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Strongly C-Lindelof Spaces

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Abstract: In this paper, we define another type of Lindelof which is called strongly c- Lindelof, and we introduce some properties about this type of Lindelof and the relationships with Lindelof , c- Lindelof and strongly Lindelof spaces.

Keywords: Lindelof , Strongly Lindelof , C- Lindelof Spaces

Introduction:

A topological space (X,τ) is said to be Lindelof space if and only if every open cover of X has a countable subcover [1]. A topological space (X,τ) is said to be c- Lindelof if and only if each closed set $A \subseteq X$, each open cover of A contains a countable subfamily W such that $\{\mathrm{cl}\,U : U \in W\}$ covers A [2].Mashhour et.al.[3] introduced preopen sets , a subset A of a space X is said to be preopen set if $A \subseteq \mathrm{int}\,(\mathrm{cl}(A))$ .Also they defined the following concepts :

1. A is called a preclosed set if and only if $(X - A)$ is preopen set.
2. The intersection of all preclosed sets in X which contain A is called the preclosure of A and
denoted by $\text{pre-cl}\,A$.
3. The prederived set of A is the set of all elements x of X satisfies the condition, that for every preopen set U contains x ,implies $U \setminus \{x\} \cap A \neq \phi$.

A topological space (X,τ) is called strongly Lindelof space if and only if every preopen cover of X has a countable subcover [2]. In this paper, we introduce the concept of strongly c-

$X = \bigcup_{i\in\Delta\subset N} \{\mathrm{cl}\,U_{\alpha_i} : U_{\alpha_i} \in W\}$.This means ,there is $x \in X$ such that $x \in \mathrm{cl}\,U_{\alpha_i}$,but $x \notin U_{\alpha_i}$ for some $i \in \Delta \subset N$.

Then $x \in U'_{\alpha_i}$ where U'_{α_i} is the derived set of U_{α_i} .Since X is a T_1- space then $\{x\}$is a closed subset of X and since $x \notin U_{\alpha_i}$,then $y \notin \{x\}\ \forall\ y \in U_{\alpha_i}, i \in \Delta \subset N$ .By regularity of X ,there are two open sets V_y and V_y^* such that $y \in V_y$, $\{x\} \subset V_y^*$ and $V_y \cap V_y^* = \phi$ for each $y \in U_{\alpha_i}$ .Now ,put $V = \bigcup_{y\in U_{\alpha_i}} V_y$,then V is an open set contains U_{α_i} .So we have V_y^* is an open set containing x such that $V \cap V_y^* = \phi$,therefore, $x \notin U'_{\alpha_i}$ which is a contradiction .Then X is Lindelof space.

Lindelof space. A topological space (X,τ) is called strongly c- Lindelof space if and only if for every preclosed set $A \subseteq X$,each preopen cover $\{U_\alpha : \alpha \in \Delta\}$ of A contains a countable subfamily W such that $\{\text{preclosure}\ U_\alpha : U_\alpha \in W\}$ covers A, we study some properties of this kind of Lindelof space. We also study the relationships among Lindelof spaces, c- Lindelof spaces, strongly Lindelof spaces and strongly c- Lindelof spaces.

Remark

Every strongly Lindelof space is Lindelof space .

Proof:

Let (X, τ) be a strongly Lindelof space and let $\{U_\alpha : \alpha \in \Delta\}$ be an open cover of X .Since each open set in X is a preopen set , then $\{U_\alpha : \alpha \in \Delta\}$ is a preopen cover of X which is strongly Lindelof space.Therefore,there exists a countable number of $\{U_\alpha : \alpha \in \Delta\} \ni$ the family $\{U_{\alpha_i} : i \in \Delta \subset N\}$ covers X . Hence every open cover of X has a countable subcover , therefore X is Lindelof space.

Remark

Every Lindelof space is c- Lindelof space .

Proof:

Let (X, τ) be a Lindelof space and let $A \subseteq X$ be any closed subset of X. Let $\{U_\alpha : \alpha \in \Delta\}$ be an open cover of A. Since A is a closed subset of X , then $X - A$ is an open subset of X , so $\{X - A\} \cup \{U_\alpha : \alpha \in \Delta\}$ is an open cover of X which is an Lindelof space. Therefore,there exists a countable number of $\{U_\alpha : \alpha \in \Delta\}$ such that $\{X - A\} \cup \{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$ is a countable subcover of $\{U_\alpha : \alpha \in \Delta\}$ for X .Since $A \subseteq X$ and $X - A$ covers no part of A , then $\{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$ is a countable subcover of A .Put $W = \{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$, then it is clear that W is a countable subfamily of $\{U_\alpha : \alpha \in \Delta\}$ such that $\{\mathrm{cl}\, U_{\alpha_i} : U_{\alpha_i} \in W\}$ covers A .Hence X is c – Lindelof space.

Definition [1]

A topological space X is a regular space if and only if whenever A is closed in X and $x \notin A$,then there are disjoint open sets U and V with $x \in U$ and $A \subset V$ .A space X is said to be a $\mathrm{T}_3 -$ space if and only if it is regular and $\mathrm{T}_1 -$space.

Remark

Every c- Lindelof and T_3 - space is Lindelof space .

Proof:

Let (X, τ) be a T_3 - c- Lindelof space. Assume X is not Lindelof space , then there is an open cover $\{U_\alpha : \alpha \in \Delta\}$ for X which has no countable subcover .Since X is c- Lindelof , then there is a countable subfamily $W = \{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$ of $\{U_\alpha : \alpha \in \Delta\}$ such that

countable subcover of X .Since $A \subseteq X$ and $X - A$ covers no part of A , then $\{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$ is a countable subcover of A .Put $W = \{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$, then it is clear that W is a countable subfamily of $\{U_\alpha : \alpha \in \Delta\}$ $\ni$ $\{\text{pre-cl}\, U_{\alpha_i} : U_{\alpha_i} \in W\}$ covers A .Hence X is strongly c – Lindelof space.

Proposition

Every strongly c-Lindelof and T_3 - space is strongly Lindelof space .

Proof:

Let (X, τ) be a T_3 strongly c- Lindelof space. Assume X is not strongly Lindelof space , then there is a preopen cover $\{U_\alpha : \alpha \in \Delta\}$ for X which has no countable subcover. Since X is strongly c- Lindelof , then there is a countable subfamily $W = \{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$ of $\{U_\alpha : \alpha \in \Delta\}$ such that $X = \bigcup_{i \in \Delta \subset \mathrm{N}} \{\text{pre-cl}\, U_{\alpha_i} : U_{\alpha_i} \in W\}$.This means ,there is $x \in X$ such that $x \in \text{pre-cl}\, U_{\alpha_i}$,but $x \notin U_{\alpha_i}$ for some $i \in \Delta \subset \mathrm{N}$ .Implies $x \in \text{pre-derived}\, U_{\alpha_i}$. Since X is a T_1- space then $\{x\}$ is a closed subset of X and since $x \notin U_{\alpha_i}$,then $y \notin \{x\}\ \forall\ y \in U_{\alpha_i}, i \in \Delta \subset \mathrm{N}$ .By regularity of X ,there are two open sets V_y and V_y^* $\ni y \in V_y$, $\{x\} \subset V_y^*$ and $V_y \cap V_y^* = \phi$ for each $y \in U_{\alpha_i}$. Now ,put $V = \bigcup_{y \in U_{\alpha_i}} V_y$,then V is an open set contains U_{α_i} .So we have V_y^* is an open set containing x such that $V \cap V_y^* = \phi$,therefore, $x \notin \text{pre-derived}\, U_{\alpha_i}$ which is a contradiction .Then X is strongly Lindelof.

Note : From remark (1.1) and remark (1.4) we have every strongly Lindelof space is a c- Lindelof space.

Theorem [2]

If the set of accumulation points of the space X is finite ,then X is strongly Lindelof ,whenever it is Lindelof space.

Theorem

Every c- Lindelof and T_3 - space is strongly Lindelof space, whenever the set of accumulation points of X is finite.

Proof:

Let (X, τ) be a T_3-c-Lindelof space such that the set of accumulation points of X is finite. Remark (1.3) gives X is Lindelof and theorem (1.5) gives X is strongly Lindelof.

Strongly C- Lindelof Spaces:

In this section, we give the definition of strongly c-Lindelof, and we also study the relationships among Lindelof spaces, c- Lindelof spaces, strongly Lindelof spaces and strongly c- Lindelof spaces.

Definition

A topological space (X, τ) is called strongly c-Lindelof space if and only if for every preclosed set $A \subseteq X$,each preopen cover $\{U_\alpha : \alpha \in \Delta\}$ of A contains a countable subfamily W such that $\{\text{preclosure } U_\alpha : U_\alpha \in W\}$ covers A.

Remark

Every strongly Lindelof space is strongly c- Lindelof space .

Proof:

Let (X, τ) be a strongly Lindelof space and let $A \subseteq X$ be any preclosed subset of X. Let $\{U_\alpha : \alpha \in \Delta\}$ be an preopen cover of A.Then $\{X - A\} \cup \{U_\alpha : \alpha \in \Delta\}$ is a preopen cover of X which is strongly Lindelof space . Therefore,there exists a countable number of $\{U_\alpha : \alpha \in \Delta\}$ such that $\{X - A\} \cup \{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$ is a

which is a contradiction. Therefore X is c-Lindelof space.

Proposition

In a T_3- space X ,if the set of accumulation points of X is finite, then the concepts of c-Lindelof and strongly c- Lindelof are coincident.

Proof:

If X is strongly c- Lindelof space then by proposition (2.6) it is c-Lindelof. Conversely, if X is a T_3 - c- Lindelof space, then by remark (1.3),it is Lindelof,and since the set of accumulation points of X is finite, then by proposition (2.5) it is strongly c-Lindelof space.

Definition [4]

Let $f : (X, \tau) \to (Y, \tau')$ be any function , f is said to be a preirresolute function if and only if the inverse image of any preopen set in Y is a preopen set in X.

Remark [4]

A function $f : (X, \tau) \to (Y, \tau')$ is a preirresolute if and only if the inverse image of any preclosed set in Y is a preclosed set in X.

Lemma [4]

A function $f : (X, \tau) \to (Y, \tau')$ is a preirresolute if and only if $\text{pre cl}(f^{-1}(B)) \subseteq f^{-1}(\text{pre cl}(B)) \ \forall B \subseteq Y$.

Corollary

Every strongly c-Lindelof and T_3- space is Lindelof space .

Proof:

If X is a T_3- strongly c-Lindelof space, then by proposition (2.3), X is strongly Lindelof,and by remark (1.4), X is Lindelof.

Proposition

If the set of accumulation points of the space X is finite ,then X is strongly c-Lindelof space whenever it is a Lindelof space.

Proof:

Let X be a Lindelof space such that the set of accumulation points of X is finite, then by theorem (1.5), X is strongly Lindelof,and by remark (2.2),it is strongly c-Lindelof space.

Proposition

Every strongly c- Lindelof space is c- Lindelof space .

Proof:

Let X be a strongly c- Lindelof space, to prove it is c-Lindelof. If not, then there is a closed set $A \subseteq X$ and an open cover $\{U_\alpha : \alpha \in \Delta\}$ for A, such that $A \neq \{\mathrm{cl}U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$. Since each open set is a preopen set , then $\{U_\alpha : \alpha \in \Delta\}$ is a preopen cover of A,then there is a countable subfamily $W = \{U_{\alpha_i} : i \in \Delta \subset \mathrm{N}\}$ of $\{U_\alpha : \alpha \in \Delta\}$ $\ni A = \bigcup_{i \in \Delta \subset \mathrm{N}} \{\text{pre - cl}\, U_{\alpha_i} : U_{\alpha_i} \in W\}$.This means there exists $x \in A$ such that $x \in \text{pre - cl}U_{\alpha_i}$ and $x \notin \mathrm{cl}U_{\alpha_i}$,for some $i \in \Delta \subset \mathrm{N}$ .Since $x \notin \mathrm{cl}U_{\alpha_i}$,then $x \notin U_{\alpha_i}$,but $x \in \text{pre - cl}U_{\alpha_i}$,then $x \in \text{pre - derived}\, U_{\alpha_i}$ .On the other hand ,since $x \notin \mathrm{cl}U_{\alpha_i}$,implies $x \notin U_{\alpha_i}$ and

$x \notin \text{derived}\, U_{\alpha_i}$. Since each open set is a preopen set ,then $x \notin \text{pre - derived}\, U_{\alpha_i}$

Theorem [4]

Every homeomorphism function is a preirresolute function.

Theorem

A strongly c- Lindelof is a topological property.

Proof:

Let (X, τ) be a strongly c- Lindelof space and let (Y, τ') be any space homeomorphic to (X, τ), then by theorem (2.12), (Y, τ') is a preirresolute image of a strongly c- Lindelof space (X, τ),and by theorem (2.11), (Y, τ') is strongly c- Lindelof.

Theorem

The preirresolute image of a strongly c- Lindelof space is a strongly c- Lindelof space.

Proof:

Let $f:(X,\tau) \to (Y,\tau')$ be a preirresolute onto function and let X be a strongly c- Lindelof space. To prove Y is strongly c- Lindelof. Let $A \subseteq Y$ be any preclosed subset of Y ,and let $\{U_\alpha : \alpha \in \Delta\}$ be a τ'- preopen cover for A .Since f is a preirresolute function, then $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a preopen cover of a preclosed subset $f^{-1}(A)$ of X, since X is strongly c- Lindelof space, then there is a countable subfamily $W = \{ f^{-1}(U_{\alpha_i}) : i \in \Delta \subset \mathrm{N} \}$ of $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ such that $f^{-1}(A) = \bigcup_{i \in \Delta \subset \mathrm{N}} \{\text{pre - cl}\,(f^{-1}(U_{\alpha_i})) : f^{-1}(U_{\alpha_i}) \in W\}$ .Then $\{ f\,(\text{pre - cl}\,(f^{-1}(U_{\alpha_i}))) : f^{-1}(U_{\alpha_i}) \in W \}$ covers A .Since f is a preirresolute function, then by lemma (2.10),we have $\{ f(f^{-1}(\text{pre - cl}(U_{\alpha_i}))) : f(f^{-1}(U_{\alpha_i})) \in f(W) \}$ covers A .Since f is onto, then $\{ \text{pre - cl}(U_{\alpha_i}) : U_{\alpha_i} \in f(W) \}$ is a countable subfamily of $\{ U_\alpha : \alpha \in \Delta \}$ for A .Therefore Y is strongly c- Lindelof space.

References

1.S.Willard (1980).General Topology. *ADDISON–WESLEY Publishing company.*Inc.United States of America.

2.M.Ganster (1989).A Note on Strongly Lindelof Spaces .Soochow J.Math.15(1):99-104.

3.A.S.Mashhour,M.E.Abd El-Monsef and S.N.El-Deeb (1982).On Pre-Continuous and Weak Pre-Continuous Mappings.Proc.Math. And Pnvs. Soc. Egypt 53:47-53.

4.E.Rana Bahjat (2004).On Semi-p-Open Sets.M.Sc.Thesis University of Baghdad, Iraq.

فضاءات فوق ليندلوف . C

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الخلاصة:

في هذا البحث ، قمنا بتعريف نوع آخر من فضاءات ليندلوف أسميناه فضاء فوق ليندلوف . C ودراسة بعض خواص هذا الفضاء والعلاقة بينه وبين فضاءات ليندلوف وليندلوف – C و فوق ليندلوف .