NUMERICAL PREDICTION OF PRIMITIVE VARIABLES FOR TWO-DIMENSIONAL SUPERSONIC FLOW OVER A WEDGE

Ahmed Kadhim Hussein **Hayder Krady Rashid**
Mohammed Y. Al-Janabi
College of Engineering-Mechanical Engineering Department
Babylon University

1.Abstract:

The paper is used to compute the primitive variables of supersonic flow based on finite difference computational fluid dynamic methods. The problem was considered is to deal with a two dimensional external, inviscid, compressible supersonic flow over a wedge body. In this work Euler equation was solved using time-marching MacCormack's explicit technique. The flow conditions are taken at sea level and Mach number was tested at **2.5**. To deal with complex shape of wedge body the so-called " **Body fitted coordinate system**" were considered and the algebraic methods were used to generate grids over a wedge body. The results showed a good agreement with another published results. The results in our work are taken at a wedge angles equal to **15 deg**. and **30 deg**. respectively.

2.List of Symbols:

Symbol	Description	Dimension
a	Speed of sound.	m/s
CFL	Courant Fridrich Lewys stability condition.	
C_ζ, C_η	Artificial viscosity coefficients in ζ and η directions respectively.	
e	Specific internal energy per unit mass.	J/kg
E_t	Total energy per unit volume.	J/m^3
E,F	Column vector in Cartesian coordinates.	
$\bar{E}, \bar{F}$	Column vector in body fitted coordinates.	
L	Length of the wedge.	m
J	Jacobian of coordinates transformation.	
M_∞	Free stream Mach number.	
P_∞	Free stream pressure.	N/m^2
$\underline{Q}$	Flux vector.	
$\bar{\underline{Q}}$	Vector of conserved variable in body fitted coordinates.	
Re	Reynold's number.	

R	Universal gas constant.	J/kg.K
SQ_1	Artificial viscosity term.	
T_∞	Free stream temperature.	K
t	Time.	sec
u	Velocity compoment in x-direction.	m/sec
U	Contravariant velocity component in ζ -direction	m/sec
v	Velocity compoment in y-direction.	m/sec
V	Contravariant velocity component in η -direction	m/sec
x , y	Cartesian coordinates	m

Greek Symbols

γ	Ratio of specific heats.	
Δt	Time step.	sec
ρ	Density.	Kg/m^3
α	Angle of attack.	Deg.
δ	Boundary layer thickness.	m
$\Delta x, \Delta y$	Spatial steps in physical domain.	
$\Delta\zeta, \Delta\eta$	Spatial steps in computational domain.	
ζ, η	Computational coordinates.	

Subscript

i, j Node symbols indicates position in x and y directions.

∞ Conditions at free stream.

o Stagnation (total) conditions.

Superscript:

n Time level t .

n+1 Time level $(t+\Delta t)$.

3.Introduction

Supersonic flow was a flow in which a Mach number is greater than 1, and this flow is very important in the design of aircraft and rockets. During the past, the experimental and analytical methods were used to simulate the properties of supersonic flow over a limited number of shapes, but for supersonic two-dimensional shapes such as a 2D wedge, the analytical method was failed due to non-linearity also to design an aircraft many thousands of tests were drawn in a supersonic wind tunnel which reqires a hard and expensive work and require a very long time. In contrast, a numerical prediction give the same result with a short-time and an accurate computation and the computer program may be changed easily to deal with any other complex shape such as wing, airfoil and missile. In the numerical solution, the complex differential equations are overcomed by replacing it with differences, calculated from a finite number of values associated with the computational nodes, which are distributed on a suitable grid over the solution domain. This work, was completed the previous work by **Fayadh [1]**, which deal with the same problem but it is restricted, since the primitive variables is not explain in detail. In this study, the predictor-corrector MacCormack's explicit finite difference method was used to predict the aerodynamic properties of two-dimensional external compressible inviscid supersonic flow, such the velocity, pressure, density, Mach number and temperature at each grid. The time- marching method was chosen to treat a wedge as a plane body. In the next section, the mathematical model was described in detail and the style, which used to produce meshes or grids are given. It is very important to refer that the primitive variable covers, velocity, density, temperature, pressure and Mach number.

4.Mathematical Model:

Supersonic flow treats with a non-viscous, non-heat conducting fluid, so it is described by Euler equation. The latter is obtained from Navier-Stokes equations by neglecting all shear stresses and heat-conduction terms, so it is a vaild approximation for flows at high speed (supersonic flow), i.e., at high Reynolds number outside the viscous region developing near solid surface. The mathematical behavior of the Euler equation is classified as hyperbolic in supersonic flow **[2]**. The solution is obtained using time-marching method. In this method three points must be noticed.

1. The grid points are generated in physical plane and transformed to computational plane before solving governing equations.
2. The solution is obtained by marching from some initial flow field through time until a steady – state is obtained.
3. The governing equation for an inviscid, non-heat conducting, external, compressible, two-dimensional supersonic flow expressed in conservation form are **[3]**:

Continuity equation:

$$\nabla.(\rho V) = \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho \upsilon)}{\partial y} \qquad \ldots(1)$$

The conservation of monemtum equation is :

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p)}{\partial x} + \frac{\partial(\rho u \upsilon)}{\partial y} = 0 \qquad \ldots(2\text{-A})$$

$$\frac{\partial(\rho \upsilon)}{\partial t} + \frac{\partial(\rho u \upsilon)}{\partial x} + \frac{\partial(\rho \upsilon^2 + p)}{\partial y} = 0 \qquad \ldots(2\text{-B})$$

The conservation of energy equation is :

$$\frac{\partial(\rho E_t)}{\partial t} + \frac{\partial}{\partial x}\left[(\rho E_t + p)u\right] + \frac{\partial}{\partial y}\left[(\rho E_t + p)\upsilon\right] = 0 \qquad \ldots(3)$$

It is suitable to put these equations in a vector form before applying a numerical scheme to these equations. The 2-dimensinal Euler equation may be arranged in vector form as :-

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0 \qquad \ldots(4)$$

where Q,E and F are column vectors defined by:

$$Q = \frac{\partial}{\partial t} = \begin{bmatrix} \rho \\ \rho u \\ \rho \upsilon \\ \rho E_t \end{bmatrix} ; E = \frac{\partial}{\partial x} = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ \rho u \upsilon \\ (\rho E_t + P)u \end{bmatrix} ; F = \frac{\partial}{\partial y} = \begin{bmatrix} \rho \upsilon \\ \rho u \upsilon \\ \rho \upsilon^2 + P \\ (\rho E_t + P)\upsilon \end{bmatrix} \ldots(5)$$

also the equation of state is given by :-

$p = \rho RT$, at ambient temperature, it becomes :-

$$\rho_\infty = \frac{p_\infty}{RT_\infty} \qquad ...(6)$$

and the free stream Mach number is given by :-

$$M_\infty = \sqrt{\gamma RT_\infty}$$

The total pressure and temperature (or stangation pressure and temperature) are given by :-

$$p_o = p_\infty\left[1+\frac{(\gamma-1)}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} \qquad ...(7)$$

$$T_o = T_\infty\left[1+\frac{(\gamma-1)}{2}M^2\right] \qquad ...(8)$$

The Reynolds number and boundary layer thickness are given by :-

$$\mathrm{Re} = \frac{\rho_\infty u_\infty L}{\mu_\infty} \text{ and } \delta = \frac{5.0L}{\mathrm{Re}_L^{0.5}} \qquad ...(9)$$

Moreover the velocity components are given by :-

$$u_\infty = V\cos\alpha \qquad ...(10\text{-}A)$$

$$\upsilon_\infty = V\sin\alpha \qquad ...(10\text{-}B)$$

where $V = M\,a_\infty$ noting that:-

M=Mach number. and a_∞ = speed of sound.

V =Fluid velocity.

δ =Boundary layer thickness.

Re=Reynolds number.

μ_∞ = Free stream dynamic viscosity.

5.Two Dimensional Mesh Generation

Grid or mesh generation is a method which is used to treat the complexity of the governing equations in supersonic fluid dynamic which is in most cases can not be solved analytically. In this work, the **algebraic grid generation** method is used to produce grid. This method generates grid points in space by means of interpolations based on given boundary data. Because of the non-uniform shape of wedge, a " body fitted coordinate system " is used for the transformation of governing equations from a cartesian system (x,y) to a general curvilinear sytem (ζ,η) and it used to transform from ***physical plane*** to the ***computational plane***. The transformation of any partial differential equations from physical plane (x,y) to computational plane (ζ,η) are defined by the following relations :-

$$\zeta = \zeta\,(x,y) \qquad ...(11\text{-}A)$$

$$\eta = \eta\,(x,y) \qquad ...(11\text{-}B)$$

The details of transformation is complex, for more details, it is recommended to see **[4]** and the results are given by:

$$J = \left[x_\zeta y_\eta - y_\zeta x_\eta\right]^{-1} \qquad ...(12)$$

where J is the jacobian of transfiormation, and it is defined as the ratio of the volumes in the physical space to that of the computational space. Also the metrics of transformation are given by (in two dinension) as follows:-

$$\zeta_x = J\, y_\eta$$
$$\zeta_y = -J\, x_\eta \qquad \text{...(13-A)}$$

$$\eta_x = -J\, y_\zeta \qquad \text{...(13-B)}$$
$$\eta_y = J\, x_\zeta$$

the physical meaning of the metrics is that. it represents the ratio of arc length in the computational space to that of the physical space. The terms x_ζ , x_η , y_ζ .... are computed numerically using forward approximation. as an example :-

$$y_\eta = \frac{\partial y}{\partial \eta} = \frac{-3y_{i,j} + 4y_{i,j+1} - y_{i,j+2}}{2\Delta\eta} \qquad \text{...(14)}$$

6.Numerical Procedure

Explicit time-dependent solution of the two-dimensional Euler equations has been performed using MacCormacks predictor- corrector finite difference technique, which is second-order accurate in both space and time. This method is very effective finite difference technique for viscous and inviscid supersonic flow. specially for unsteady flow shock capturing. By using this technique a computer program is constructed to predict the shock wave which consists from the following steps:

1. a two-dimensional domain is chosen over a wedge , which consists from (**im=53, jm=63**).where:-

im = maximum number of grids in x-direction.
jm = maximum number of grids in y-direction.

2. a grid generation is performed in two direction and the Jacobian and different metrics are calculated.
3. a flow conditions such as (u ,v ,T, ρ and P) are computed at the surface (J=1) and then there are computed in the domain except at the surface where (I=1 to I=im, j=2 to j=jm).
4. a time step calculation is performed. the time step employed in this work is designed so that it is not exceed the maximum step size permitted by stability. In this study the inviscid **CFL** conditions **[5]** is used which is given by the following relation:-

$$\Delta t)_{CFL} \le \left[\frac{|u|}{\Delta x} + \frac{|v|}{\Delta y} + a * \left[\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} \right]^{\frac{1}{2}} \right]^{-1} \qquad \text{...(15)}$$

5. a changing of primitive variables to fluxes is occurred which causes to compute the values of flux vectors for all grid points at time step (n).
6. a forward predictor version of MacCormack`s which is given by **[4]** :-

$$\overline{Q}^{n+1}_{i,j} = \overline{Q}^{n}_{i,j} - \frac{\Delta t}{\Delta\zeta}\left[\overline{E}^{n+1}_{i+1,j} - \overline{E}^{n+1}_{i,j}\right] - \frac{\Delta t}{\Delta\eta}\left[\overline{F}^{n+1}_{i,j+1} - \overline{F}^{n+1}_{i,j}\right] \qquad \text{...(16)}$$

is used inside the domain where (I=2 to im-1, j=2 to jm-1) where
n is the time level (t) and n+1 is the time level (t+dt).

7. in order to make our numerical scheme accurate and stable. since we deal with high velocities (high Reynolds number) the following expression for explicit **artificial viscosity** is added to the predictor step. where $SQ^{n+1}_{i,j}$ is a fourth-order artificial viscosity term, defined by **[5]**:-

$$SQ^{n+1}_{i,j} = C_\xi \frac{\left|P^n_{i-1,j} - 2P^n_{i,j} + P^n_{i+1,j}\right|}{P^n_{i-1,j} + 2P^n_{i,j} + P^n_{i+1,j}} * \left[(Q_1^-)^n_{i-1,j} - 2(Q_1^-)^n_{i,j} + (Q_1^-)^n_{i+1,j}\right] +$$

$$C_\eta \frac{\left|P^n_{i,j-1} - 2P^n_{i,j} + P^n_{i,j+1}\right|}{P^n_{i,j-1} + 2P^n_{i,j} + P^n_{i,j+1}} * \left[(Q_1^-)^n_{i,j-1} - 2(Q_1^-)^n_{i,j} + (Q_1^-)^n_{i,j+1}\right] \quad \ldots(17)$$

The main advantage of artificial viscosity is to provide some mathematical dissipation analogous to the ***real viscous effects*** inside the ***shock wave***.

8. a decoding is occurred which is used to produce our parameters from fluxes, also at this step, the contravarient velocity components which defined by :-

$$U = \xi_x u + \xi_y \upsilon$$

$$V = \eta_x u + \eta_y \upsilon \quad \ldots(18)$$

is computed .The **contravarient velocity components U** and **V** represent velocity components which are perpendicular to planes of constant η and ξ

9. a corrector step is computed, where the value of fluxes (E , F) are computed at each grid in the intermediate level (n+1) depending on the values of primitive variables from previous step, so the computations occurs inside the domain [i=1 to i=im, j=1 to j=jm].
10. a backward corrector version of MacCormacks method which is given by **[4]** is then applied :-

$$Q^{n+1}_{i,j} = (1/2) * \left\{Q^n_{i,j} + \overline{Q}^{n+1}_{i,j} - \frac{\Delta t}{\Delta\xi}\left[\overline{E}^{n+1}_{i,j} - \overline{E}^{n+1}_{i,j-1}\right] - \frac{\Delta t}{\Delta\eta}\left[\overline{F}^{n+1}_{i,j} - \overline{F}^{n+1}_{i,j-1}\right]\right\} \quad \ldots(19)$$

which is used inside the domain (i=2 to im-1, j=2 to jm-1) and, also a fourth-order artificial viscosity term [5] at corrector step is added to **Eq. (19)**. This expression is given by:

$$SQ^{n+1})_{i,j} = C_\xi \frac{\left|P^{n+1}_{i+1,j} - 2P^{n+1}_{i,j} + P^{n+1}_{i-1,j}\right|}{\left[P^{n+1}_{i+1,j} - 2P^{n+1}_{i,j} + P^{n+1}_{i-1,j}\right]} * \left[(\overline{Q}_1)^{n+1}_{i+1,j} - 2(\overline{Q}_1)^{n+1}_{i,j} + (\overline{Q}_1)^{n+1}_{i-1,j}\right] +$$

$$C_\eta \frac{\left|P^{n+1}_{i,j+1} - 2P^{n+1}_{i,j} + P^{n+1}_{i,j-1}\right|}{\left[P^{n+1}_{i,j+1} - 2P^{n+1}_{i,j} + P^{n+1}_{i,j-1}\right]} * \left[(\overline{Q}_1)^{n+1}_{i,j+1} - 2(\overline{Q}_1)^{n+1}_{i,j} + (\overline{Q}_1)^{n+1}_{i,j-1}\right] \quad \ldots(20)$$

11. after a corrector step is completed, a ***decoding step*** began where our parameters such as (u,v,P,T and M) are computed.
12. our parameters are computed under different boundary conditions which can be explained as follows:-
    i. up-stream boundary condition (i=1, j=2 to jm) and at the edge of the body (j=1,i=1 to i=im)
    ii. solid boundary condition (j=1,i=2 to im)
    iii. down stream boundary condition (i=im, j=2 to jm-1).
    iv. upper plane of symmetry(i=2 to im and j=2 to jm).
    v. plane of symmetry (j =jm, i=2 to im).
13. The ***convergence*** of the solution is examined, knowing that the last flow field variable to be convergence is the **density**, therefore, the following convergence criterion was established at every point in the flowfield from one time step to the next, where **[6]:**

$$\text{error} = \frac{\rho_{old} - \rho_{new}}{\rho_{old}} \le 1 * 10^{-8} \quad \ldots(21)$$

7. Results and Discussion

Figures (1) and (2) show a mesh generation over a two-dimensional wedge in supersonic inviscid flow with wedge angles equals to **(15 degree)** and **(30 degree)** respectively. The mesh points are produced by using an **algebraic grid generation** with (53 x 63) grid points in both cases. The explicit technique has required about **(1500)** time steps to achieve the converged solution.

Figure (3) and **figure** (8) show a Mach number contours for supersonic wedge with wedge angle=**15 degree** and **30 degree** respectively, and the free stream Mach number = 2.5. In both figures, the shock wave can be noticed clearly. From these figures, the flow pattern near the leading edge of wedge where the incoming supersonic flow undergoing a sudden change in flow direction resulting a continuous compression wave. The angle of the shock wave depends on wedge angle and free stream Mach number. The shock is observed to be detached from wedge angle and flow behined the shock near the trailing edge of wedge area becomes subsonic. Also, in **figure** (8) the shock strength increase with increase the wedge angle from **(15 degree)** in **figure** (3) to **(30 degree)** in **figure**(8) until the flow becomes subsonic behined the shock. This prediction gives a good agreement with the **experimental results** dealing with the same problem as indicated in [7].

Figures (4) and (9) show a temperature contours for supersonic wedge angles equal to **(15deg.)** and **(30 deg.)** for free stream Mach number = **2.5** . These figures indicate that the temperature distribution occurs at the region between the wedge surface and the shock wave. Also, the temperature increases as wedge angle increase due to shock wave strength and is decreased gradually toward the free stream value.

Figures (5) and (10) show a density contours over a supersonic wedge with wedge angles equal to **(15 deg.)** and **(30 deg.)** respectively. From these figures, the density values increases with increasing the wedge angle and this is due to increasing in shock wave strength.

Figures (6) and **(11)** explain a pressure contours over a wedge at wedge angles equal to **(15 deg.)** and **(30 deg.)** respectively. From both figures, the pressure values increases a head of shock and then decreased toward its free stream value away from the shock. This is agree with [8] also, the pressure values is increased dramatically as wedge angle increase.

Figures (7) and (12) show the velocity contours over a two-dimensional wedge with wedge angle =**15 deg**. and **30 deg**. respectively. Both figures show clearly the shock wave prediction and again the velocity values increase with increasing wedge angle; so that values of velocity in **figure** (7) is less than the corresponding values in **figure** (12). This increasing is connected with increasing in the Mach number values, which as explain above increase with increasing wedge angle. Also, from these figures, the reduction in velocity clearly noticed down stream of the shock. This is due to change in flow behaviour from supersonic flow a head of the shock to subsonic flow downstream of the shock.

8. Conclusions

The following conclusions can be drawn from the results of the present work :-

1. a solution of Euler equation for wedge converges at a range of (1000-1500) iteration. The range of iteration is related to the procedure of mesh producing.
2. for capturing the flow field parameters ; a more grid points are required near the surface.
3. geometry and wedge angle have an important effect on the flow field pattern; where in the supersonic flow the change in flow direction due to geometry induced a type of drag. This is called wave drag.

4. since the mesh generation has been separated from explicit solver any mesh type can be used. The stability of the solution depends on the number of grid points and **CFL** condition.
5. shock angle increases as the wedge angle increases at the same free stream Mach number.
6. The rise in the value of temperature at the **stagnation point** is due to flow nature change from supersonic a head of the shoch to subsonic down stream of the shock. This change will reduce the kinetic energy and at the same time, this reduction gives an increase in internal energy and as a result increase the temperature.
7. The time-marching solution which is used to deal with a planer body as a **wedge**, which can be extended to deal with an axis-symmetry body such as a **rocket**.
8. from the results obtained, the hydrodynamic properties such as pressure, temperature and density are increased as the wedge angle is increased.
9. The software constructed explains, that the convergence depends on the optimum values of the grid points, artificial viscosity and **CFL.**

9.Referencers

1. Fayadh AL-Dulaimy, "**Numerical Prediction of Supersonic Inviscid and Viscous Flow over Arbitrary Configurations**", Ph.D. thesis, University of Technology, Baghdad , 2002.
2. Zlatko, P., Stupar, S., " **Fundamental Equations of Aerodynamics**", Mechanical Eng. Faculty, Belgrade ,1997.
3. Denton,J.D., " **An Improved Time-Marching Method for Turbomachinery Flow Calculation**" ASME Journal of Eng. for power, Vol.105, pp.515-524 , 1985.
4. Hoffmann, A.K., " **Computational Fluid Dynamics for Enginering** ", University of Texsas, 1989.
5. Anderson, J.D.," **Computational Fluid Dynamics, The Basic with applications**" , McGraw-Hill, New York , 1995.
6. MacCormack, R.W., " **Current Status of Numerical Solutions of the Navier Stokes Equations**", AIAA paper 85-0032, 1985.
7. David, F., Haering, E. and Aguilar, J., " **Determination of Sun Angles for Observations of Shock Waves on a Transport Aircraft** ", NASA / TM-1998-206551, pp: 1-14,1998.
8. Alauzet , F., Frey, P. and Mohammadi , B.," **An isotropic Mesh adaptation: Applications to Transient CFD Problems** ", Computational Fluid and Solid Mechanics, pp.1851-1854 , 2003.

الخلاصة

في هذا البحث تم إظهار أهمية الحل العددي في أيجاد حل للمشاكل الأيروديناميكية عند السرع الصوتية العالية أي (**supersonic flow**) عندما يكون رقم ماخ اعلى من واحد. تم اللجوء الى الأسلوب العددي لتعذر حل المشكلة الهندسية تحليلياً وكذلك فأن حل أي مشكلة أيروديناميكية تحتاج الى نفق هوائي (**supersonic wind tunnel**) وهو غير متوفر في العراق ويتطلب وقت زمني كبير جداً لاجراء التجارب. في الدراسة الحالية تم التعامل مع برنامج هندسي متكامل وتطويره للتعامل مع الجريان فوق الصوتي وإيجاد المتغيرات الأساسية (**primitive variables**) وهي السرعة والضغط ودرجة الحرارة والكثافة ورقم ماخ عند أي نقطة . تضمن أسلوب الحل ، حل معادلة أويلر وتم توليد مجاميع النقاط بواسطة تقنية (**grid generation**) وتم أختيار الـ **Wedge body** كحالة للدراسة. الحل العددي تم بأستخدام طريقة ماكارموك(MacCormack's Method) وهي طريقة كفوءه عند استخدامها لدراسة الجريان الخارجي فوق الصوتي وتم اختيار رقم ماخ للجريان الخارجي (Free stream Mach number) بقيمة (2.5) كحالة دراسة وايضاً تم دراسة تأثير تغيير Wedge angle على حساب المتغيرات الأساسية للجريان وكذلك على تخمين موجات الصدمة (shock waves).

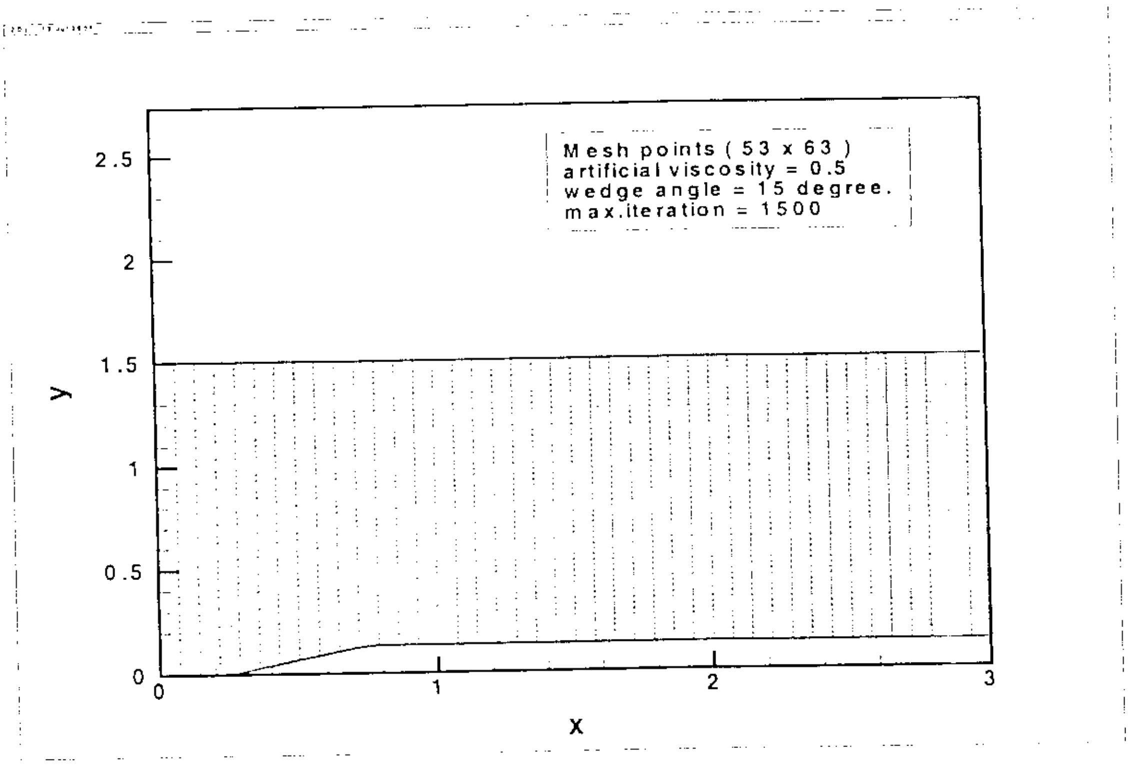

Fig.1 Mesh generation over a two-dimensional wedge in supersonic inviscid flow.

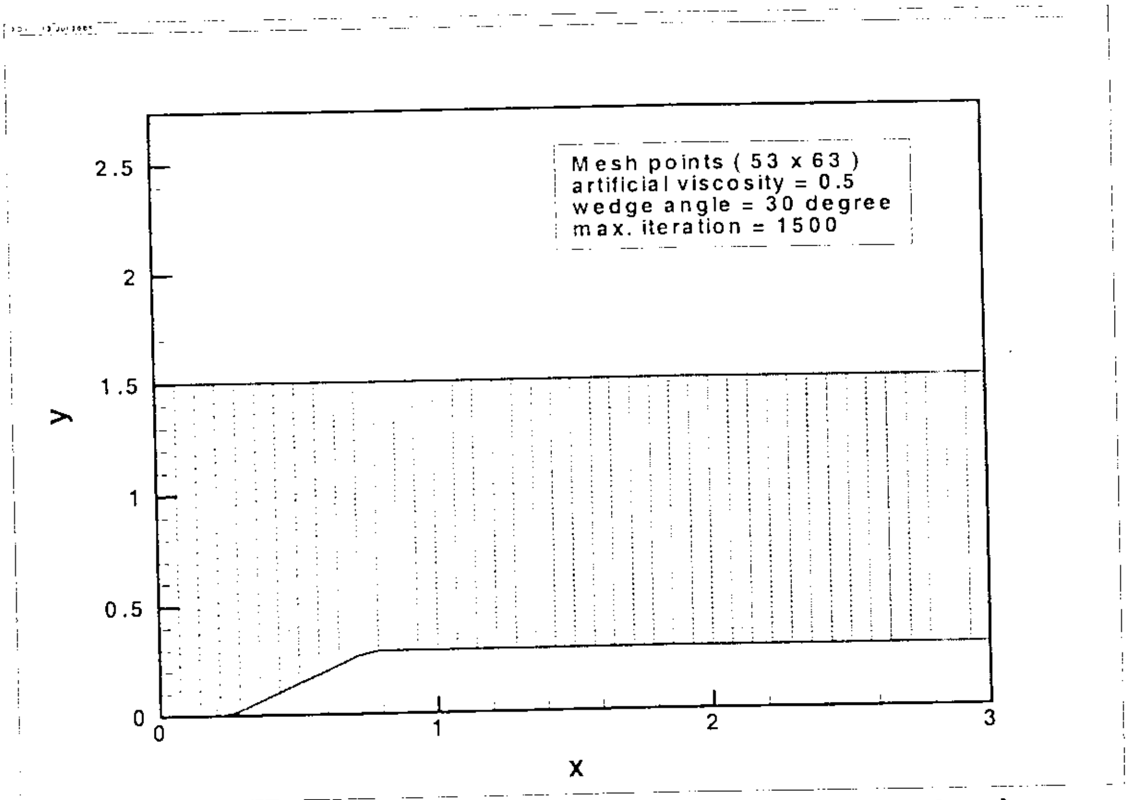

Fig.2 Mesh generation over a two-dimensional wedge in supersonic inviscid flow.

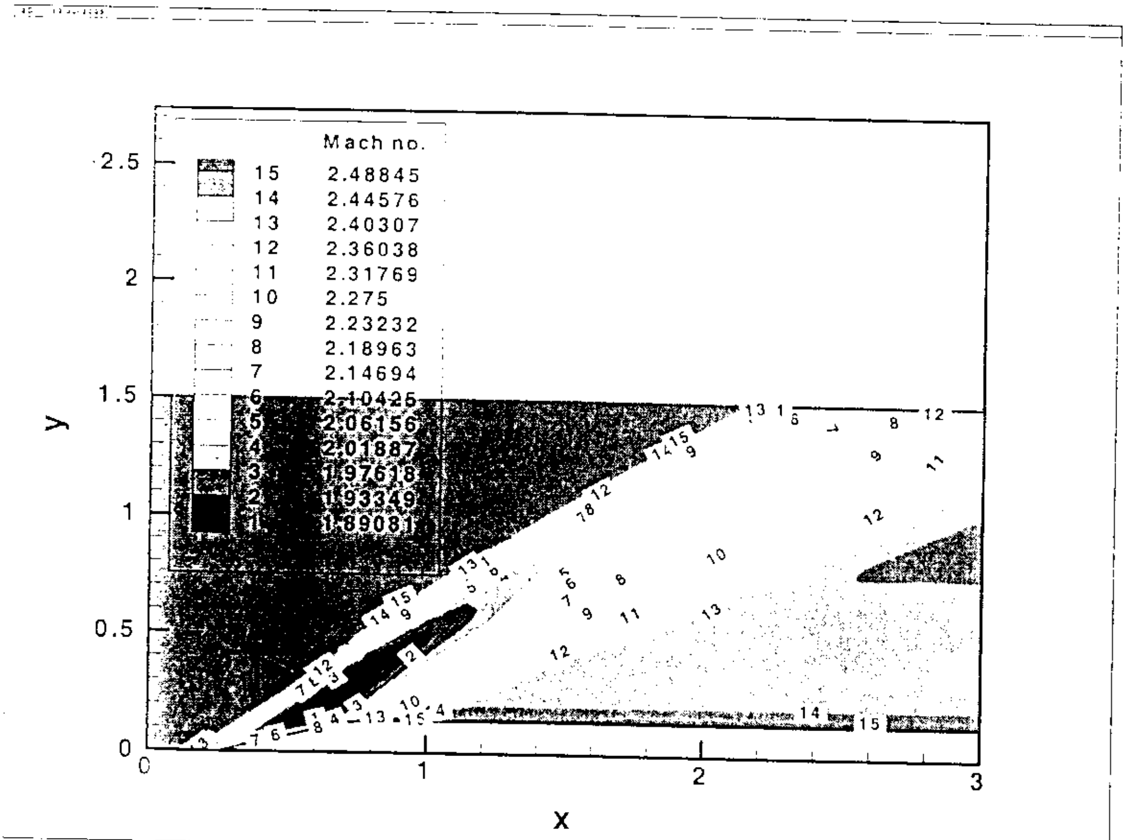

Fig.3 Mach number contours for supersonic wedge with wedge angle = 15 deg. and free stream Mach number = 2.5

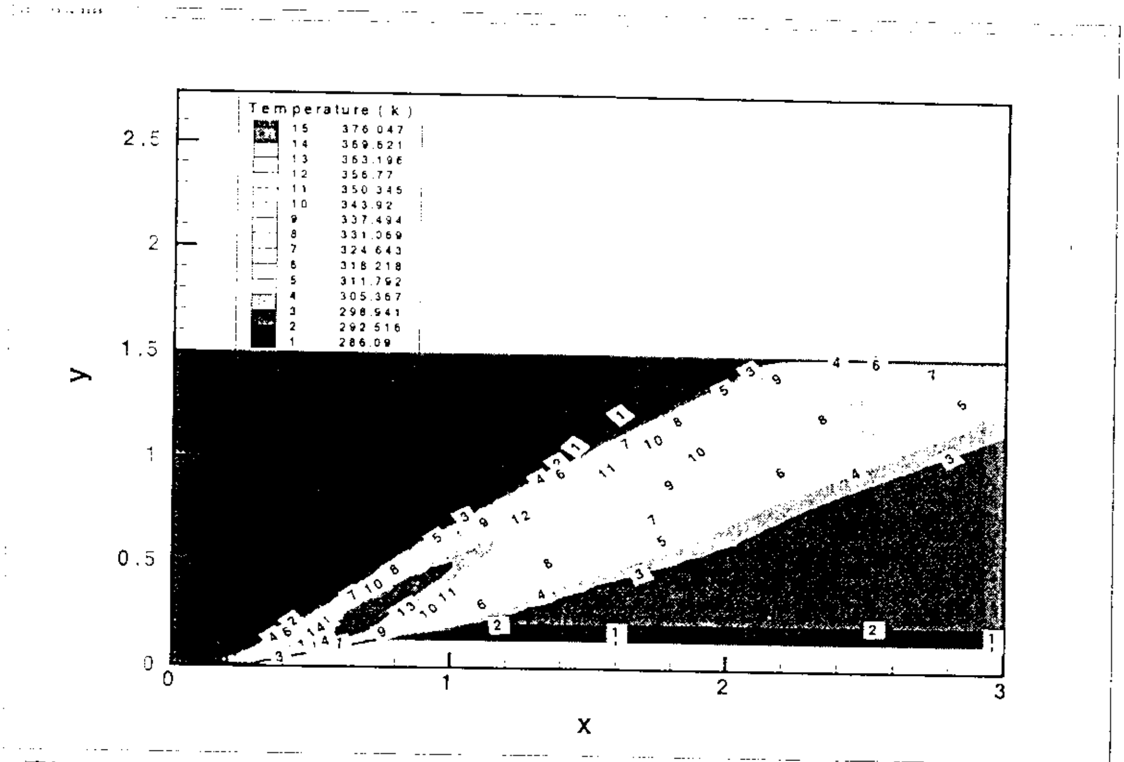

Fig.4 Temperature contours for supersonic wedge with wedge angle = 15 deg. and free stream Mach number = 2.5

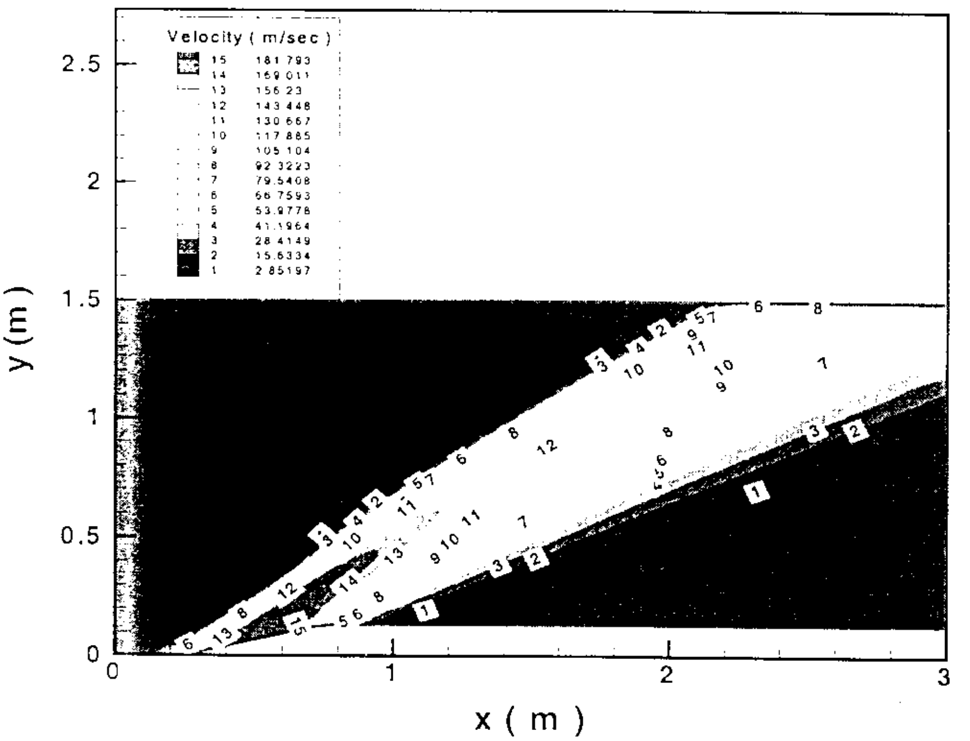

Fig.7 Velocity contours over a two-dimensional wedge with a wedge angle = 15 deg. and free stream Mach number = 2.5

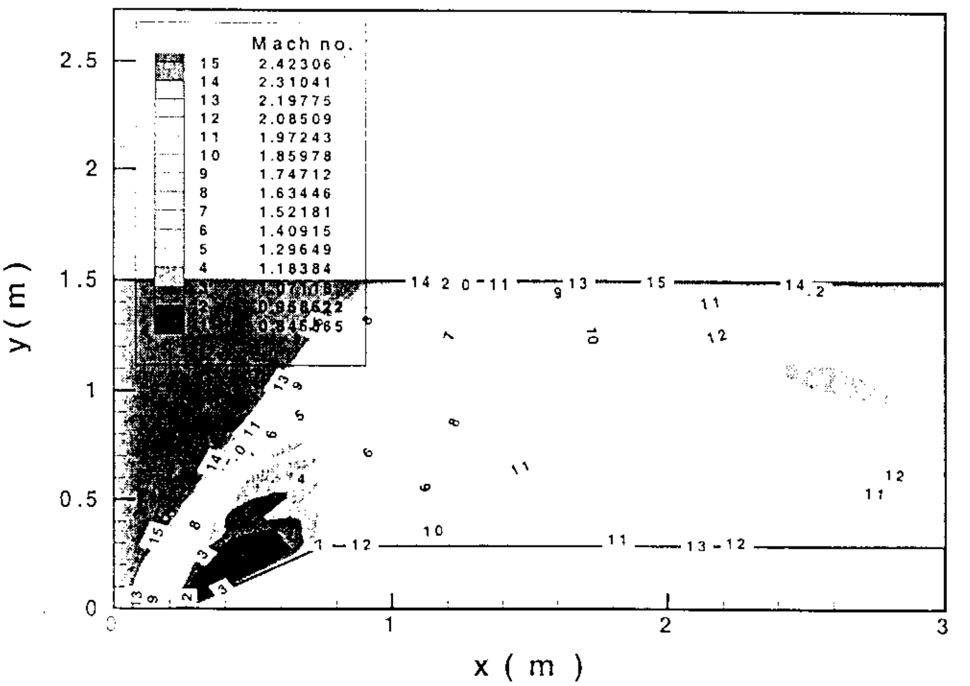

Fig.8 Mach no. contours over a two-dimensional wedge with a wedge angle = 30 deg. and free stream Mach number = 2.5

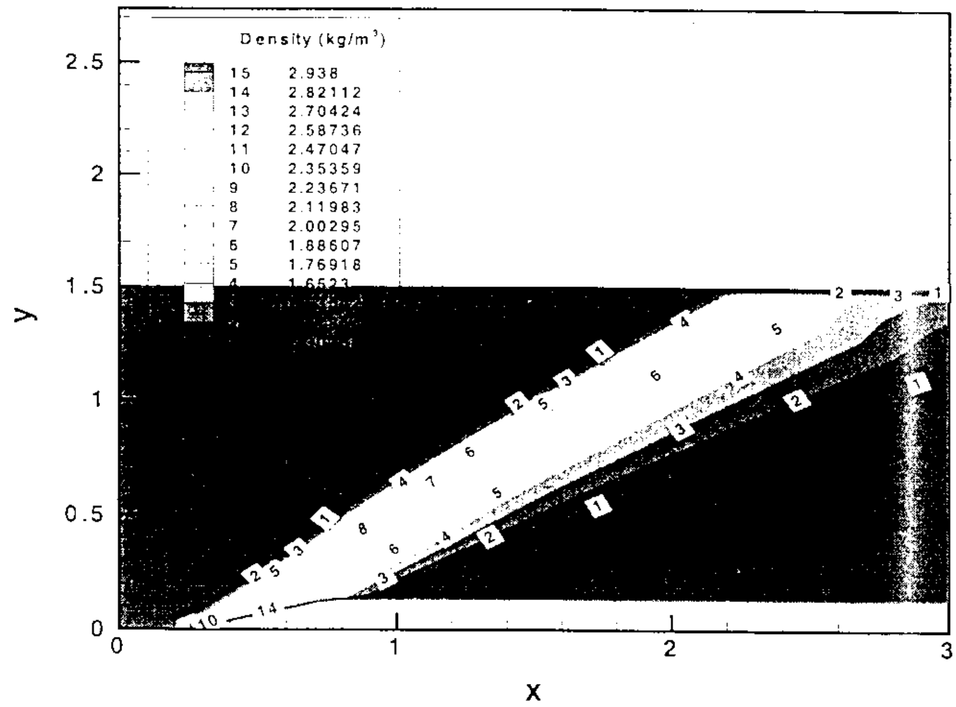

Fig.5 Density contours over a two-dimensional wedge with wedge angle = 15 deg. and free stream Mach no. = 2.5.

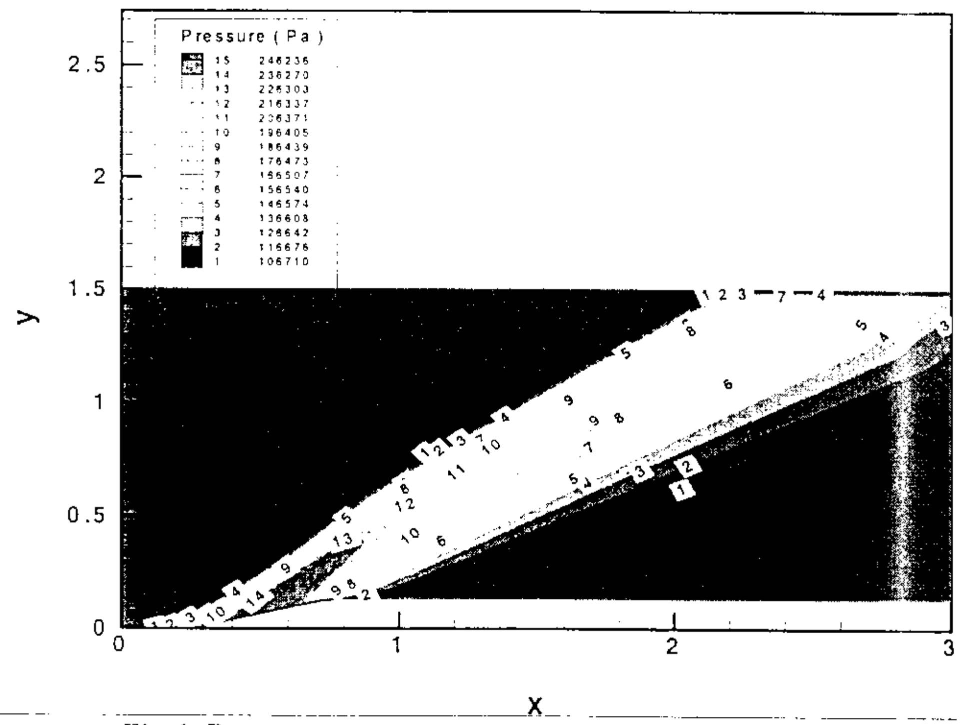

Fig.6 Pressure contours over a two-dimensional wedge with wedge angle = 15 deg. and free stream Mach no. = 2.5.

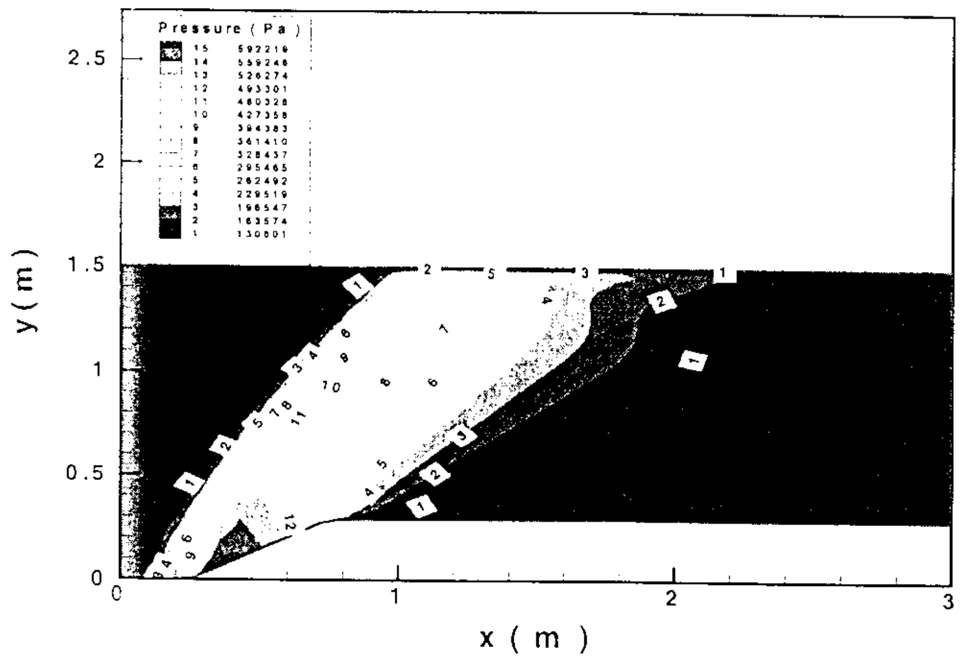

Fig.11 Pressure contours over a two-dimensional wedge with wedge angle = 30 deg. and free stream Mach no. = 2.5.

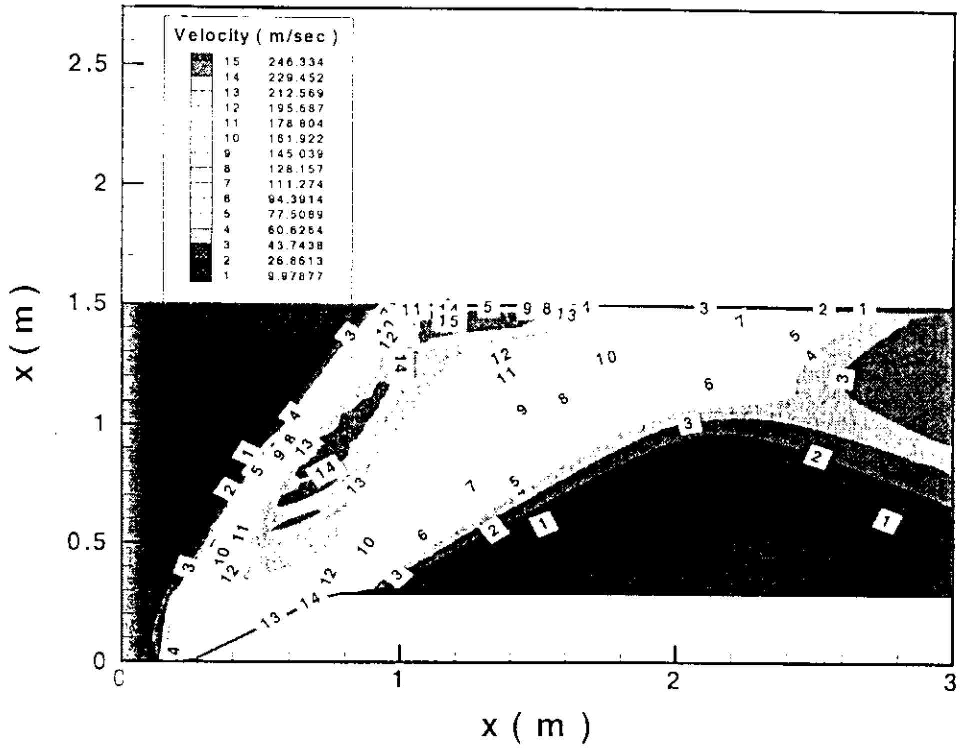

Fig.12 Velocity contours over a two-dimensional wedge with wedge angle = 30 deg. and free stream Mach no. = 2.5.

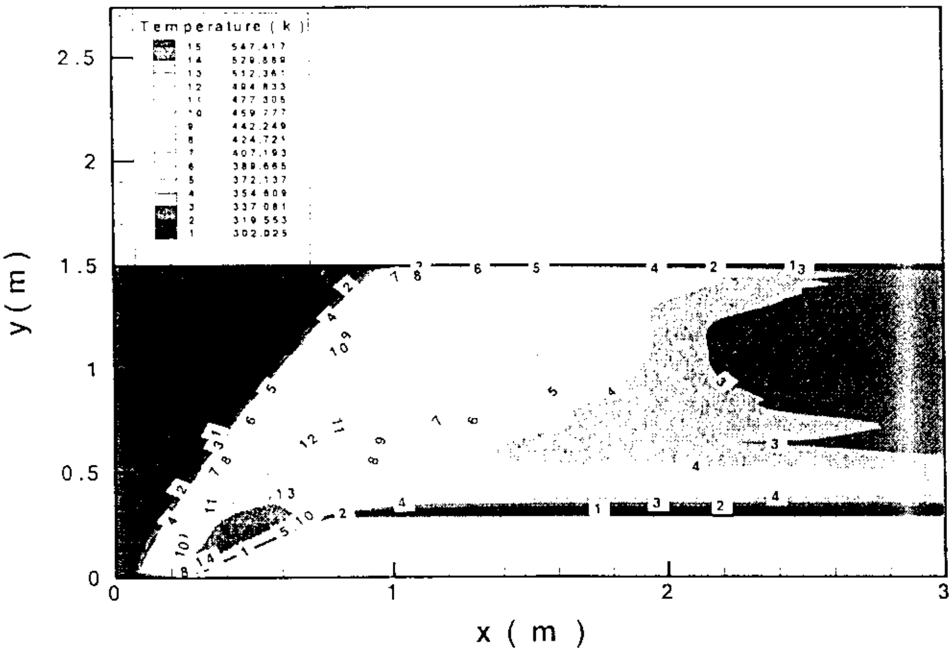

Fig.9 Temperature contours for supersonic wedge with wedge angle = 30 deg. and free stream Mach number = 2.5

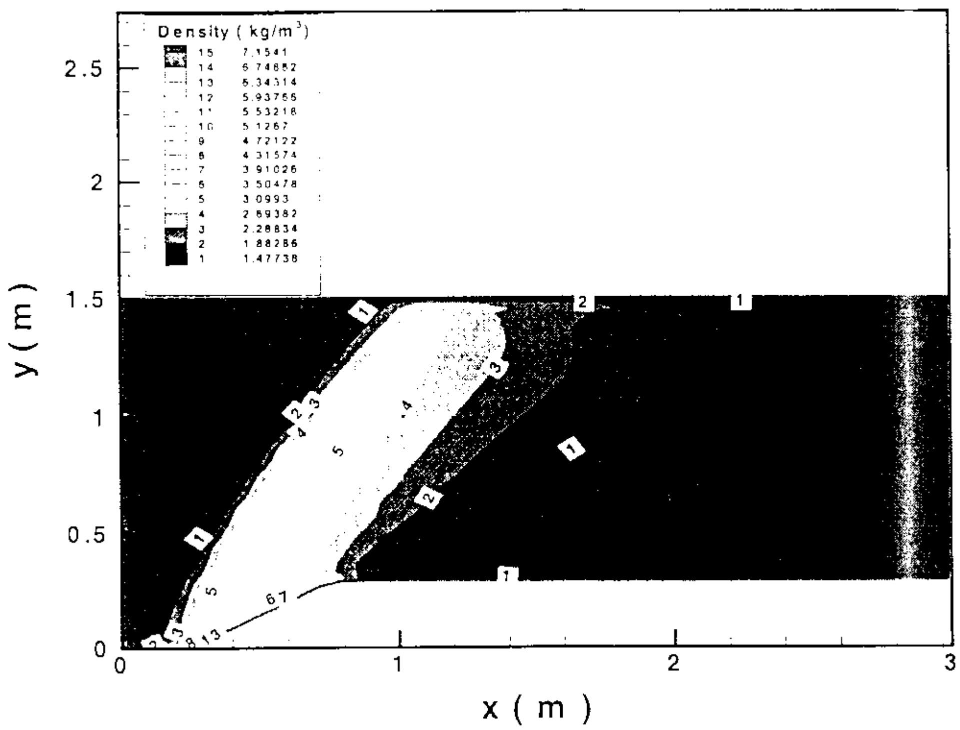

Fig.10 Density contours for supersonic wedge with wedge angle = 30 deg. and free stream Mach number = 2.5