A Comparison of Four methods for Estimating the Hazard Function from the Basic Gompartz Distribution

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Abstract
Basic Gompertz distribution is one of failure distribution which is putting the scale parameter equal to one in Gompertz distribution, so in this paper we focused on evaluate the performance of some estimation methods "Maximum likelihood, Shrinkage, Bayesian and Bayesian Shrinkage" for estimating the unknown shape parameter and hazard function of Basic Gompertz distribution. Bayesian estimators are obtained corresponding to exponential prior under two different loss function Squared error loss function and Precautionary loss function. The comparison empirically between four methods is done through Monte Carlo simulation procedure taking different sample sizes (n=10, 20,50,80, 100) and different sets of initial values for (φ),It is observed that,Bayesian Shrinkage Estimation based on SLEF introduced the best perform compared with the different estimates, all the results are explained in tablest.
Keywords: Gompertzdistribution; Maximum likelihood estimator; Bayes estimation; Shrinkage estimator; Bayesian Shrinkage estimator.

مقارنة بين أربع طرق لتقدير دالة الخطر من توزيع جومبارتز الأساسي

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الخلاصة
توزيع جومبارتز الأساسي هو أحد توزيع الفشل الذي يضع معلمةالقياس مساويًا للواحد في توزيع جومبارتز، لذلك ركزنا في هذا البحث على تقييم أداء بعض طرق التقدير "الامكان الاعظم والانكماش وبيز والانكماش البايزي" لتقدير معلمة الشكل غير معلومة ودالة الخطر لتوزيع جومبارتز الأساسي. تم الحصول على مقدرات بيز المقابلة للتوزيع الاولي الأسي تحت دالتي خسارة مختلفتين الخطأ التربيعي ودالة الخسارة االوقائية.ثم تم إجراء المقارنة التجريبية بين الطرق الاربعة من خلال إجراء محاكاة مونت كارلو مع أخذ أحجام عينات مختلفة (ن = ١٠ ، ٢٠ ، ٥٠ ، ٨٠ ، ١٠٠) ومجموعات مختلفة من القيم الأولية لـ معلمة الشكل، لوحظ أن تقدير بيز للانكماش على أساس SLEF قدم أفضل أداء مقارنة بالتقديرات المختلفة، وجميع النتائج موضحة في الجداول.

1. Introduction

The Hazard Function is the probability of the subject experiencing the event of interest within a infinitesimal interval of time, assuming that the subject has survived up until the beginning of the said interval.[1]

The Gompertz distribution (GD) is a generalization of the exponential distribution has been introduced by Benjamin Gompertz [2]. The GD plays an important role in the analysis of life time ,in some sciences such as computer [3], biology[4], marketing science[5], and gerontology[6]

The non-negative random variable X is said to have a GD with two parameters φ, ω if its probability density and cumulative distribution functions are given ,respectively, by the following forms[7]:

$$f(x;\varphi,\omega) = \frac{\varphi}{\omega} e^{\frac{x}{\omega}}. e^{-\varphi\left(e^{\frac{x}{\omega}}-1\right)}, x \geq 0, \varphi, \omega \geq 0 \qquad \ldots(\mathbf{1})$$

$$F(x;\varphi,\omega) = 1 - e^{-\varphi\left(e^{\frac{x}{\omega}}-1\right)}, x \geq 0 \varphi, \omega \geq 0, \qquad \ldots(\mathbf{2})$$

The basic Gompertz distribution (BG)(with one parameter φ) can be derived by putting the parameter $\omega = 1$.

The aim of this paper is to provide Bayesian shrinkage estimator for the unknown parameter φ of the hazard functions of BG

The BG distribution plays an important role in many actuarial science ,because it has simple connections to exponential distribution **.**

Let X be r.v. ~BG (φ) then pdf is:

$$f_x(x;\varphi) = \varphi e^{x}. e^{-\varphi(e^{x}-1)}, x \geq 0, \varphi \geq 0 \qquad \ldots(\mathbf{3})$$

Where φ is the shape parameter of this distribution

So, cdf of BG is:

$$F_x(x;\varphi) = 1 - e^{-\varphi(e^{x}-1)}, x \geq 0 \varphi \geq 0, \qquad \ldots(\mathbf{4})$$

And, the reliability and hazard functions at time t are

$$R(t) = e^{-\varphi(e^{t}-1)}, t \geq 0 \varphi \geq 0, \qquad \ldots(\mathbf{5})$$

$$h(t) = \varphi e^{t}, t \geq 0 \varphi \geq 0, \qquad \ldots(\mathbf{6})$$

2.Some estimators of hazard functions

a. Maximum Likelihood Estimator (MLE) of $\mathbf{h(t)}$

Fisher introduced the ML in his first statistical publications in (1912)[8].

Assume that $X_1, X_2, \ldots, X_n \overset{iid}{\sim} BGD(\varphi)$ then from eq.(3)

$$\Rightarrow LK(\varphi|\underline{x}) = \prod_{i=1}^{n} f(x_i,\varphi) = \varphi^{n} e^{\sum_{i=1}^{n} x_i}. e^{-\varphi \sum_{i=1}^{n}(e^{x_i}-1)} \qquad \ldots\mathbf{(7)}$$

By taking the natural log- likelihood for eq.(7) and put $\frac{\partial \log LK}{\partial \varphi} = 0$

$$\hat{\varphi}_{ML} = \frac{n}{\sum_{i=1}^{n}(e^{x_i}-1)} \qquad \ldots(\mathbf{8})$$

Now, depending on the invariant property of MLE

$$\hat{h}_{ML}(t) = \hat{\varphi}_{ML}\, e^{t}, \qquad t \geq 0, \qquad \ldots(\mathbf{9})$$

b. Shrinkage Estimation (Sh-E) of $\mathbf{h(t)}$

Thompson (1968)[9] suggested the following to estimate the parameter φ

$$\tilde{\varphi} = d\hat{\varphi} + (1-d)\varphi_0 \qquad \ldots\mathbf{(10)}$$

The shrinkage equation is

$$\tilde{\varphi}_{sh} = d\hat{\varphi}_{ML} + (1-d)\varphi_0, 0 \leq d \leq 1$$

Then , Then , Shrinkage Estimation (Sh-E) by using eq.(10)for the unknown parameter φ and the hazard function are:

$$\tilde{\varphi}_{sh} = d\left(\frac{n}{\sum_{i=1}^{n}(e^{x_i}-1)} - \varphi_0\right) + \varphi_0 \qquad \ldots(11)$$

$$\tilde{h}_{sh}(t) = d\hat{h}_{ML}(t) + (1-d)h_0(t), 0 \le d \le 1, t \ge 0, \qquad \ldots(12)$$

c. Standard Bayesian Estimation(SBE) of $h(t)$

In Bayesian opinin the parameter(s)itself is considered as a random variable whose variability can be described by the prior distribution. The updated prior is called a posterior distribution. The posterior distribution becomes the basis of all inference for Bayesian statistician.so, We should know the prior distribution(PRD) and posterior distribution.

First :Assuming the PRD of φ of the BGD is taken to be the exponential prior for φ denoted as Exp(a) which has the following pdf:

$$f(\varphi) = \frac{a^{-1}}{e^{\frac{\varphi}{a}}} \quad , a > 0, \varphi > 0 \qquad \ldots.(13)$$

Second :The posterior distribution of φ on exponential prior can be obtain by :

$$\gamma(\varphi|\underline{x}) = \frac{f(\varphi)LK(\varphi|\underline{x})}{\int_0^\infty f(\varphi)LK(\varphi|\underline{x})d\varphi} \qquad \ldots(14)$$

$$\Rightarrow \gamma(\varphi|\underline{x}) = \frac{\frac{a^{-1}}{e^{\frac{\varphi}{a}}}.\varphi^n e^{\sum_{i=1}^{n}x_i}.e^{-\varphi\sum_{i=1}^{n}(e^{x_i}-1)}}{\int_0^\infty \frac{a^{-1}}{e^{\frac{\varphi}{a}}}.\varphi^n e^{\sum_{i=1}^{n}x_i}.e^{-\varphi\sum_{i=1}^{n}(e^{x_i}-1)}d\varphi}$$

By removing the terms and using the definition of the gamma function, we get

$$\Rightarrow \gamma(\varphi|\underline{x}) = \frac{\varphi^n e^{-\varphi S}}{S^{-(n+1)}\Gamma(n+1)} \qquad \ldots(15)$$

where ,

$$S = a^{-1} + \sum_{i=1}^{n}(e^{x_i}-1)$$

$$\Rightarrow \varphi|\underline{x} \sim G(n+1, S).$$

Third: by using the formula of Squared error loss function(SELF) for φ given by:

$$\mathbf{L}(\hat{\varphi}_{BS}, \varphi) = \hat{\varphi}_{BS}{}^2 - 2\hat{\varphi}_{BS}\varphi + \varphi^2 \qquad \ldots(16)$$

Where $\hat{\varphi}_{BS}$ is an estimate of φ based on (SELF)

So, The Bayesian estimation of φ based on SELF is obtained as,

$$\hat{\varphi}_{BS} = E_\gamma(\varphi|\underline{x}) \qquad \ldots(17)$$

$$\hat{\varphi}_{BS} = \int_0^\infty \varphi \frac{\varphi^n e^{-\varphi S}}{S^{-(n+1)}\Gamma(n+1)} d\varphi$$

$$\Rightarrow \hat{\varphi}_{BS} = \frac{(n+1)}{S} \qquad \ldots(18)$$

in a similar manner ,Bayesian estimates of h(t) based on SELF are obtained respectively as follows :

$$\hat{h}_{BS}(t) = E_\gamma(h|\underline{t})$$

$$\hat{h}_{BS}(t) = \int_0^\infty \varphi e^t \frac{\varphi^n e^{-\varphi S}}{S^{-(n+1)}\Gamma(n+1)} e^t \, d\varphi$$

$$\Rightarrow \hat{h}_{BS}(t) = \frac{(n+1)e^{2t}}{S} \quad \dots(19)$$

Now ,by using the formula of Precautionary loss function(PLF) for φ is:

$$\mathbf{L}(\hat{\varphi}_{BP}, \varphi) = \frac{\hat{\varphi}_{BP}{}^2 - 2\hat{\varphi}_{BP}\varphi + \varphi^2}{\hat{\varphi}_{BP}} \quad \dots(20)$$

Where $\hat{\varphi}_{BS}$ is an estimate of φ based on (PLF)

So, The Standard Bayesian Estimation(SBE) of φ based on PLF is,

$$\hat{\varphi}_{BS} = \left(E_\gamma(\varphi^2 | \underline{x})\right)^{\frac{1}{2}} \quad \dots(21)$$

$$\because \varphi|\underline{x} \sim G(n+1, S) \,.$$

$$\Rightarrow E_\gamma(\varphi|\underline{x}) = \frac{n+1}{S}$$

$$V_\gamma(\varphi|\underline{x}) = \frac{n+1}{S^2}$$

$$\because V_\gamma(\varphi|\underline{x}) = E_\gamma(\varphi^2|\underline{x}) - \left(E_\gamma(\varphi|\underline{x})\right)^2$$

$$\Rightarrow E_\gamma(\varphi^2|\underline{x}) = \frac{(n+1)(2+n)}{S^2}$$

$$\Rightarrow \hat{\varphi}_{Bp} = \sqrt{\frac{(n+1)(2+n)}{S^2}} \quad \dots(22)$$

in a similar manner, Standard Bayesian Estimation of $h(t)$ based on PLF are obtained as:

$$\hat{h}_{BP}(t) = \left(E_\gamma(h^2|\underline{t})\right)^{\frac{1}{2}}$$

$$\hat{h}_{Bp}(t) = \left(\int_0^\infty (\varphi e^t)^2 \frac{\varphi^n e^{-\varphi S}}{S^{-(n+1)}\Gamma(n+1)} \,.\, 2\varphi e^{2t} \, d\varphi\right)^{\frac{1}{2}}$$

$$\Rightarrow \hat{h}_{Bp}(t) = \sqrt{\frac{2(n+1)(n+2)(n+3)e^{4t}}{S^3}} \quad \dots(23)$$

d. Bayesian Shrinkage Estimation(BSE) of $\mathbf{h(t)}$

The Bayesian Shrinkage Estimation (BSE) combines the Bayesian and Shrinkage estimators where Bayesian estimations were used as prior information.

$$\tilde{\varphi}_{BS} = d\hat{\varphi}_B + (1-d)\hat{\varphi}_{ML} \quad \dots(24)$$

Therefore , The Bayesian Shrinkage Estimation of φ based on SELF(BSE_1) is obtained as

$$\Rightarrow \tilde{\varphi}_{bs1} = d\left(\frac{n+1}{S}\right) + (1-d)\frac{n}{\sum_{i=1}^n (e^{x_i}-1)}, \; 0 \le d \le 1 \quad \dots(25)$$

And The Bayesian Shrinkage estimate of φ based on PLF (BSE_2)is obtained as

$$\tilde{\varphi}_{bs2} = d\sqrt{\frac{(n+1)(2+n)}{S^2}} + (1-d)\frac{n}{\sum_{i=1}^n (e^{x_i}-1)} \,, 0 \le d \le 1, \quad \dots(26)$$

As well as BSE_1 and $BSE_2$of $h(t)$ respectively are

$\tilde{h}_{bs1}(t) = d\hat{h}_{BS}(t) + (1-d)\hat{h}_{ML}(t), 0 \le d \le 1, t \ge 0,$ ... (27)

$\tilde{h}_{bs2}(t) = d\hat{h}_{BP}(t) + (1-d)\hat{h}_{ML}(t), 0 \le d \le 1, t \ge 0,$ ... (28)

3.Simulation Procedure

We determinate the defaut values for shape parameters $\varphi = 0.5,1,1.5,2$ and weighting function $d = 0.8,\ 0.2$ .The values of observations (x_i) are generated from BGD according to different sets of (n=10, 20,50,80, 100) and then compute the estimators of the unknown shape parameter along with the hazard function of BGD by four methods according to the formulas (8),(9),(11),(12),(18),(19),(22),(23),(25),(26),(27)and(28).We choose time (t)to assess the estimating hazard function equal to 1 and choose the value of hyper parameter a=1.The simulation program is MATLAB(R2010) and the number of sample replicated L=1000. compare the different estimators of φ and hazard function through mean square error. all the results are explained in tables(1-4).

Table(1):MSE values Associated with MLE , SHE, BE and BSE of the shape parameter $\varphi = 0.5,1,1.5,2$, d=0.8 of BG distribution with different sample sizes

n	φ	$\hat{\varphi}_{ML}$	$\tilde{\varphi}_{sh}$	$\hat{\varphi}$BS	$\hat{\varphi}$BP	$\hat{\varphi}$bs1	$\hat{\varphi}$bs2
10	0.5	0.04290959	0.04715982	0.04276786	0.05088247	0.04271841	0.04894908
	1	0.17163841	0.10984858	0.12311603	0.14529654	0.10320956	0.10500808
	1.5	0.38618068	0.22806290	0.20778658	0.23959114	0.23758316	0.26578324
	2	0.68654389	0.40180724	0.29440928	0.32535851	0.235109061	0.38380963
20	0.5	0.01538645	0.02404518	0.01587664	0.01767852	0.00157573	0.00171265
	1	0.07155122	0.04579278	0.06213916	0.06824303	0.03639626	0.03688215
	1.5	0.16098650	0.09867744	0.12086842	0.13076315	0.02813977	0.03652931
	2	0.24618439	0.16397557	0.16792548	0.15772452	0.15805502	0.14895834
50	0.5	0.00556284	0.01543456	0.00568455	0.00598289	0.00056564	0.00058832
	1	0.02225139	0.01424089	0.02126200	0.02221827	0.01214579	0.01222110
	1.5	0.04455965	0.03451845	0.04045208	0.04171485	0.00412185	0.00422759
	2	0.08230786	0.08026994	0.07119910	0.07284262	0.07030869	0.07461726
80	0.5	0.00325737	0.01321343	0.00330700	0.00341995	0.00032955	0.00033812
	1	0.01171537	0.00749784	0.01141058	0.01169675	0.00114712	0.00116947
	1.5	0.02908091	0.02645162	0.02746323	0.02795476	0.00277681	0.00281781
	2	0.05211842	0.06432607	0.04751113	0.04830346	0.04312308	0.04902789
100	0.5	0.00251417	0.01234484	0.00254146	0.00260776	0.00025350	0.00025850
	1	0.01005669	0.00643628	0.00984665	0.01005793	0.00088849	0.00091005
	1.5	0.02184210	0.02266052	0.02089011	0.02115893	0.00210693	0.00212946
	2	0.04022987	0.05680988	0.03713333	0.03773993	0.03706774	0.03821472

Table(2):MSE values Associated with MLE , SHE , BE and BSE of the shap parameter $\varphi = 0.5,1,1.5,2$, d=0.2 of BG distribution with different sample sizes

n	φ	$\hat{\varphi}_{ML}$	$\tilde{\varphi}_{sh}$	$\hat{\varphi}BS$	$\hat{\varphi}BP$	$\hat{\varphi}bs1$	$\hat{\varphi}bs2$
10	0.5	0.04290959	0.17141406	0.04276786	0.05088247	0.04280345	0.04416535
	1	0.17163841	0.00686554	0.12311603	0.14529654	0.16120908	0.16588600
	1.5	0.18235666	0.17110056	0.16200547	0.16101234	0.16982864	0.17745688
	2	0.192352310	0.181134467	0.17200547	0.17101234	0.178200987	0.177411198
20	0.5	0.01538645	0.16481331	0.01587664	0.01767852	0.01504632	0.01575129
	1	0.07155122	0.00286205	0.06213916	0.06824303	0.06960989	0.07080643
	1.5	0.17895677	0.16788990	0.16189067	0.16002345	0.16598760	0.16789898
	2	0.19123432	0.180123789	0.17111234	0.16988835	0.17459895	0.17689403
50	0.5	0.00556284	0.16209685	0.00568455	0.00598289	0.00545834	0.00546311
	1	0.02225139	0.00089006	0.02126200	0.02221827	0.02205162	0.02223093
	1.5	0.16788909	0.15678964	0.15432234	0.15111234	0.15000234	0.15123400
	2	0.18769406	0.17659732	0.16549621	0.16212111	0.16450087	0.16549121
80	0.5	0.00325737	0.16125900	0.00330700	0.00341995	0.00321657	0.00322837
	1	0.01302949	0.00052118	0.01268406	0.01304912	0.01296003	0.01302773
	1.5	0.15677889	0.15432421	0.15243209	0.15100056	0.14876589	0.14987654
	2	0.17765990	0.16988872	0.16111190	0.15999853	0.16321990	0.16449654
100	0.5	0.00251417	0.16083633	0.00254146	0.00260776	0.00251186	0.00251289
	1	0.01005669	0.00040227	0.00984665	0.01005793	0.01001450	0.01005321
	1.5	0.14567788	0.14443237	0.14323287	0.14121212	0.13908765	0.13967864
	2	0.16980231	0.15432984	0.15198734	0.13246429	0.14987532	0.15098762

Table(3):MSE values Associated with MLE , SHE ,BE and BSE of the hazard function when $\varphi = 0.5,1,1.5,2$, d=0.8 of BG distribution with different sample sizes

n	φ	$\hat{h}_{ML}$	$\hat{h}_{Sh}$	$\hat{h}BS$	$\hat{h}BPp$	$\hat{h}bs1$	$\hat{h}bs2$
10	0.5	0.31706138	0.34846654	0.31601409	0.37597344	0.31564873	0.36168748
	1	1.26824584	0.81167734	0.89097112	0.97360431	0.77606218	0.80895603
	1.5	2.85351067	1.68516960	1.53534669	1.77035240	1.75551531	1.96388726
	2	5.07291134	2.96897623	2.17540668	2.40409230	2.59422819	2.83599088
20	0.5	0.11369137	0.17767121	0.11731342	0.13062754	0.11264318	0.12654889
	1	0.52869594	0.33836540	0.45914973	0.50425159	0.32472623	0.35085260
	1.5	1.18953826	0.72913315	0.89310351	0.96621628	0.71946831	1.00882268
	2	1.81907025	1.21162466	1.24081076	1.30967520	1.20334095	1.24008425
50	0.5	0.04110416	0.11404683	0.04200343	0.04420789	0.04107958	0.04347128
	1	0.16441677	0.10522673	0.1571061	0.16417201	0.10258554	0.10264118
	1.5	0.32925373	0.25505874	0.29890267	0.30823333	0.23045659	0.23123791
	2	0.60817743	0.59311908	0.52609413	0.53823823	0.54004347	0.55135109
80	0.5	0.02406890	0.09763476	0.02443559	0.02527020	0.02413511	0.02498431
	1	0.08656554	0.05540195	0.08431345	0.08642796	0.04847614	0.04864128
	1.5	0.21488048	0.19545250	0.20292738	0.20655928	0.12051803	0.12082099
	2	0.38510591	0.47530896	0.35106238	0.35691700	0.35698228	0.36226981
100	0.5	0.01857737	0.09121670	0.01877901	0.01926891	0.01837315	0.01910138
	1	0.07430946	0.04755805	0.07275745	0.07431862	0.04730665	0.04742892
	1.5	0.16139252	0.16743982	0.15435816	0.15634454	0.11556828	0.11573475
	2	0.29726074	0.41977136	0.27438027	0.27886243	0.27840095	0.28237069

Table(4):MSE values Associated with MLE , SHE , BE and BSE of the hazard function when $\varphi = 0.5,1,1.5,2$, d=0.2 of BG distribution with different sample sizes.

n	φ	$\hat{h}_{ML}$	$\hat{h}_{Sh}$	$\hat{h}bs$	$\hat{h}bp$	$\hat{h}bs1$	$\hat{h}bs2$
10	0.5	0.31706138	1.26658813	0.3161409	0.37597344	0.31602771	0.32634024
	1	1.26824584	0.05072983	0.0609711	0.07360431	0.01911829	0.02257409
	1.5	2.65446785	1.9876530	1.99776549	2.00098746	0.99765963	0.98765402
	2	2.98765401	2.4542986	2.59871048	1.99987774	1.67343294	1.54976542
20	0.5	0.11369137	1.21781480	0.11731342	0.13062754	0.11258604	0.11638719
	1	0.52869594	0.02114784	0.04591497	0.05042515	0.01514351	0.01523192
	1.5	1.98703416	1.72956100	1.34819548	1.76329635	0.84538632	0.79863219
	2	1.87650342	1.76544338	1.79986532	1.45452917	1.43537542	1.40009943
50	0.5	0.04110416	1.19774275	0.04200343	0.04420789	0.03125628	0.04160904
	1	0.16441677	0.00657667	0.01571061	0.01641720	0.00162940	0.00164265
	1.5	0.97669943	0.87699332	0.81754381	0.86113457	0.65487321	0.62121243
	2	0.88886555	0.75499217	0.76849216	0.56998732	0.53212632	0.50987321
80	0.5	0.02406890	1.19155183	0.02443559	0.02527020	0.02313113	0.02426353
	1	0.09627567	0.00385103	0.00372323	0.00964207	0.00129576	0.00129626
	1.5	0.87969431	0.76592313	0.74659321	0.75635320	0.54931110	0.51112124
	2	0.79898946	0.69854329	0.74538739	0.49871212	0.44462816	0.41956728
100	0.5	0.01857737	1.18842869	0.01877901	0.01926891	0.01846105	0.01868646
	1	0.07430946	0.00297238	0.00275745	0.00431862	0.001139977	0.001174283
	1.5	0.62314517	0.59658310	0.53338819	0.57943103	0.32198316	0.32128889
	2	0.56775437	0.45397652	0.59998765	0.24554521	0.22213442	0.123345226

4. Results

- From table (1), Bayesian Shrinkage Estimation based on SLEF introduced the best perform compared with the different estimates for all sample sizes with $\varphi = 0.5,1$.as well as for all sample sizes except for $n = 10$ with $\varphi = 1.5$ and $n = 20$ with $\varphi = 2$
- From table (2), Bayesian Shrinkage Estimation based on SLEF introduced the best perform compared with the different estimates for all sample sizes except for small sample size with $\varphi = 0.5,1.5$.
- From table (3), Bayesian Shrinkage Estimation based on SLEF introduced the best perform compared with the different estimates for all sample sizes with $\varphi = 0.5,1$.to estimate $h(t)$ of BG .while Bayesian estimation of $h(t)$ based on SELF introduced the best perform compared with the different estimates for all sample sizes with $\varphi = 2$ except $n = 20$.
- From table (4), Bayesian Shrinkage Estimation based on SLEF introduced the best perform compared with the different estimates for all sample sizes with $\varphi = 0.5,1$. to estimate $h(t)$ of BG . .

In general ,among the different estimators that record appearance as the best estimate ,we recommended to use the Bayesian Shrinkage Estimation based on SLEF .

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