Single Degree of Freedom Analysis Method forSteel BeamsUnder Blast PressureUsing Nonlinear Resistance Function with Strain Rate Effects

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Abstract

This paper presents and validates a single degree of freedom (SDOF)analysis method of steel beamssubjected to airs blast loadsinduced by explosions. The method usesa quasi-static non-linear resistance function of beams under uniformly distributed pressure.The non-linear resistance function has been implemented in a single degree of freedom analysis procedure to determine the nonlineardisplacement –time history of the steel beams subjected to air blast pressure.Strain rate effects have been accounted for usingthe well-known Cowper Symonds equation by differentiating the corresponding strain rate equations. Thesuggested SDOF analysis method was validated against the available experimental tests results using two W sections steel beams subjected to different values of the blast pressures and impulses. The validation results have indicated the accuracy and reliability of the suggested method to predict the nonlinear response of steel beamsunder transverse air blast pressure.

Key words: Blast pressure, SDOF method, steel beams,nonlinear response, resistance function, experimental tests

الخلاصة

يهدف هذا البحث الى عرض و تدقيق طريقة للتحليل الديناميكي للعتبات الفولاذية المعرضة الى ضغط عصف الهواء الناجم من الانفجارات وذلك باستخدام طريقة التحليل ذات درجة حرية احادية (SDOF) . تستخدم الطريقة المقترحة دالة مقاومة غير خطية للاحمال العرضية الموزعة بانتضام بالاعتماد على التحليل شبه الاستاتيكي و من ثم توضيف هذه الدالة في طريقة تحليل ديناميكي احادي درجة الحرية لايجاد علاقة غير خطية للازاحة الجانبية للعتبات الفولاذية مع الزمن . تم ادخال تاثير نسبة الانفعال ضمن التحليل الديناميكي وذلك باستخدام دالة Cowper Symonds equation المعروفة عالميا . تم التحقق من دقة الطريقة المقترحة وذلك بمقارنة النتائج المستحصلة من الطريقة المقترحة مع النتائج المختبرية المتوفرة لعتبتين فولاذيتين بمقاطع W و معرضة لقيم مختلفة من الضغط و النبضات الناجمة من العصف . بينت نتائج المقارنةدقة و موثوقية الطريقة المستخدمة في توقع و ايجاد التصرف غير الخطي للعتبات الفولاذية تحت الضغط العرضي لعصف الهواء الناجم من الانفجار.

الكلمات الدالة :ضغط الانفجار، طريقة SDOF، العتبات الفولاذية، الاستجابة غير الخطية، دالة المقاومة، النتائج المختبرية.

1. Introduction

During the last decades, many buildings and structural members have been severely damaged due to air blast and fire induces by explosions caused by terrorist attack.Sincethese terrorist attacks events are increasing at considerable rate, the awareness of civil engineering community has increased to the importance of developing strategies and design methods for blast protection of structures that are prone to such loading conditions.

The single degree of freedom (SDOF) analysis method has been proven to be a simpleand powerful tool for predicting the dynamic response of structural members under blast load with reasonable results. One of the major parameters that an equivalent single degree of freedom system relies uponis the resistance functionof the structural member (Biggs, 1964). The resistance function must accurately represent the nonlinear resistance-deflection behaviourof the selected degree of freedom for the actual system with accenting for the strain rate effects. However, for steel beams under blast load, no such nonlinear resistance function has yet been developed.

The UFC manual (USDOD, 2008) has suggested simple equations to calculate the elastic-plasticresistancefunctions of one-way and two-way structural members tobe used for a single degree of freedom systemof either simple or complex structures subjected to blast pressures as shown in Table 1. However, as it will be shown in this study, the suggested simple equations do not consider the nonlinear elastic-plastic behavior of the beam resistance which may result overestimated strength of the steel beams.

Table 1: Resistance functions of beams under uniform distributedblast pressure [USDOD, 2008]

Support condition	*Simply supp.*	*Fixed sup.*	*Fixed-pinned*
Elastic-Plastic (Ultimate) resistance, r_p	$r_p = \frac{8M_P}{L^2}$	$r_P = \frac{8(M_N + M_P)}{L^2}$	$r_P = \frac{4(M_N + 2I}{L^2}$

Cartaand Stochino(2013 and 2014)have recently presented studies inwhich a SDOF method wasused to assess the dynamic response of reinforced concretebeams subjected to uniformly distributed blast load. The elastic and plastic resistance functions were derived from the static equilibrium equations in conjunction withthe linear elastic bending theory.The strain rate effect was incorporatedinto the analysis by calculating the differentiation of the elastic and plastic strain-curvature relationships with respect to time.

(Shope ,2006)has developed a simplified model for a steel beam-column under static axial force subjected to a blast load using energy conservation principle with quasi-static approximation of the beam-column behavior. It has been assumed that the dynamic system behaves as a single degree of freedom model in an elastic perfectly plastic manner. However, the suggested method has not been validated against experimental data or numerical results.

(Nassr *et.al.*,2012)have also used the equivalent SDOF analysis to analyzethe response of steel beamsunder blast loads taking into account the material nonlinearity. The strain rate effect has been accounted for in the moment-curvature response of the steel beam using Cowper–Symonds equation to trace the nonlinear stress distribution over the section. It has been concluded that the assumption of a constant dynamic implication factor(DIF) as suggested by current designcodes may lead to conservative estimation of the dynamic effect of the blast pressure when compared to that incrementally calculated based on the updated strain rates during the analysis.

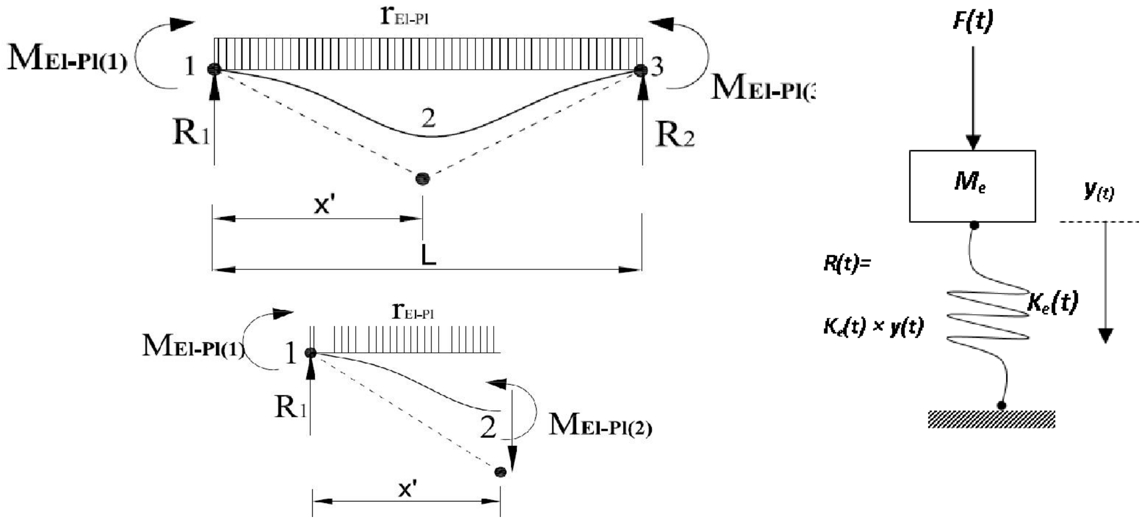
(Astarliogluaetal ,2013)haveemployed a single degree of freedom (SDOF) model to study the effect of axial compressive load on the resistance function of reinforced concrete (RC) columns subjected to axial load and blast induced transverse loads. The effects of flexural, diagonal shear, and tension membrane behaviours were also included in the column behaviour. It has been shown that the level of axial compressive load has a significant influence on the behaviour of RC columns when subjected to transverse blast-induced loads.

(Anderson and Karlsson ;2012) used SDOF system to design of reinforced concrete members subjected to explosions. In their SDOF model, Anderson and Karlsson have introduced the concept of time dependent transformation factors at which the transformation factors come from an assumed deformation shapewhich in turn governed by the wave propagation rather than form a fixed deformation shape. This concept is based on the energy conservation principle and makes the deformations in the SDOF approach more accurate.

It is the aim of this paper to present a single degree of freedom analysis method for steel beams subjected to transverse blast pressure. The development focuses on deriving a nonlinear resistance function of the steel beams under transvers pressure based on a quasi-static approximation of the steel beam behaviour taking into account the strain rate effects. Afterward, the developed analysis method will be validated against the availableexperimental test results

2.Modified single degree of freedom (SDOF)method.

Figure 1(A) showsa steel beam subjected to blast pressure. The steel beam can be idealized into an equivalent SDOF model asshown in**Figure 1** (B).



(A) **(B)**

Figure 1: Steel beam model (A) with the equivalent SDOF model (B)

The suggestedsimplified SDOF model shown in Figure 1consists of equivalentmass, M_e, equivalent resistanceR(t) and equivalent blast load F(t). The equivalent resistance is equal to equivalent stiffness (K_e) multiplied by the displacement ($y_{(t)}$). The damping effects were neglected in model because the duration of the blast event is very small compared to the natural time period of the column (Thilakarathna *et.al.*, 2010).

The accuracy of the SDOF model significantly depends on the accuracy of the adopted resistance function R(t). The resistance function must simulatethe actual behaviour of the columntaking into account the nonlinear elastic-plastic behavior of the steel material. In the following sections, a modified nonlinear resistance function of the steel beamof the SDOF systemwill be derived. The modified resistance function allows for tracing the full elastic-plastic response of the steel beam.

2.1 Derivations of the resistance functions

Figure 1 (A) shows the assumed elastic- plastic deformation shape of the steel beam under transverse blast pressure.For simple material modelling, the elastic-perfectly plastic behaviour can be assumed with yielding being concentrated at the plastic hinge locations.The equilibrium equationsof the beam are derived based on the quasi-static state(Al-Thairyand Wang, 2011).It is further assumed that the steelbeamhas a compact cross section andthe elastic deformed shape of the beamfollows the beamelastic buckling shape. The intermediate plastic hinge location was assumed to beclose to the position of the maximum lateral displacement of the beam.

Replacing the dynamic blast load by a nominal elastic-plastic quasi-static resistance unite (load/unite length), r_{El-Pl}, and referring to Fig.1(A- top), the reaction of the column at end 1, (R_1)can be determined by assuming the quasi-static equilibrium condition and taking moment about end 3:

$$R_1 = \frac{r_{El-Pl} \times L^2 / 2 - M_{El-Pl(1)} + M_{El-Pl(3)}}{L} \quad ....1$$

The relationship between the equivalent elastic-plastic quasi-static resistance, r_{El-Pl},and the intermediate plastic hinge location (x'),can be determined by the quasi-static moment equilibrium condition of Fig.1(bottom) as follows:

$$R_1 \times x' - r_{El-Pl} \times x'^2 / 2 + M_{El-Pl(1)} - M_{El-Pl(2)} = 0 \quad ....2$$

Substituting the value of $R_1$from Eq.1into Eq.2and solving for r_{El-Pl} gives the following equation:

$$r_{El-Pl} = \frac{2\left(M_{El-Pl(1)} (\frac{x'}{L} - 1) + M_{El-Pl(2)} - \frac{M_{El-Pl(3)}}{L} x' \right)}{Lx' - x'^2} \quad ....3$$

By calculating the values of $M_{El-Pl(1)}$, $M_{El-Pl(2)}$ and$M_{El-Pl(3)}$ corresponding to each lateral displacement of the steel beam behaviour, Eq.3can be used to determine the equivalent nonlinear elastic –plastic resistance functions against thetransverse blast load.

2.2 Derivation of the elastic-plastic bending resistance about major (x-axis)

Figure 2 (A) shows I shaped steel section subjected to bending moment causes bending about the major (x-x) axis of the section. The section is assumed to be in the elastic-plastic phase of deformation where strains at top and bottom layers of the section are beyond the yielding strainlimitof steel $\geq \varepsilon_y$ whereas strains at other layers are still elastic $< \varepsilon_y$. The extend of yielding over the cross section depth in compression and tension zones are denoted here as y_c and $y_t$respectively calculated from the top and bottom layers of the steel section respectively. The strains at the top and bottom layers of the elastic part of the section,corresponding to each value of transverse displacement (u+Δu), can be calculated as a function of the elastic curvature, section depth, and depth of yielding at compression and tension zonescorresponding to the previous value of transverse displacement (u),using the elastic bending theory as follows, seeFigure 2(A).

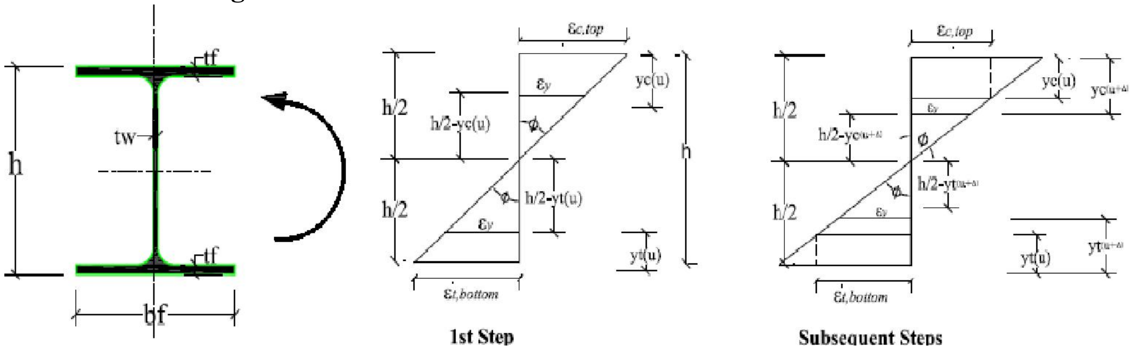
$$\varepsilon_{Top\,(u+\Delta u)} = \phi \times (h/2 - y_{top\,(u)}) \leq \varepsilon_f \quad ....4$$

$$\varepsilon_{Bottom(u+\Delta u)} = -\phi \times (h/2 - y_{bottom(u)}) \geq -\varepsilon_f \quad ....5$$

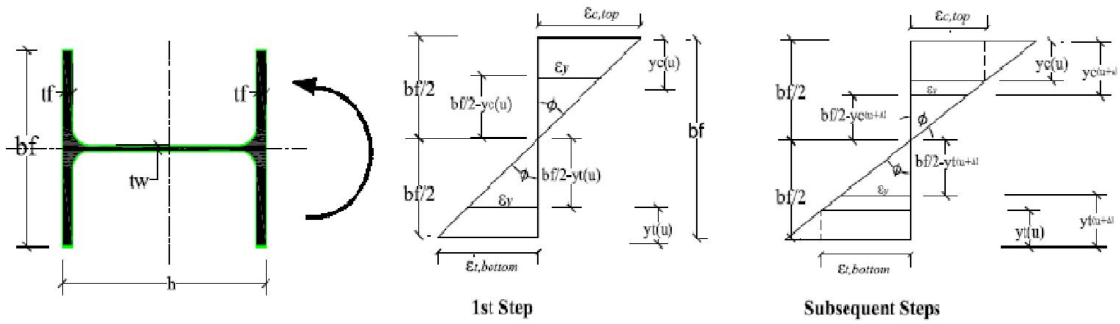
Where u is the accumulative value of the transverse displacement at the previous analysis step; Δu is the incremental value of the transverse displacement used in the analysis; *h* is the cross section depth; $y_{top(u)}$, $y_{bottom(u)}$ are depths of yielding calculated from the top and bottom layers of the section respectively atthe previous analysis step (u) as shown in Figure 2(A); ε_f is the fracture strain of the steel material; ϕ is the elastic curvature of the beam at the location where moment is to be calculated. ϕ is calculatedfor the elastic part of the elastic- plastic section using the elastic bending theory by the equation:

$$\phi = \frac{\partial^2 u(x)}{\partial x^2} \qquad \text{...6}$$

Where$u_{(x)}$ isthe lateral displacement of the beam at the distance (x) along its axis.Assuming that the displacement shape of the steel beam during the elastic phase is as shown in **Figure 3** and **Table2.**



A. bending about major (x-axis)



B. bending about minor (y-axis)

Figure 2: Strain distribution over a cross section of the steel column

Eq. 6 must be evaluated for the locations of $M_{El-Pl(1)}$, $M_{El-Pl(2)}$ and $M_{El-Pl(3)}$ in Eq.3. For example, for the simplysupported steel beam, the equation of elastic displacement shapes is as follows (Timoshenko and Gere, 1961):

$$u_{(x)} = U \times Sin(\lambda x) \qquad \text{... 7}$$

Where $\lambda = \frac{\pi}{L}$

SubstitutingEq. (7) into Eq. (6), the elastic curvature at locations (x=0 for $M_{El-Pl(1)}$, x=0.5$L$for $M_{El-Pl(2)}$ and x=$L$for$M_{El-Pl(3)}$) are:

$$\phi_{x=0} = \left(\frac{\partial^2 u(x)}{\partial x^2}\right)_{x=0} = 0 \qquad \text{...8}$$

$$\phi_{x=0.5L} = \left(\frac{\partial^2 u(x)}{\partial x^2}\right)_{x=0.5L} = -\frac{\pi^2}{L^2}U \quad \text{.......9}$$

$$\phi_{x=L} = \left(\frac{\partial^2 u(x)}{\partial x^2}\right)_{x=L} = 0 \quad \text{.......10}$$

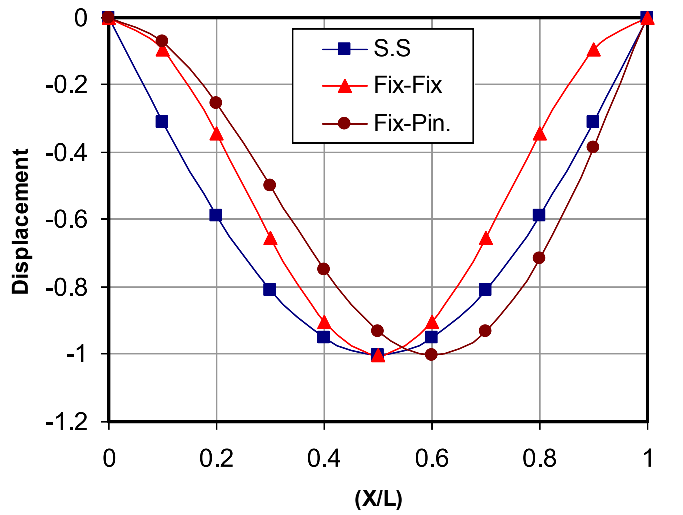


Figure 3: Assumed displacement shapes of steel beams for three cases of boundary conditions

The yieldingof the section is initiated when values of $\varepsilon_{Top\,(u+\Delta u)}$ and $\varepsilon_{Bottom\,(u+\Delta u)}$ calculated from Eqs.4 and5 exceed the yielding strain, ε_y . Values of y_{top}, y_{bottom}arecalculated accumulatively according to the following incremental equations derived using trigonometric symmetry assuming a linear strain distribution over the elastic part of the section; see Figure 2 (A).

$$y_{top(u+\Delta u)} = y_{top(u)} + (h/2 - y_{top(u)}) \times \left(1 - \frac{\varepsilon_y}{\varepsilon_{Top\,(u+\Delta u)}}\right) \quad \text{.......11}$$

$$y_{bottom(u+\Delta u)} = y_{bottom(u)} + (h/2 - y_{bottom(u)}) \times \left(1 - \frac{\varepsilon_y}{\left|\varepsilon_{bottom(u+\Delta u)}\right|}\right) \quad \text{.......12}$$

Eqs. 11and 12can be used to trace the gradual spread of yielding over the cross section depth of the steel beam due to the increasing of the transverse displacement(u+Δu).

The equation for theelastic-plastic bending resistance of a beam-column section at locations 1, 2 and 3 in **Figure 1**corresponding to each value transverse displacement (u+Δu) can be calculated by summing up the bending resistance of all the layers overthe section depth as follows, see Figure 2(A):

$$M_{El-Pl} = \sum_{0}^{h/2} (\varepsilon_1 \times E) \times \Delta h \times b \times [h/2 + \kappa - \Delta h] + \sum_{0}^{h/2} (\varepsilon_2 \times E) \times \Delta h \times b \times [h/2 - \kappa + \Delta h]$$

.......13

Where $\varepsilon_{1,}\varepsilon_2$ are values of strainsateach layer of compression and tension zones of the section respectively, andΔh is the selected layer depth which must be small enough to give accurate results. Values of $\varepsilon_{1,}\varepsilon_2$ can be calculated from the following equations assuming a linear strain distribution over the cross section depth as shown in Figure 2(A):

$$\varepsilon_1 = \phi \times \kappa_1 \times U \leq \varepsilon_y \quad \text{.......} 14$$

$$\varepsilon_2 = -\phi \times \kappa_2 \times U \geq -\varepsilon_y \quad \text{.......} 15$$

κ isthe distance from theneutral axis (N.A.) to the corresponding layer in the compression and tension zones respectively;bis the layer width which is taken according to the location of the layer over the cross section as follows, see Figure 2 (A):

b=t_w.......................when $h/2 - t_f \geq \kappa \geq 0$ 16

b=b_f ....when $h/2 \geq \kappa \geq h/2 - t_f$ 17

2.3 Derivation of the elastic-plastic bending about minor axis (y-axis)

Using same procedure described above, the bending resistance of the steel beam section about y-axis can be determined. However, two adjustmentshave to be made to account for the change in the direction of bending; these are,see Figure 2(B):

1. The dimension **h i**n Eqs. (4-5 and 11-17) must be replaced by the dimension **b_f**.
2. The bending resistance equation is calculated using the following equation.

$$M_{El-Pl} = \left[\sum_0^{b_f/2} (\varepsilon_1 \times E) \times \Delta h \times 2t_f \times (\mathrm{b_f}/2 + \kappa - \Delta h) + \sum_0^{b_f/2} (\varepsilon_1 \times E) \times \Delta h \times 2t_f \times (\mathrm{b_f}/2 - \kappa + \Delta h) \right]$$
$$+ (\varepsilon_{cw} \times E) \times t_w \times (h - 2t_f) \times b_f / 2 \quad \text{.......} 18$$

Where ε_{cw} is the compressive strain at the web calculated by:

$$\varepsilon_{cw} == \frac{|\varepsilon_2| \times (\mathrm{y_c} - \mathrm{b_f}/2) + \varepsilon_1 \times (\mathrm{b_f}/2 - \mathrm{y_t})}{(\mathrm{b_f} - \mathrm{y_c} - \mathrm{y_t})} \quad \text{.......} 19$$

Figure 4 shows the elastic-plastic moment resistance of steel beamwhen it is bent bout major and minor axes (i.e. Eq. 13and Eq.18).

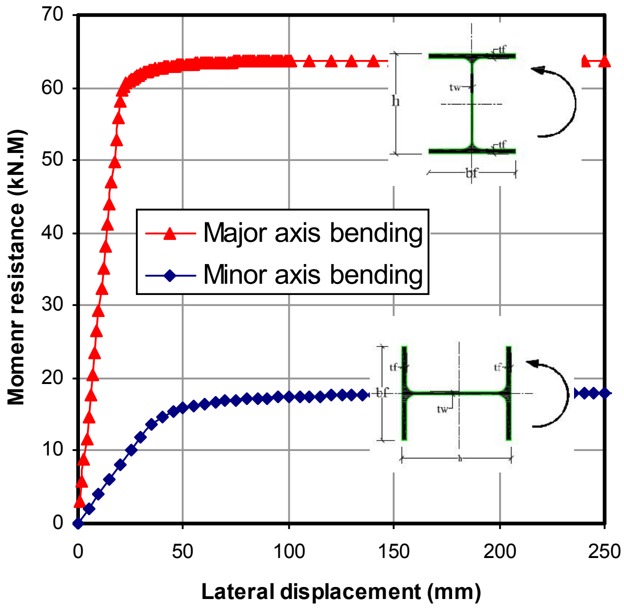


Figure 4: Elastic Plastic moment –displacement relationship ofsteel beams sections bent about major and minor axes.

Finally, substituting$M_{El\text{-}Pl(1)}=0$, $M_{El\text{-}Pl(2)}=M_{El\text{-}Pl}$ and $M_{El\text{-}Pl(3)}=0$and x'=0.5L in Eq.3gives the following equationfor the elastic-plastic resistance function for the simply supported beam:

$$r_{El-Pl}=\frac{2M_{El-Pl}}{(0.5-0.25)L^2}=\frac{8.0M_{El-Pl}}{L^2} \qquad \text{......20}$$

The elastic-plastic resistance functions for any boundary conditions can be derived using the same above procedure.

Table2: Assumed elastic -displacement shape($u_{(x)}$) and intermediate plastic hinge location (x')of steel beam

B.C.	Elastic displacement equation	(x')
S-S	$u_{(x)}=U\times Sin(\lambda x)$	$0.5L$
F-F	$u_{(x)}=\frac{U}{2}\times[1-Cos(2\lambda x)]$	$0.5L$
H-F	$u_{(x)}=\frac{U}{6.3}\times\left[Sin(1.432\lambda x)-1.432.\lambda LCos1.432\lambda.x+1.432.\lambda L(1-\frac{x}{L})\right]$	$0.58L$

Where $\lambda=\frac{\pi}{L}$

Figure 5demonstrates the nonlinear elastic-plastic behavior of the resistance function of asimply supported beamaccording to Eqs. 20

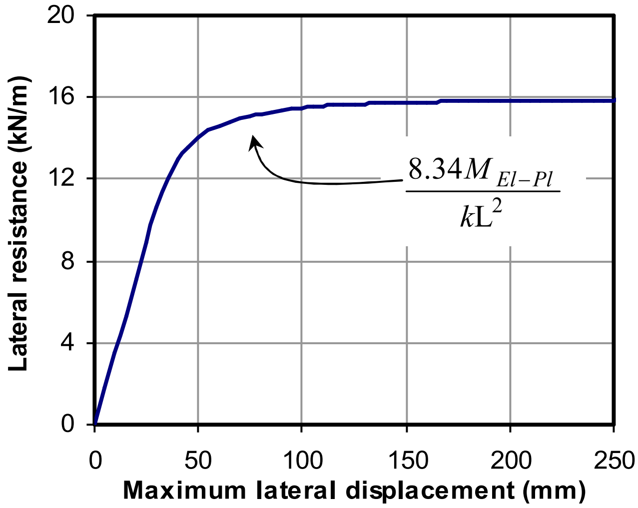


Figure 5: Nonlinear elastic-plastic behavior of the resistance function of a simply supported beam

2.2Equivalent SDOF system parameters

The total mass $M_t$and stiffness $K_t$of a steel beam with a uniform cross section along with the blast pressure $P_t$can be idealized to equivalent mass M_e, equivalent stiffness and equivalent load P_e respectively fora SDOF model bythe following equations(Biggs, 19964, USDOD, 2008 and ,Timoshenko and Gere, 1964):

$$M_e = M_t \int_0^L u^2(x)\, dx \quad \text{.......} 21$$

$$K_e = K_t \int_0^L u(x)\; dx \quad \text{.......} 22$$

$$P_e = P_t \int_0^L u(x)\; dx \quad \text{.......} 23$$

Where $u(x)$ is the elastic and plastic shape functions of the steelbeam which should satisfy loading and boundary conditions.The elastic deflection shapeand the linear plastic shapes of the beamwere used in the present study for the elastic and plastic phases respectively.The integrals $\int_0^L u^2(x)\, dx$and $\int_0^L .u(x).$ are referred to as the mass transformation factor, $K_M$and load transformation factor,K_L respectively.. However, it is common that both transformation factors are put together in one factor which can be used in an equation of motion and is referred to as the load-mass transformation factor,K_{LM} using the following integral:

$$K_{LM} = \frac{\int_0^L u^2(x)dx}{\int_0^L u(x)dx} \quad \text{.......} 24$$

The integral $\int_0^L u^2(x)dx \Big/ \int_0^L u(x)dx$ was evaluated for the assumed elastic and plastic deformation shapes of the steel beam for the three cases of boundary conditions listed in Table 1.Table 3presents the calculated results.

Table 3: Elastic and plastic equivalent mass of the steel column

B.C.	S-S	F-F	H-F
K_{LM} (Elastic)	0.78	0.77	0.78
K_{LM} (Plastic)	0.66	0.66	0.66

2.3The idealized blast load-time relationship

The blastload-time function generated byexplosions canbe idealized to a blast load -time function to be used in the SDOF analysis as shown in Figure 6According to Figure 6, the idealized load-time function,$P_{(t)}$can be expressedbythe following function derived by assuming a triangular impulse shape:

$$P(t) = P_{\max}\left(1 - \frac{t}{t_d}\right) 0<t<t_d \text{......} 25$$

$$P(t) = 0 \; t_d<t<T \text{......} 26$$

Where$P_{max}$is the maximum value of the blast load; t_d is the positive phase time duration of the blast load-time history; T is the total time required for the analysis and $t$is the time at which the blast load is to be calculated.

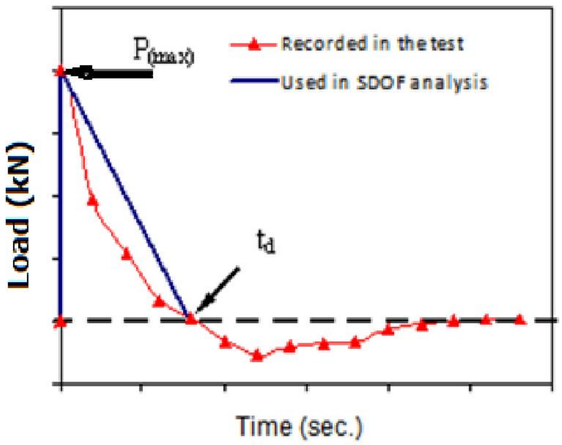


Figure 6: The idealized blast load-time function used in the equivalent SDOF model

2.4The dynamic equation of motion

The dynamic equation of motion intheequivalent SDOF system of steel beam subjected to lateral blast pressure can be expressed as follows:

1. For the elastic and elastic- plastic response:

$$M_e\ddot{U}(t) + r_{el-pl}L = P(t) \; 0<U(t)<U_{ult} \text{......} 27$$

Where U_{ult} is the required ultimate displacement at which the analysis is to be stopped.

2. When unloading or displacement rebound occurs, the following equation may be used to determine the beam response:

$$M_e\ddot{U}(t) + r_{el-pl(u_R)}L - r_{el-pl(u_R-u)}L = P(t)\ U(t) < U_R \qquad 28$$

Where U_R is the displacement at which unloading occurred.

When a reloading occurs after unloading, Eq. 28can be used for $U(t) < U_R$ and Eq. 27can be used for $U_R < U(t) < U_{ult}$

As mentioned earlier, the damping termhas been neglected in the above equation becauseit has a minor effect on the behaviour of steel beam subjected to a blast load with very short time duration compared to the natural period of the structure's system (Thilakarathna *et.al.,* 2010, Jones, 1997).Figure 7shows the three phases of steel column behaviour.

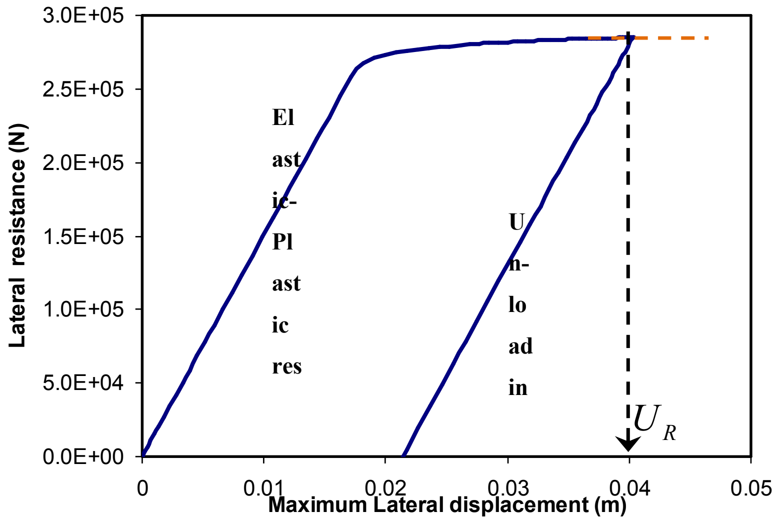


Figure 7: Generalized non-linear resistance-displacement relationship of steel beams

2.6Calculation ofstrain rate effect

Strain rate has a considerable effect on the material behaviour of steel members subjected to short duration dynamic loading (Cowper and Symonds, 1957).The strain rate effect has been accounted for in the present researchby calculating the dynamic amplification factor (DIF) which is determined using the following constitutive relationshipsuggested by Cowper-Symonds [14]:

$$DIF = 1 + (\frac{\dot{\varepsilon}^{pl}}{D})^{\frac{1}{n}} \quad \text{...29}$$

Where $\dot{\varepsilon}^{pl}$ is the uniaxial plastic strain rate.***D*** and ***n***arematerial parameters which can be obtained from the uniaxial compression test [Jones, 1997, Cowper and Symonds, 1957].For structural steel, values of *D* and *n*are suggested to be D=40.4 and n=5 data [ones, 1997].

Hence, the dynamic yield stress (F_{yd}) at each time step of the SDOF analysis can be determined by multiplyingthe static yield stress (F_y)by the dynamic amplification factor (DIF)

The equivalentplastic strain rate $\overset{\bullet}{\varepsilon}^{pl}$ can be calculated by numerical differentiating of the axial strain equation (Eqs. 4and5) with respect to time as follows:

$$\overset{\bullet}{\varepsilon}_{c,Top} = \frac{\Delta\varepsilon_{c,Top}}{\Delta t} = \frac{[\phi\times(h/2-y_{top\,(u)})]_{t+\Delta t} - [\phi\times(h/2-y_{top\,(u)})]_{t}}{\Delta t} \qquad ....30$$

$$\overset{\bullet}{\varepsilon}_{t,Bottom} = \frac{\Delta\varepsilon_{t,Bottom}}{\Delta t} = \frac{[\phi\times(h/2-y_{bottom\,(u)})]_{t+\Delta t} - [\phi\times(h/2-y_{bottom\,(u)})]_{t}}{\Delta t} \qquad ....31$$

Up to the yielding point, the strain rate effect has no effect (DOF=1). After yielding, the plastic strain rate iscalculated using Eqs.30 and 31at each time increment and the dynamic yield stress is evaluated accordingly.Figure 8 shows the relationship between the dynamic amplification factor and the plastic strain rate ($\overset{\bullet}{\varepsilon}^{pl}$).

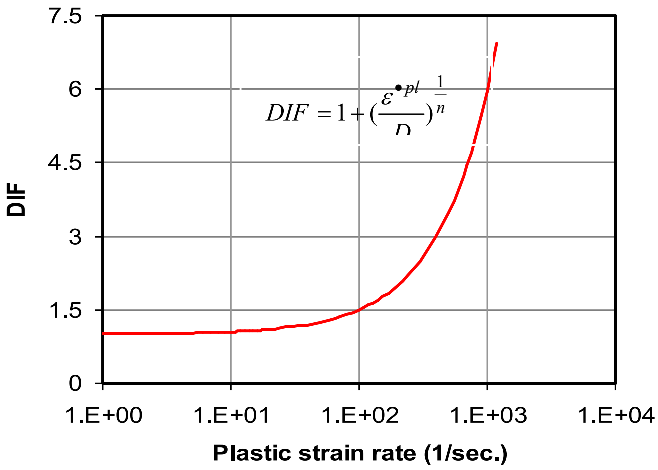


Figure 8: Dynamic amplification factor (DIF) as a function of the plastic strain rate ($\overset{\bullet}{\varepsilon}^{pl}$)

3.Validation of the SDOF method.

The suggested SDOF model was validated against the experimental tests' results of Nassret. al (Nassr *et.al.,*2012,Nassr.,2012)who have conducted experimental tests on full scale simply supported W-shape steel beamsusing two different section sizes, W150×24 and W200×71, and a total length of 2.413m. The beams were subjected to a direct explosion to cause bending about major and minor axes of the columns. Table 4shows geometrical properties and blast pressure parameters of the test cases used in the validation examples.

Table 4: Geometrical properties andblast loads parameters of steel beamsused inthe validation examples (**Nassr *et. al*, 2012**).

Test case	*Section*	*Blast direction*	*KL/r*	*P_{max} (N/mm^2)*	*t_d (ms)*	*I ((N/m^2)×sec.)*
1	W150×24	x-x	36.5	0.307	7.3	715
2	W150×24	y-y	98.1	0.623	6.0	1279

3	W150×24	x-x	36.5	1.560	6.2	2130
5	W200×71	x-x	26.3	2.098	8.4	3144

P_{max}= Maximum blast pressure, t_d= Positive blast pressure time duration,

The test beams shown in Table 4 have been analysed using the SDOF analysis method presented in previous sections. A numerical and incremental analysis procedure has been used to solve the dynamic equations of motions (Eq. 27 and 28) to determine the beam response, $y_{(t)}$ at each time increment (Δt).The results of analysis has been validated against experimental test results of Nassir (Nassr*et.al.,* 2012,Nassr., 2012) and the comparison results are shown in Figures 10 to 14.

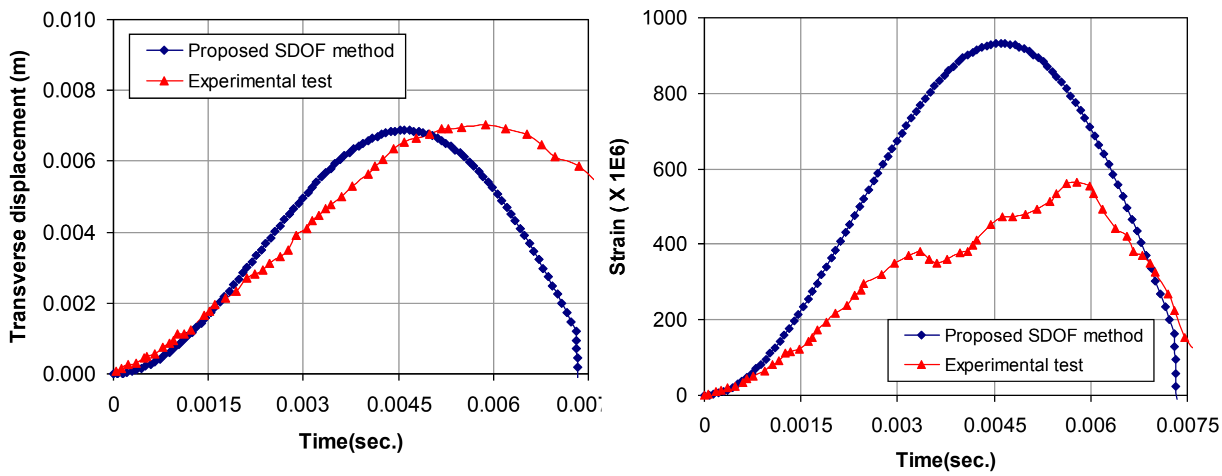


Figure 9:Comparison between the proposed SDOF method and the experimental testresults of the test caseNo.1(Nassr *et. al,* 2012,Nassr., 2012)

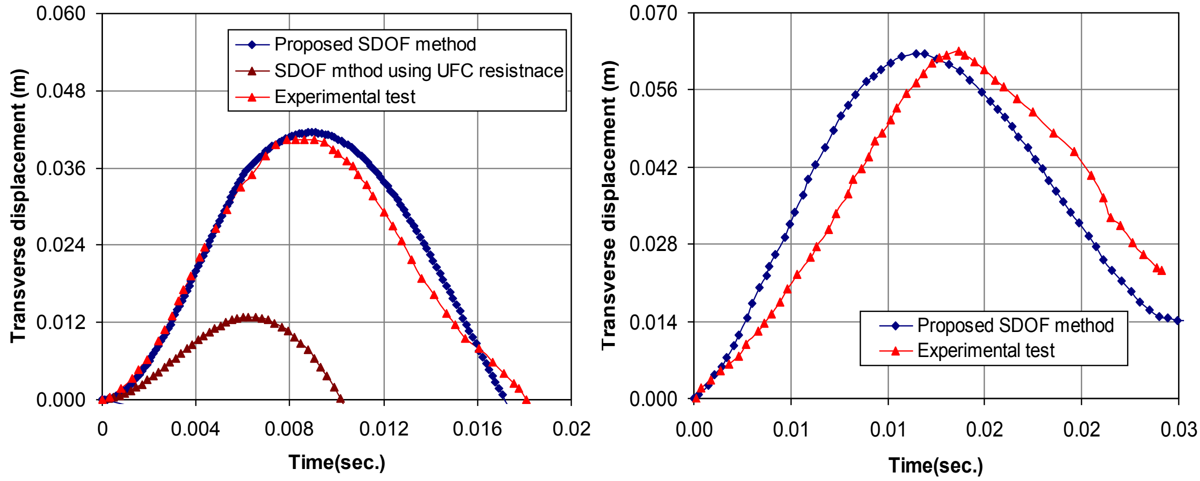


Figure 10: Comparison between the proposed SDOF method and the experimental test results of the test case No. 2 (A) and test case No. 5(Nassr *et. Al.*, 2012,Nassr., 2012)

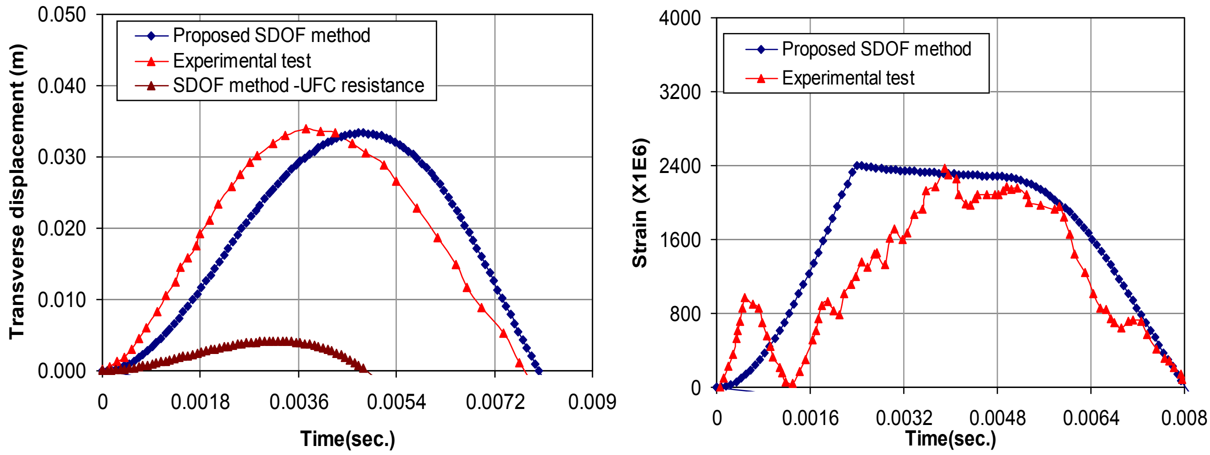
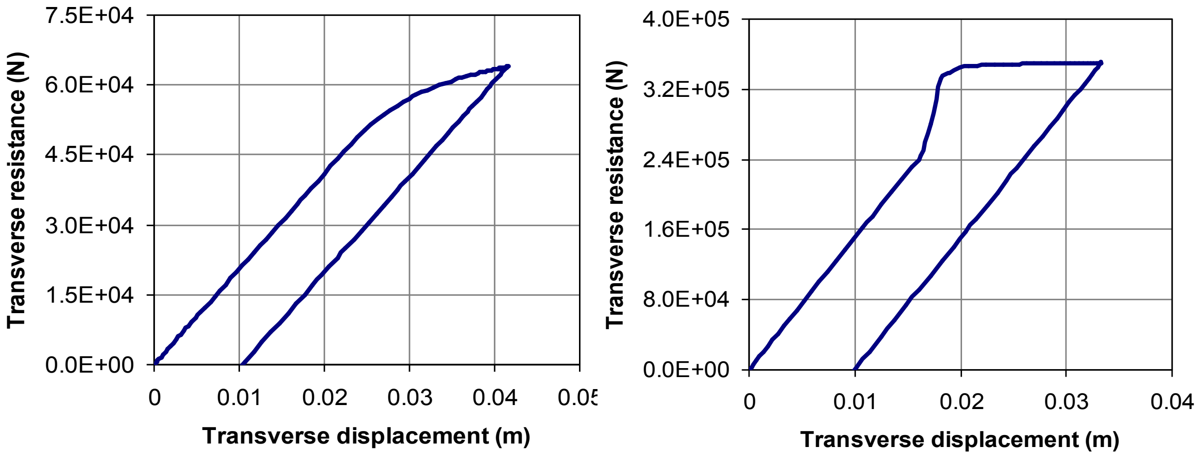


Figure 11Comparison between the proposed SDOF method and the experimental test results of the test case No.3(Nassr *et. al.*, 2012,Nassr., 2012)



(A) (B)

Figure 12: Resistance function-mid span displacement behavior of the steel beam.(A) test case 2;(B) test case 3

Figs. 10 to 12 show the displacement and the strain time histories of the tested beams. Reasonable agreement can be seen between the experimentally recorded (Nassr, 2012-2) and the analytically calculated results. The experimental test results have shown that test beam 1 has experienced elastic displacement while the tested beams 2, 3 and 5 have shown inelastic displacement (Nassr *et. al,* 2012). It is clearly shown from Fig. 10-12 that the suggested SDOF method has reasonably captured the elastic and plastic nonlinear behaviour of all tested beams as shown from the nonlinear resistance behaviour of these cases (Fig 13(A, B and C). However, the extracted strain time history in Figs. 10(B) has shown some divergence compared with the experimental results. Nevertheless, as the beam response is more important for design purposes, the divergence in strain does not affect the reliability and applicability of the suggested SDOF method.

Figs. 11(A) and 12(A) show a comparison of the mid-span displacement history of test beams 2 and 3 calculated form SDOF analysis when using a linear resistance function suggested by UFC-304-2 (USDOD, 2008). It can be seen from these figures that a much lower displacement values were obtained when using the linear simple resistance functions compared with the experimental results and compared with the displacement values obtained using the suggested nonlinear resistance functions which may results overestimate the beam strength.

On the other hand, Fig.14) shows the calculated lateral displacement time history when the strain rate effect was not accounted for in the SDOF analysis of test beam No.3. The figure has clearly demonstrated that when strain rate effect is no included in the analysis, the beam behaviour becomes more flexible and the SDOF method gives high values of the beam maximum displacement compared with the experimental results.

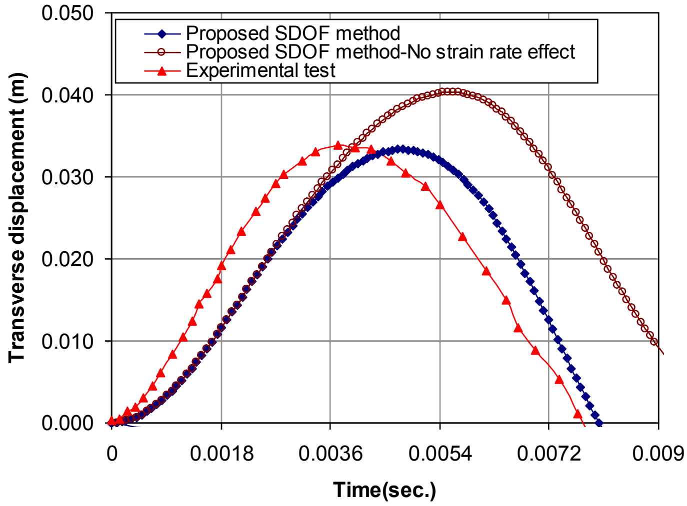


Figure 13:Comparison of the transverse mid span displacement time history of the tested case No.3between the proposed equations (with and without the inclusion of the strain rate effects)and the experimental testresults(Nassret. al, 2012)

4.Conclusions

The presentstudy has suggested and validated a single degree of freedom method for the analysis of steel beamssubjected to lateral blast loads. The suggested method has employeda non-linear elastic-plastic resistance function ofsteel beams under transverse blast loadderived based on a quasi-static approximation of the beam behaviour taking into accountthe strain rate effect.The accuracy of the nonlinear resistance function was validated against experimental tests resultsusing two simply supported W-shape steel beamssubjected to a different blast pressures to cause bending about major and minor axes of the beam. The validation results haveshown the validity of the suggested method to capturethe response steel beams under transverse blast load.The following conclusions may be extracted from the present study:

1.The nonlinear behaviour of the resistance function of steel beams under transverse blast has a remarkable effect on the beam response.
2.The linear equations of the resistance function suggested by the currents standards and codes to be used in the SDOF analysis do not represent the realistic behaviour of steel beams resistance to transverse blast. Using such equations in the SDOF analysis resultsin inaccurate behaviour of the beam and leads to underestimate the beam displacement which may be unsafe for the design purpose.
3.Strain rate behaviour of steel material has a considerable effect on the dynamic response of steel beams and must not be neglected in the SDOF analysis of steel members subjected to dynamic loads during shorttime duration.

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