DETERMINATION OF LUBRICATION FORCE IN THE ELASTOHYDRODYNAMICALLY LUBRICATED LINE CONTACTED SURFACES

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Abstract

In this research the elastohydrodynamic force on two slightly deformable bodies approaching one another head on (squeeze motion) is calculated in the limit of small gap size and small deformations by combining Hertz's theory of deformation and lubrication theory based on solving Reynolds equation in the line contact form. An analytical approach is used to offer an expression for the lubricating force. The effect of the gap size (the lubricant film thickness), the elasticity and the velocity of approach on the lubrication force is demonstrated. Further, the lubrication force with a pressure dependent viscosity is calculated.

Introduction

Elastohydrodynamic lubrication (EHL) occurs in lubricated contacts that are formed between heavily loaded machine components such as gear teeth and rolling element bearings. In this lubrication regime high pressure are generated within the contact and induce both large elastic deformations of the contacting surfaces and a huge viscosity increase in the lubricant (piezo-viscous effect).

A hydrodynamic force between two elastic bodies deforms these objects changing their surfaces profile which in turn leads to a modification of the hydrodynamic force. If the distance between the surfaces is small enough, i.e. typically much smaller than the radii of curvature of surfaces of the bodies, Reynolds equation can be used to calculate the hydrodynamic pressures in the gap with respect to ambient pressure. If the deformations are not too large, the Hertzian approach [1,2] can be used to calculate the counteracting elastic pressures.

Zaid [3] employed the finite element method to solve the Reynolds equation while the boundary element method is used to determine the elastic deformation of the gear teeth. The thermal effects on both the lubricant and the elastic body are studied by solving the energy equation using the finite element method in coupling with the boundary element method to determine the heat transferred to the gear teeth.

Robert, H. Davis et al. [4] used the Hertzian approach to calculate the deformation and the hydrodynamic pressure numerically using an implicit time-stepping routine based upon the finite difference method.

Win Kam Liu et al. [5] presented a finite element model for mixed EHL of journal bearing systems. The mixed EHL consists of partial lubrication region where both EHL and boundary lubrication coexist. The asperity effects on contact and lubrication at large eccentricity ratios are modeled. The elastic deformation due to both hydrodynamic and contact pressure and the cavitations of the lubricant film are considered in the model system.

Jabault, L. et al. [6] applied the Raman micro spectrometry to in situ pressure measurements in an EHL point contact. An experimental set-up coupling a Raman spectrometer and an elastohydrodynamic ball on disc test rig had been developed. Pressure profiles and maps had been recorded under pure rolling conditions for a wide range of applied loads, rolling speeds and lubricant temperatures. The measured pressure profiles are compared quantitatively with numerical solutions of the isothermal EHL point contact problem, and good agreement obtained.

The additional assumptions made in this research to calculate the lubrication force analytically are an explicit reference to the surface profile of the deformed body (it is assumed to consist of two parts: a flat central region and a curved outside region which is not deformed), to the elastic pressure profile (semi-elliptic) and to the squeeze velocity (constant in time and space).

Theoretical analysis

(a) isoviscous case

In Fig.(1) the principal geometrical parameters for the surfaces of the solid bodies are presented.

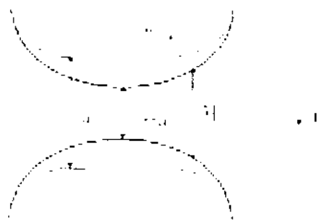

Fig.(1): Geometry of two curved surfaces before deformation (thick line) and after deformation (thin line)

The reduced radius for two curved surfaces with radii of curvature R_1 and R_2 in a tangentially parallel orientation is defined as [2]

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad ....(1)$$

For length scales small compared to the radii of curvature the distance H between the unreformed surfaces is expressed as a parabolic function in r (see Fig.1) as follow:

$$H = H_0 + \frac{r^2}{2R} \quad ....(2)$$

where H_0 is the distance of closest approach between the surfaces for r=0. For deformed surfaces the above equation is replaced by

$$h = H_0 + \frac{r^2}{2R} + w(r) \quad ....(3)$$

where h denotes the distance between deformed surfaces and $w(r) = w_1(r) + w_2(r)$ denotes the deformation of the surfaces at position r. In the present research it is assumed that the deformability is Hertzian, i.e. a flat region for $r \le a$ and a curved outside region for $r > a$ can be distinguished if both surfaces are deformed where a is the radius of the flat central region. The distance between the flat regions on both surfaces for $r = 0$ with $w(0) = \delta$ is

$$h_0 = H_0 - \delta \quad ....(4)$$

The distance between the flat regions on both surfaces for $r \le a$ is

$$h = h_0 \quad ....(5)$$

and the distance between the deformed surfaces for $r > a$ is [7]

$$p(r)-p(\infty)=-(\frac{3\eta V}{h_0})[(\frac{a}{h})^2-(\frac{r}{h})^2+(\frac{R}{h})] \quad ....(14)$$

and (2) for the flat region, i.e. $r > a$ the pressure will be

$$-[p(r)-p(\infty)]=\int_r^{\infty}(\frac{\partial p}{\partial r})_{outside}\,dr \quad ....(15)$$

leads to

$$p(r)-p(\infty)=-3\eta V\frac{R}{h^2} \quad ....(16)$$

The contribution to the lubrication force for $r \le a$ is obtained by integrating equation (14) over the area of the flat region of the two surfaces

$$F_{in}(r \le a)=2\pi\int_0^a[p(r)-p(\infty)]r\,dr \quad ....(17)$$

Working the above equation out gives

$$F_{in}(r \le a)=-3\pi\eta Vh_0[\frac{a^4}{h^4}+\frac{Ra^2}{h^3}-\frac{2}{3}\frac{a^3}{h^3}] \quad ...... (18)$$

The contribution to the lubrication force for $r > a$ is obtained by integrating equation (16) over the area outside the flat region of the two surfaces

$$F_{out}(r > a)=2\pi\int_a^{\infty}[p(r)-p(\infty)]r\,dr \quad ....(19)$$

Working the above equation out results in

$$F_{out}(r > a)=-6\pi\eta V\frac{R^2}{h} \quad ....(20)$$

The lubrication force $F_{lub} = F_{in}(r \le a)+F_{out}(r > a)$ is

$$F_{lub}=-3\pi\eta Vh_0[(\frac{a}{h})^4+\frac{Ra^2}{h^3}-\frac{2}{3}(\frac{a}{h})^3+2(\frac{R}{h_0})^2] \quad ....(21)$$

The radius a of the flat region is determined by balancing the contribution to the lubrication force for $r \le a$, equation (18) with the Hertzian force, equation (9). That this contribution and not the total lubrication force is equated with the Hertzian force is in line with the approximation that the surfaces outside the flat region are not deformed. Further, it should be noted that the parabolic approximation for the tangential distances used in both the calculation of the Hertzian force and the lubrication forces only valid for $\delta << R$ and $H << R$.

With the dimensionless quantities $H^* = \frac{H}{R}$, $\delta^* = \frac{\delta}{R}$, and $D^* = 3\pi\eta V(\frac{D}{R})$ the dimensionless force is defined as $F^* = \frac{F}{3\pi\eta VR}$. Inserting these dimensionless quantities into equation (21) leads to the dimensionless lubricating force

$$F^*_{lub}=-[\frac{2}{H^*}+\frac{5\delta^*}{H^{*2}}-(\frac{2}{3})\frac{\delta^{*3/2}}{H^{*}}+4\frac{\delta^{*2}}{H^{*}}-(\frac{2}{3})\frac{\delta^{*}}{H^{*}}][1+\frac{\delta^*}{H^*}]^2 \quad ..... (22)$$

The above equation reduces to the expression for the squeeze lubrication force on an undeformable body in the limit $\delta^* = 0$

$$\frac{(h-h_0)R}{a^2} = \frac{1}{\pi}\left[\left(\frac{r^2}{a^2}-2\right)\arccos\left(\frac{a}{r}\right)+\sqrt{\frac{r^2}{a^2}-1}\,\right] \quad \ldots\ldots(6)$$

In this research it is assumed that the above expression can be replaced by the approximation

$$\frac{(h-h_0)R}{a^2} = \frac{r^2}{2a^2}-\frac{1}{2} \quad \ldots\ldots\ldots\ldots(7)$$

The difference between the R.H.S. of equations (6) and (7) is small which can be understood from the fact that they are both continuously increasing functions of $\frac{r}{a}$ and their limits for $\frac{r}{a} \downarrow 1$ and $\frac{r}{a} \to \infty$ are equal.

Differentiating both sides of Eq.(7) gives

$$\frac{\partial h}{\partial r} = \frac{r}{h} \quad \ldots\ldots\ldots\ldots\ldots(8)$$

The force which arises due to small compressions, $\delta << R$, is modeled as the Hertzian force[1] as follow

$$F_{Hertz} = \delta^{\frac{3}{2}}\frac{R^{\frac{1}{2}}}{D} \quad \ldots\ldots\ldots\ldots(9)$$

where D is the Hertzian elastic constant $(\frac{m^2}{N})$. The relation between a and the compression δ is [1]

$$a = \sqrt{\delta R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

To calculate the lubrication force on the two bodies it is assumed that the Reynolds equation expressed in polar coordinates in the line contact form is restricted to the squeeze motion

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r h(r)^3\frac{\partial p}{\partial r}\right] = 12\eta V \quad \ldots\ldots\ldots(11)$$

where V is the velocity along the line of centers of the surfaces in (m/sec). η is the dynamic viscosity of the lubricant between the surfaces in (Pa.sec) and p is the pressure between the surfaces in (Pa=N/m^2). In the derivation of the Reynolds equation it is assumed that the lubricant is Newtonian and the inertia and tension forces are negligible compared with the viscous forces[2]. This is justified for $Re\,h_0/R << 1$, where the Reynolds number is defined by $Re = \frac{\rho V}{\eta}$ and ρ is the lubricant mass density in Kg/m^3. The viscosity and the velocity are considered to be constant through the film thickness of the lubricant. Integrating Eq.(11) once for both the flat and parabolic region leads to

$$\frac{\partial p}{\partial r} = 6\eta V\frac{r}{h^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

The pressure profile between the deformed surfaces in the present approximation consists of two parts. (1) for $r \le a$ the pressure with respect to the ambient pressure at $r = \infty$ is

$$-[p(r)-p(\infty)] = \int_r^a\left(\frac{\partial p}{\partial r}\right)_{flat}dr + \int_a^\infty\left(\frac{\partial p}{\partial r}\right)_{parabolic}dr \quad \ldots\ldots\ldots(13)$$

integrating the above equation gives

In order to determine the radius a of the flat region the contribution to the lubrication force for $r \le a$ should be balanced with the Hertzian force, as for the isoviscous case.

Results and Discussion

The deformation δ^* is larger for larger values of D^* and decreases for increasing values of H^* as shown in figures (2) and (3). Higher values of δ^* corresponds to smaller values of the lubrication force, as reflected by the curves in figure (4). For the limiting value of $D^* = 0$ the hard body limit is reached.

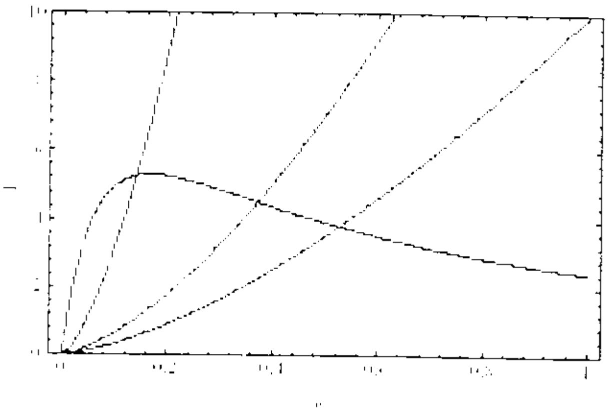

Fig.(2): Lubrication force, $-F^*_{lub}(r \le a)$ and the Hertzian force, F^*_{Hertz} as a function of the deformation δ^* (film thickness $H^* = 0.2$. Monotonically increasing curves are for F^*_{Hertz} with D^*=0.01,0.05 and 0.1 from left to right.

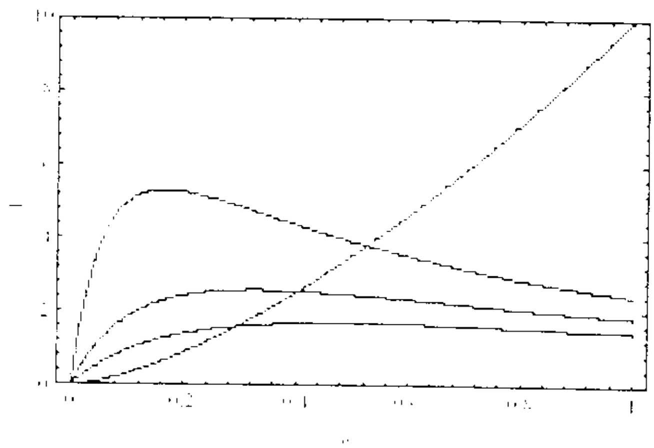

Fig.3: Lubrication force, $-F^*_{lub}(r \le a)$ and the Hertzian force, F^*_{Hertz} as a function of the deformation δ^* with the elastic constant $D^* = 0.1$. The curves which decreases for large δ^* are for $-F^*_{lub}(r \le a)$ with H^*=0.2,0.4 and 0.6 from top to bottom.

$$F^*_{lubr} = -\frac{2}{H^*} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

The dimensionless Hertzian force is

$$F^*_{Hertz} = \frac{\delta^{*3/2}}{D^*} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

For small values of δ^* an approximative expression of δ^* is obtained by equating an expansion to third order in $\sqrt{\delta^*}$ of $-F^*_{lubr}(r \le a)$ with F^*_{Hertz}:

$$\delta^* \approx \frac{25}{H^{*3}}\left[\frac{1}{\left\{\frac{1}{D^*}+\left(\frac{2}{3}\right)\left(\frac{1}{H^{*2}}\right)\right\}^2}\right] \qquad \ldots\ldots\ldots(25)$$

Since this approximation is better for smaller δ^* its accuracy improves for smaller D^* and increasing values of H^*. To leading order in both D^* and H^* the deformation is $\delta^* \approx \frac{25D^{*2}}{H^{*3}}$.

To leading order in δ^* the lubrication force is $F^*_{lubr} \approx -\left(\frac{2}{H^*} - \frac{\delta^*}{H^{*2}}\right)$. The resulting approximative equation for the lubricating force is

$$F^*_{lubr} \approx -\left(\frac{2}{H^*} - 25\frac{D^{*2}}{H^{*5}}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(26)$$

(b) Piezoviscous case

In this case the lubrication force is calculated with a pressure dependent viscosity according to the relation [2]

$$\eta = \eta_o \exp(\alpha p) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

where η_o is the dynamic viscosity of the lubricant at atmospheric pressure in (Pa.sec) and α is a constant depending on the oil, called pressure viscosity coefficient of units (m^2 N⁻Pa⁻¹).

The procedure to do so is well known [2] and can be based on the equations derived in the theory section of isoviscous case.

Replacing $p(r) - p(\infty)$ on the left hand side of equations (14) and (16) by $[1 - \exp\{-\alpha(p(r) - p(\infty))\}]/\alpha$ and calculating $p(r) - p(\infty)$ from these equations results, after integration, in the equations for the contributions to the lubrication force with $r \le a$ and $r > a$ analogous to the equations (18) and (20) for the isoviscous case.

$$F_{lubr}(r \le a) = -\frac{A}{\alpha^* B}\left[-Ba^{*2} - (C - Ba^{*2})\log(C - Ba^{*2}) + C\log(C)\right] \quad \ldots(28)$$

where $\alpha^* = \alpha\frac{3\eta_o V}{R}$, $a^* = \frac{a}{R}$, $A = 3\pi\eta_o VR$, $B = \frac{\alpha^*}{H^{*3}}$ and

$$C = 1 + \frac{\alpha^*}{H^*}\left\{\left(\frac{a^*}{H^*}\right)^2 - \frac{1}{H^*}\right\}$$

and

$$F_{lubr}(r > a) = -\frac{2A}{\alpha^*}\left[\pi\sqrt{\alpha^*} - 2\sqrt{\alpha^*}\arctan\left(\frac{H^*}{\sqrt{\alpha^*}}\right) - H^*\log\left(1 + \frac{\alpha^*}{H^{*2}}\right)\right] \quad \ldots(29)$$

For not too large values of δ^* (small D^* and /or larger values of H^*) the approximation of δ^* with Eq.(25) and the lubrication force with Eq.(26) for the force leads to a good approximation of the numerically calculated force as illustrated in Fig.(5).

Further it should be noted that the lubrication force is not linear in the velocity as for hard bodies. For small deformations this can be demonstrated by putting the dimensions back into Eq.(26)which for the highest order in the velocity leads to $F_{z} \approx V^{2}$.

Concluding Remarks

The present treatment of elastohydrodynamic forces is only valid for $\delta^* << 1$ due to the use of Hertzian theory for the deformation of the bodies and for $H^* << 1$ due to the use of the lubrication theory. Therefore the precise validity range of the present results will have to be established by other, e.g. experimental works.

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حساب قوة التزييت للسطوح ذات التلامس الخطي المزيتة بنظام التزييت الهيدروديناميكي المرن

الخلاصة

في هذا البحث تم حساب القوة الهيدروديناميكية المرنة على جسمين قابلين للتشوه قد لا يقترب أحدهما من الآخر بصورة رأسية (حركة عصرية) في حدود صغيرة من الفجوات والتشوهات القليلة بواسطة ربط نظرية هيرتز للتشوه مع نظرية التزييت المبنية على حل معادلة رينولدز في شكلها في التلامس الخطي. استخدمت طريقة تحليلية لتقديم تعبير لقوة التزييت. ان تأثير حجم الفجوة(سمك غشاء الزيت)، المرونة وسرعة الوصول على قوة التزييت قد تم اختبارها. إضافة إلى ذلك فقد تم حساب قوة التزييت المعتمدة على الضغط.

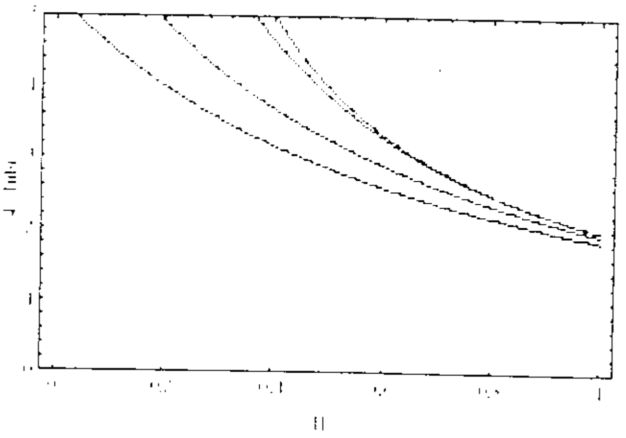

Fig.(4): Lubrication force, $-F^*_{lub}$ as a function of the film thickness H^*. Curves are from left to Right for values of the elastic constants D^* =0.1,0.050.01 and 0.0

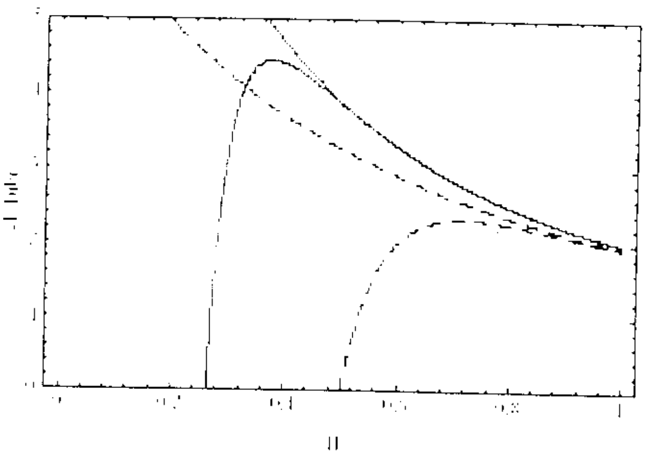

Fig.(5): Lubrication force, $-F^*_{lub}$, as a function of the film thickness H^*. Upper curves are numerically calculated, lower curves are obtained using Eq.(26) valid for small deformations. Solid curves for the elastic constant D^*=0.01; dashed curves for D^*=0.05.

These observations can be clarified for small values of δ^* using Eq.(26). It shows that the lubrication force for a deformable body is smaller than for an undeformable one. Further for small values of $\frac{D^{*2}}{H^{*3}}$

The lubrication forces will all go to the same hard body limit, as is observed in Fig.(4).