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The method of Weighted Multi objective Fractional Linear Programming Problem (MOFLPP)

Waleed Khalid Jaber **Zeanab k. jabar**
Thi- Qar University- College of Computer science and mathematical

Abstract: More theories and algorithms in non-linear programming with titles convexity (Convex). When the objective function is fractional function, will not have to have any swelling, but can get other good properties have a role in the development of algorithms decision problem.In this work we focus on the weights method- (one of the classical methods to solve Multi objective convex case problem). Since we have no convex or no concave objective functions, and this condition is essential part on this method implementation, we these valid conditions under method as generator sets efficient and weakly efficient this problem. This raises the need to a detailed study of pseudoconvex idea, cause convex idea, Invex, pseudoinvex idea,…, etc. concepts. Offer a numerical example to show the valid by the conditions previously set generate all weakly efficient set our problem.

Keywords : Weighted ,Multi objective Fractional Linear Programming Problem , (MOFLPP)

1Introduction

The possibility of mod conditioned by the reality is complex and, in most cases, the best representing model is determined by possibility consideration of more than one conflict objective. This leads to the problem no longer obtain an optimal solution and becomes a problem of decision making in.

Through functions represented by a ratio. Of course, a quotient of functions is simply a nonlinear function. However, the structure ratio leads to establish some special properties that do not share the nonlinear functions in general, which motivates the study of such functions separately from the non-linear. Generally the functions are expressed of any other ratio as neither convex nor concave .That is why the result are great importance to extend the basic results of convex programming to less restrictive assumptions. Moreover within these functions, the most relevant those with associated with numerator and denominator. These dues to the good properties they possess.

$$\left.\begin{array}{l} \min \left\{ \varphi_1(x) = \dfrac{c_1^t x + \alpha_1}{d_1^t x + \beta_1}, \ldots, \varphi_p(x) = \dfrac{c_p^t x + \alpha_p}{d_p^t x + \beta_p} \right\} \\ s.t. \quad Ax \le b \\ \qquad\quad x \ge o \end{array}\right\} \text{(MOFLP)...(1)}$$

where** $c, d \in R^n$, α_i , $\beta_i \in R$, $A \in M_{mxn}(R)$ **and** $b \in R^m$.* We call ***X the set of opportunities of this problem, ***(i.e.*** $X = \{ x \in R^n / Ax \le b, x \ge 0 \}$ ***).***

Which is bounded. In this problem, ranging from definitions of natural and efficient point weakly efficient point are given in Multi objective Programming. One way to solve this problem is to determine the set of efficient and weakly efficient solutions in a strictly technical, way not incorporating into the analysis any information about the preferences of central decision-maker. The purpose of such methods is to provide sufficient information of the efficient or the weakly efficient structure of the whole problem. One method of generation the classic efficient Multi objective linear problem is the Weights method.

The method on converting the problem that scalar construct an objective function is sum of the objective functions starting weighted relative weight assigned to each of them. Thus, for each possible weight you get a problem subject to the restrictions of the original problem consisting a minimizing scale resulting function.[5] & [11].

Well, in this work problem. To do this, we have divided the work in to five sections as what is follow:

In the next section we shall briefly show the method that we encountered in wanting to apply it to the case fractional linear. Section 3 provides the literature review of theory basis on generalized convexity which is necessary to section fourth in which we establish the main result that, due to the non convexity of functions, the conditions under which they can ensure that the solutions of a weighted problem are Optimal solution of multi objective the original problem are rather weaker than those given in the case of convex functions. This also is reinforced by a counter example.

Finally we will see, in section five, the conclusions of this work.

2. Method of Weights

To give the problem of Multi objective fractional linear programming problem (MFLP) Vector objective function, with $\varphi(x) = (\varphi_1(x), ..., \varphi_p(x))$, and to give a weight vector $\lambda \in R^{p+}$ as not identically zero, considering the problem as the following weighted problem (P_λ)

$$\left.\begin{array}{ll} \min \lambda^t \varphi(x) & \\ s.t. & Ax \leq b \\ & x \geq 0 \end{array}\right\} \quad ...(p_\lambda)$$

If we denote by $S(P_\lambda)$ set of solutions to this problem is clear $S(P_\lambda)$ coincides with $S(P_{\alpha\lambda})$ for all $\alpha > 0$. Therefore, loss of generality usually takes the standard weight vector. That is,

if $\lambda = (\lambda_1, ..., \lambda_p)$, then $\lambda_i \geq 0$, for all $i = 1, ..., p$ and $\sum_i \lambda_i = 1$.

The following results are easy to show and can be found among others. To be set all together under one statement, including the following theorem.[9]

2.1.Theorem 1.

If x^* is solution of the problem (P_λ), then x^* is a weak point efficient of the original problem.

If x^* is solution of the problem (P_λ) with all weights strictly positive that is, if $\lambda_i > 0$ for all $i = 1, ..., p$, then x^* is optimal solution of problem Multi objective original. If x^* is unique solution of problem (P_λ), then x^* is the optimal solution of original problem.

Proof.

The weakness of this method is found in the reciprocal of these results, since not all efficient solutions will be obtained under the scalar problems weighting. When we have secured the convexity of all functions objective reaches a certain reciprocal states that if $x^* \in X$ is Pareto optimal, then there exists a weight vector is not necessarily strictly positive such that x^* is the solution of the weighting scalar problem associated with this vector. ***[9].***

However, the functions that make up our multi-objective problem are not convex functions and therefore we can ask a question whether we can found under our conditions some kind of mutual, albeit weak, of previous results. We will do this by a result that is established in terms of low efficiency and problems without restrictions. This is possible thanks to special form of our functions are concave but not convex nor do enter within a classification to be more relaxed pseudolinear.

3. Generalized Convexity

Consider a general mathematical programming problem of minimizing whose objective function differentiable assume $S \subseteq R^n$,

$\min \varphi(x)$

$s.t.\ x \in S$

A first immediate generalization that emerges from the familiar definition of convexity is the quasiconvex idea.

1.3. Definition 1.

It is said that φ is quasiconvex if $\forall x, y \in S$, $\forall \lambda \in (0,1)$, we have:

$$\varphi(\lambda x + (1 - \lambda) y) \leq \max(\varphi(x), \varphi(y))$$

A function is quasiconcave if $-\varphi$ is φ and is said quasiconvex if quasiconvex and quasiconcave at a time.

Since we are dealing with differentiable functions, we can make another generalization of convexity which is based on the characterization of convex functions differentiable. We are referring to the concept of pseudoconvexidad.

2.3. Definition 2.

It is said that φ is a function quasiconvex *on S if* $\forall x, y \in S$ *with* $\varphi(y) < \varphi(x)$ *then necessarily* $\nabla \varphi(x)^t (y - x) < 0$.

As with the quasiconvex idea, a function is pseudoconcave if your pseudolinear opposite is quasiconvex and if both pseudoconcave and quasiconvex.

Generally, the pseudoconcave idea property is stronger than quasiconcave when the functions are differentiable.

Turning to the case, which is the fractional linear programming, when we have the ratio of two related functions,

$\varphi(x) = f(x) / g(x)$,

Where denominator $g(x)$ we assume strictly greater than zero, then the ratio $\varphi(x)$ is a function pseudolinear and, consequently, quasilinear. [3]

Many of the properties of linear programming in range natural functions are such fractional linear programming. Among them one of the most important, is the generalization of the sufficient conditions of optimality Kuhn-Tucker for such functions as shown in the following theorem.

3.3.Theorem 2.

Let S be a nonempty open set of R^n and are φ, h_i with $i = 1, ..., m$
functions defined actual S, that is, $\varphi, h_i : S \subset R^n \rightarrow R$.
Suppose the problem

$$\left.\begin{array}{l} \min \varphi(x) \\ s.t. \quad x \in X \end{array}\right\} \quad \dots (1)$$

Where $X = (x \in S \subset R^n \,/\, h_i(x) \leq 0,\ i = 1, ..., m)$.
Suppose x^* a workable solution
call $I = (i \,/\, h_i(x^*) = 0)$.
Let φ pseudoconvex at x^* and hi quasiconvex and differentiable at x^* *for* $i \in I$. If x^* verifies the Kuhn-Tucker conditions for ***(1), then*** x^* is a global optimal solution of the problem.

Proof. [7]

As we know, another property of convexity is the fact that it must
$\varphi(x) - \varphi(x^*) \geq (x - x^*)^t \nabla\varphi(x^*)$ *for all* $x, x^* \in S$.
However, considered a class of functions for which there is a vector function [6]
$\eta: S \times S \rightarrow R^n$ *such that* $\varphi(x) - \varphi(x^*) \geq \eta(x, x^*)^t \nabla\varphi(x^*)$ *for all* $x, x^* \in S$. More Craven later (1981) named these functions as functions INVEX.[2]

4.3. Definition 3.

Let $\varphi: S \subseteq R^n \rightarrow R$ a differentiable function in the set S open.
Then INVEX if φ is a function $\eta: S \times S \rightarrow R^n$ such that for all $x, x^* \in S$, has to $\varphi(x) - \varphi(x^*) \geq \eta(x, x^*)^t \nabla\varphi(x^*)$.
A generalization of the functions INVEX is in the definition of pseudoconvex defined functions.[1]

5.3. Definition 4.

Let $\varphi: S \subseteq R^n \rightarrow R$ a differentiable function in the set S open.
Then φ ***as*** pseudoconvex if there exists a function $\eta: S \times S \rightarrow R^n$ such that ***for all*** $x, x^* \in S$ ***have that*** $\eta(x, x^*)^t \nabla\varphi(x^*) \geq 0$ ***implies*** $\varphi(x) - \varphi(x^*) \geq 0$.
In the same way that the functions pseudoconvex generalize the convex, the pseudoconvex INVEX generalize the functions. Martin (1985) established the most important result related to the functions INVEX, in which characterized these functions as follows:[8]

a differentiable function φ ***is INVEX on S if and only if each critical point of*** φ ***is global minimum of in S.***
Because this function is verifying property pseudoconvex so, it is clear that we have the following implications:
convex ⇒ pseudoconvex ⇒ INVEX ⇒ Pseudoconvex
Therefore, in conclusion, as the functions are linear fractional pseudoconvex, we can also ensure that a function is a linear fractional INVEX function and, consequently, pseudoconvex.
Let in the following section we establish the desired result of mutual of ***Theorem 1*** for which we will use this property of fractional linear functions.

4. Method of Fractional Programming Weightings

Let's see Multi objective problem:
when we apply the method of Weights.
Given the Multi objective fractional linear problem (MFLP), as seen previously, each objective function φ_i is a pseudoconvex function, INVEX and pseudoconvex result. Naturally generalize the concepts of function and pseudoconvex INVEX for vector functions, and suppose now are an unrestricted vector problem:

$$\begin{array}{l} \min (\varphi_1(x), \dots, \varphi_p(x)) \\ s.t \quad x \in S \end{array} \quad \dots(2)$$

where S is the open set of R^n ***in which*** φ ***is defined***. [10]
pseudoinvex idea is imposing the condition for the vector objective function (2), theorem from the alternative of Gordan, arrive at a characterization of pseudoconvex functions. Furthermore, in the same way, due characterization to that, another result that exposes these authors is a certain converse of Theorem 1is states:

1.4.Theorem 3.

Let x^* weakly efficient solution to the problem (2). If $\varphi(x)$ is a INVEX function in S, then there exists a $\lambda \geq 0$ ***such that*** x^* is an optimal solution of weighted problem

$$\left.\begin{array}{l} \min \lambda^t \varphi(x) \\ s.t. \quad x \in S \end{array}\right\}$$

Proof. [10]

To with the Multi objective fractional linear problem and to suppose that we have a problem with no restrictions.

$$\left.\begin{array}{l} in\left\{\varphi_1(x)=\dfrac{c_1^t x+\alpha_1}{d_1^t x+\beta_1},\ldots\varphi_p(x)=\dfrac{c_p^t x+\alpha_p}{d_p^t x+\beta_p}\right\} \\ s.t. \quad x\in S \end{array}\right\} \quad (MFLPP)$$

Where S is the set of defining the vector function $\varphi = (\varphi_1, ..., \varphi_p)$ which is a R^n open. We recall that the functions of this problem are pseudoconvex, In which it implies, as already mentioned, INVEX functions. Therefore we can say that (MFLPS) is a problem whose objective function is a vector INVEX function and pseudoconvex.
So it makes sense in this particular case to apply Theorem 3 which characterizes weakly efficient points of the problem. For which you have to get a converse of Theorem 1 for this problem.

2.4. Theorem 4.

$x^* \in S$ is a weakly efficient point of problem (MFLPS), then exists $\lambda \in R^p$, $\lambda \geq 0$ is not identically zero such that x* is the solution of the problem scalar weights ***(*** P_λ ***).***
Proof.
Given the (MFLPS), the objective functions of it are all fractional linear therefore, they are particularly, pseudoconvex. This implies, as mentioned, all functions are INVEX. Therefore the objective function of the problem, a vector view is a INVEX function. Therefore, we faced a problem of Multi objective unrestricted INVEX function aims. This implies, according to Theorem 3, ***if*** $x^* \in S$ is a weakly efficient point in the problem, what is more there exists ***a*** $\lambda \in R^p$, $\lambda \geq 0$ as not identically zero such that x^* is solution of the problem scalar weights ***(*** P_λ ***).***
Therefore, using the weights method, we can reach conclusion that the same guarantees we obtain efficient solutions that always weights are all non-zero or the resulting solution was unique. Otherwise, the solution obtained may not be efficient but weakly efficient. The weakness of method is that, even by varying the weights in all weights possible, not assured of obtaining the whole efficient. If the problem has restrictions, then all the theory outlined above, we would achieve find the set of weak optimal solution of the problem by this method.
However, we able to obtain this result even having a problem with restrictions?: The answer, as intuition, is negative. That is, given a fractional problem Multi objective linear restrictions, is not generally true that any weaknesses comes as efficient as optimal weighted problem. The rationale is clear:
While the problem (MFLP) is a problem that's set of opportunities, ***X***, is a coming convex set expressed by linear constraints, the set image ***of X,*** φ ***(X),*** not necessarily a convex set in R^+. The absence vector of weights λ ***such that x* solve (*** P_λ ***)*** is equivalent to the nonexistence of a supporting hyper plane of the feasible region in objective space φ ***(X)*** of problem ***x****. And this condition, a Multi objective fractional linear problem is quite common.

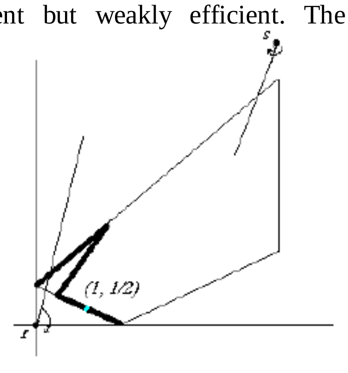
Example 1.

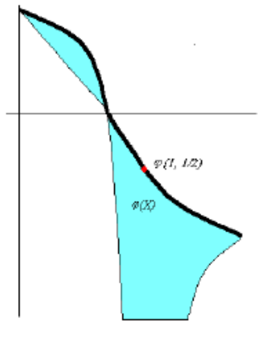
Consider the Multi fractional linear programming problem (maximum) :

$$\max\left\{\frac{x_1}{x_1+x_2}\;,\;\frac{-3x_1+2x_2}{x_1-x_2+3}\right\}$$

$$\begin{aligned} s.t. \quad & 2x_1-4x_2\leq 4 \\ & -x_1-2x_2\leq -2 \\ & -x_1+\ \ x_2\leq 1 \\ & x_1\leq 6 \end{aligned}$$

The point (1, 1 / 2) is an efficient point, and in particular weakly efficient. Figure 1b look at the whole picture of this problem. The image of all weakly efficient frontiers is φ ***(X)*** that is thicker in the figure and we can see that image of point (1, 1 / 2) is part of the non-convex frontier.





Figures 1a and 1b. Efficient point set of the problem of Example 1 in the decision space and space goals.

As already mentioned, the point ***(1, 1 / 2)*** is an efficient point, and in particular weakly efficient. However, this point cannot be obtained as solution weighting no problem. ***Si (1, 1 / 2)*** out of a problem solution ***(P_λ)*** then, in particular, verify the Kuhn-Tucker conditions of this problem.

The general weighted problem is of the form:

$$\max \quad \lambda_1 \frac{x_1}{x_1+x_2} + \lambda_2 \frac{-3x_1+2x_2}{x_1-x_2+3}$$

$$\begin{aligned} s.t. \quad & 2x_1-4x_2 \le 4 \\ & -x_1-2x_2 \le -2 \\ & -x_1+x_2 \le 1 \\ & x_1 \le 6 \end{aligned}$$

We can see that the point (1, 1 / 2) cannot be solving any of these problems since it verifies the Lagrange conditions thereof.

If we notice the multipliers Lagrangian for this problem as ***μ_i , $i = 1, ..., 4$*** to avoid confusion with the weights of the objective function, the Kuhn-Tucker conditions, which must ensure the optimum, taking into account that the unique active constraint in (1, 1 / 2) is the second, they become the next system:

$$\left.\begin{aligned} & \lambda_1 \frac{x_2}{(x_1+x_2)^2} + \lambda_2 \frac{x_2+9}{(x_1-x_2+3)^2} + \mu_2 = 0 \\ & \lambda_1 \frac{-x_1}{(x_1+x_2)^2} + \lambda_2 \frac{-x_1+6}{(x_1-x_2+3)^2} + 2\mu_2 = 0 \\ & \mu_2 \ge 0 \end{aligned}\right\}$$

Substituting these conditions in (1 1 / 2), this system of equations becomes:

$$\left.\begin{aligned} & \tfrac{2}{9}\lambda_1 + \frac{-34}{49}\lambda_2 + \mu_2 = 0 \\ & -\tfrac{4}{9}\lambda_1 + \frac{20}{49}\lambda_2 + 2\mu_2 = 0 \\ & \mu_2 \ge 0 \end{aligned}\right\}$$

This system, combined with the fact that the vector ***(λ_1, λ_2)*** is a vector of weights assume standard

($\lambda_1 + \lambda_2 = 1$) imply that $\lambda_1 = 99/148$, $\lambda_2 = 49/148$ and also that $\mu_2 = 3 / 37$. Therefore, if the point ***(1, 1 / 2)*** is a solution of a problem of weighting just to show that these weights must necessarily be ***$\lambda_1 = 99 / 148$ and $\lambda_2 = 49/148$.***

But is it really the point ***(1,1 / 2)*** to solve this problem?.

Given that the value of the objective function of this problem ***(P_λ)***

with λ = (99/148, 49/148), in (1,1 / 2) is worth (0.256757) and that in (1 / 2, 3 / 2), which is a feasible point of the problem, the objective function takes the value ***0.4155448*** since we are maximizing, we conclude that the point ***(1, 1 / 2)*** is not solution of this problem ***(P_λ).*** Therefore, since these were the only weights possible to verify that the necessary conditions of optimal point ***(1, 1 / 2)*** we can say that this point cannot be found as optimal for any weighting problem remains, however, an efficient point of problem multi objective.

5. Conclusions:

In short, we want to establish the conclusions are established in this work of weights

$\lambda = (\lambda_1 ,..., \lambda_p)$ with $\lambda_i > 0$ and $\sum_i \lambda_i = 1$

The converse of these results has been established in the literature for problems convex. It has been to consider what happens in the case of no convex fractional functions when there is work just to develop. According to results, we have a problem only when we have ensured unrestricted Using the method of weights will travel throughout the whole weak optimal solution.

It should be noted that, in solving a problem of the weighting, when the targets are fractional linear, while avoid the Multi objective nature of the problem, the nature of the linear fractional functions disappears. That is, the objective function of a problem ***(P_λ)*** is a function fractional but leaves verified the linearity of the numerator and denominator.Therefore, to be able to apply the method of weights to (MFLPP), we must be able to solve nonlinear fractional problems which can become a large scale by complication of the objective function. Using specific properties of linear fractional functions, [4] published a method to find the optimum of a sum of linear fractional functions.

So, to find efficient solutions (MFLP), must be resolved problems (Pλ) for different families of strictly positive weights. To carry implement this resolution, we propose the use of the algorithm of [4].Anyway, this paper non-convexity of our functions which prevents us to ensure that we get this way the whole efficient. If we settle for weakly efficient points, allow some weight is zero (not all at once) and also, as seen, if our problem is unrestricted by this method will succeed in obtaining the full range of weak optimal solution.

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طريقة الاوزان لمسائل البرمجة الخطية الكسرية المتعددة

وليد خالد جابر   زينب كاظم جبار

E.mail: dean_coll.science@uoanbar.edu.iq

الخلاصة :

أكثر النظريات والخوارزميات في البرمجة غير الخطية تحمل عناوين التحدب (Convex). عندما تكون دالة الهدف دالة كسرية ، سوف لن يكون لدينا أي انتفاخ ، ولكن يمكن الحصول على خصائص جيدة أخرى يكون لها دور في تطوير خوارزميات مشكلة القرار. في هذا البحث نركز على طريقة الأوزان، وتكون دالة الهدف غير محدبة او غير مقعرة ، وهذا الشرط هو جزء أساسي في طريقة التنفيذ ، لدراسة هذه الطريقة بعمق ، لاثبات صحة هذا الشرط لابد من دراسة مفصلة لفكرة التحدب (Convex)، والـ Invex function ... الخ من المفاهيم.