f:$(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ be an identity mapping, f is fuzzy almost pairwise semi-pre open but not fuzzy pairwise semi-pre open.

Theorem 3-4:

1. Let f:$(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ be a fuzzy almost pairwise semi-pre open mapping. If λ is a fuzzy set in Y and μ is a fuzzy (τ_i,τ_j)-regular closed set in X containing $f^{-1}(\lambda)$. Then there exists (τ_i,τ_j)-fuzzy semi-pre closed set Ч in Y such that $\lambda \leq$ Ч and f^{-1}(Ч) $\leq \mu$.
2. Let f:$(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ be a fuzzy almost pairwise semi-pre closed mapping. If λ is a fuzzy set in Y and μ is (τ_i,τ_j)-fuzzy regular open set in X

containing $f^{-1}(\lambda)$. Then there exists (τ_i,τ_j)-fuzzy semi-pre open set Ч in Y such that $\lambda \leq$ Ч and f^{-1}(Ч) $\leq \mu$.

Proof:

1) Since μ is (τ_i,τ_j)- fuzzy regular closed

$\Rightarrow \mu^c$ is (τ_i,τ_j)- fuzzy regular open

Since f is a fuzzy almost pairwise semi-pre open

$\Rightarrow f(\mu^c)$ is (δ_i, δ_j)-semi-pre open in Y

Let Ч$=(f(\mu^c))^c$

So Ч is (δ_i, δ_j)-fuzzy semi-pre closed in Y

Since $f^{-1}(\lambda) \leq \mu$

$\Rightarrow \mu^c \leq f^{-1}(\lambda^c)$

$\Rightarrow f(\mu^c) \leq f(f^{-1}(\lambda^c)) \leq \lambda^c$

$\lambda \leq (f(\mu^c))^c=$Ч $\Rightarrow \lambda \leq$ Ч

f^{-1}(Ч)$=f^{-1}((f(\mu^c))^c)=(f^{-1}(f(\mu^c)))^c \leq (\mu^c)^c=\mu$

$\Rightarrow f^{-1}$(Ч)$\leq \mu$

2) Similarly proved.

Theorem 3-5:

Let $f:(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ be a fuzzy almost pairwise semi-pre mapping then $f^{-1}(\text{spcl } \mu) \leq \text{cl } f^{-1}(\mu)$, for every fuzzy set μ of Y.

Theorem 3-6:

1. A mapping $f:(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ is a fuzzy almost semi-pre open if and only if $f(\tau_i\text{-int}\lambda) \leq \text{spint } f(\lambda)$, for every fuzzy set λ of X.
2. A mapping $f:(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ is a fuzzy almost semi-pre closed if and only if $\text{spcl } f(\lambda) \leq f(\tau_i\text{-cl}\lambda)$, for every fuzzy set λ of X.

Proof:

1) Suppose that f is a fuzzy almost semi-pre open and λ is a fuzzy set of X.

$\Rightarrow \tau_i\text{-int}\lambda$ is a fuzzy τ_i-fuzzy open.

Since f is a fuzzy almost pairwise semi-pre open

$\Rightarrow f(\tau_i\text{-int}\lambda)$ is (τ_i,τ_j)- fuzzy pairwise semi-pre open

$\Rightarrow f(\tau_i\text{-int}\lambda)=\text{spint } f(\tau_i\text{-int}\lambda)$

Since $\tau_i\text{-int}\lambda \leq \lambda$

$\Rightarrow \text{spint } f(\tau_i\text{-int}\lambda) \leq \text{spint } f(\lambda)$

Therefore $f(\tau_i\text{-int}\lambda) \leq \text{spint } f(\lambda)$

2) Similarly proved.

Definition 3-7:

1. A mapping $f:(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ is said to be fuzzy irresolute semi-pre continuous if $f^{-1}(\lambda)$ is a fuzzy semi-pre open set for any (δ_i, δ_j)-fuzzy semi-pre open set λ in Y.
2. A mapping $f:(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ is said to be fuzzy irresolute semi-pre open (semi-pre closed) in Y for any (τ_i,τ_j)- fuzzy semi-pre open (semi-pre closed) set μ in X.

Theorem 3-8:

Let $f:(X,\tau_1,\tau_2) \rightarrow (Y,\delta_1,\delta_2)$ and $g(Y,\delta_1,\delta_2) \rightarrow$ (Z,Ч$_1$,Ч$_2$) be two fuzzy mapping then:

1. If f is a fuzzy almost pairwise semi-pre open (semi-pre closed) and g is a fuzzy pairwise open (semi-pre closed), then gof is almost pairwise semi-pre open (semi-pre closed).

2. **If f is a fuzzy pairwise irresolute semi-pre continuous and g is almost pairwise semi-pre continuous then gof is almost pairwise semi-pre continuous mapping.**
3. **If f is a fuzzy almost pairwise semi-pre continuous and onto and gof is a fuzzy pairwise irresolute semi-pre open (semi-pre closed) then g is almost pairwise semi-pre open (semi-pre closed).**

Proof:

1) Let μ is a fuzzy (τ_i,τ_j)- fuzzy regular open set in X
Since f is almost pairwise semi-pre open mapping
$\Rightarrow$ f(μ) is (δ_i, δ_j)-fuzzy semi-pre open set in Y.
Since g is a fuzzy irresolute semi-pre open mapping
$\Rightarrow$ g(f(μ)) is (τ_i,τ_j)- fuzzy regular open set in Z
So (gof)(μ) is (τ_i,τ_j)- fuzzy regular open set in Z
Therefore gof is almost pairwise semi-pre open set.

2) Let μ be (Ч$_i$,Ч$_j$)-fuzzy regular open set in Z
Since g is a fuzzy almost pairwise semi-pre continuous mapping
$\Rightarrow g^{-1}(\mu)$ is (б$_i$,б$_j$)-fuzzy semi-pre set in Y
Since f is a fuzzy irresolute semi-pre continuous mapping
$\Rightarrow f^{-1}(g^{-1}(\mu))$ is (τ_i,τ_j)-fuzzy semi-pre open set in X
So $(gof)^{-1}(\mu)$ is (τ_i,τ_j)-fuzzy semi-pre open set in X.
Therefore gof is almost pairwise semi-pre continuous mapping.

2) Let μ be (Ч$_i$,Ч$_j$)-fuzzy regular open set in Z
Since g is a fuzzy almost pairwise semi-pre continuous mapping
$\Rightarrow g^{-1}(\mu)$ is (б$_i$,б$_j$)-fuzzy semi-pre set in Y
Since f is a fuzzy irresolute semi-pre continuous mapping
$\Rightarrow f^{-1}(g^{-1}(\mu))$ is (τ_i,τ_j)-fuzzy semi-pre open set in X
So $(gof)^{-1}(\mu)$ is (τ_i,τ_j)-fuzzy semi-pre open set in X.
Therefore gof is almost pairwise semi-pre continuous mapping.

3) Let μ is (б$_i$,б$_j$)-fuzzy regular open set in Y
Since f is fuzzy almost pairwise semi-pre continuous
$\Rightarrow f^{-1}(\mu)$ is (τ_i,τ_j)-fuzzy semi-pre open set in X
Since gof is a fuzzy pairwise irresolute semi-pre open
$\Rightarrow (gof)^{-1}(f^{-1}(\mu))$ is a (τ_i,τ_j)-fuzzy semi-pre open set in X
Since f onto $\Rightarrow$ So $f^{-1}(f(\mu))=\mu$
$\Rightarrow (gof)^{-1}(f^{-1}(\mu))=g(\mu)$
So g(μ) is (τ_i,τ_j)-fuzzy semi-pre open
Therefore g is a fuzzy almost pairwise semi-pre open.

الدوال شبه – pre على الأكثر مستمره والدوال شبه – pre على الأكثر مفتوحه (مغلقه)

محمد جاسم محمد
قسم الرياضيات / كلية ألتربيه
جامعة ذي قار

الملخص:

في هذا البحث تم دراسة المجموعات شبه – pre المفتوحه في التبولوجيا الثنائيه بالاضافه الى دراسة الدوال شبه – pre على الاكثر مستمره وكذلك في البحث تمت دراسة الدوال شبه – pre على الاكثر مفتوحه (مغلقه) كما تم برهان مجموعة من المبرهنات المتعلقه بهما .

References

[1] K.K. Azad, on fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, J.Math. Anan. Appl. 82 (1981) 14-32.
[2] C. L. Chang, fuzzy topological spaces, J. Math. Anal. Appl. 24(1963) 182-190.
[3] J. C. Kelly, bitopological spaces, prol. London Math. Soc. 13(1963) 71-89.
[4] A. Knadil, biproximaties and fuzzy bitopological spaces, simon steven 63(1989).
[5] B. Krsteska, Fuzzy strongly pre open sets and fuzzy strong pre continuity, Mat. Vesnikyso (1998), 111-123.
[6] B. Kresteska, Fuzzy pairwise almost strong pre continuity, Kragujevac J. Math. 28(2005) 193-206.
[7] S. Sampath Kumar, On fuzzy pairwise α-continuity and fuzzy pairwise pre continuity, Fuzzy sets and systems, 62(1994) 231-238.
[8] S. Sampath Kumar, Semi open sets, semi-continuity and semi open mappings in bitopological spaces, Fuzzy sets and systems 64(1994) 421-426.
[9] S. Sampath Kumar, θ-closure and δ-closure operators on fuzzy bitopological spaces, fuzzy sets and systems 87(1997) 235-242.
[10] S. Sampath Kumar, Some near fuzzy pairwise continuous mappings between fuzzy bitopological spaces, Fuzzy sets and systems 109(2000) 251-262.
[11] S. S. Thakur, and S. Singh, On fuzzy semi-pre open sets and fuzzy semi-pre continuity, Fuzzy sets and systems 98(1998) 383-391.
[12] L. A. Zadeh, Fuzzy sets, Inform control 8(1965) 338-353.