

## **The Condensation Energy and Pseudogap Energy Scale of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ from the Electronic Specific Heat and Entropy**

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### ***Abstract***

From the analysis of the electronic specific heat  $C_{es}$  and the entropy  $S_s$  of the superconducting phase of nine samples of high temperature superconductor (HTS)  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (La:214) of Sr doping rates ( $0.03 \leq x \leq 0.24$ ), it is shown that the normal state pseudogap opens abruptly in the weakly overdoped region at the critical doping  $p_{crit}$  (Where  $x=0.20$  and the hole concentration in  $\text{CuO}_2$  planes is 0.19 holes/ $\text{CuO}_2$ ) and increases as  $E_g(p) \sim 1500(1-p/p_{crit.})(K)$  at lower doping. The condensation energy  $E_c(0)$  peaks strongly at the optimal doping  $p_{opt}$  (where  $x=0.17$  and the hole concentration in  $\text{CuO}_2$  planes is 0.16 holes/ $\text{CuO}_2$ ), exhibiting strong BSC like superconductivity ( $E_c(0)/\gamma_n T_c^2 \sim 0.52$  for  $p > p_{opt}$ ) and weak superconductivity below  $p_{dopt}$ . It can be concluded that the pseudogap is a property of the normal state NS density of state DOS and is not related to precursor superconducting fluctuations.

Keywords: HTS, Electronic Specific Heat, Entropy, Superconducting Condensation Energy and Pseudogap



$S_s$                        $C_{es}$   
 (HTS)                      (La:214)  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$   
 pseudogap                      (0.03 < x < 0.24)                      Sr  
    overdoped  
 $\text{CuO}_2$                       x=0.20                      ) critical doped  
                         (p<sub>crit</sub> ~0.19 holes/ $\text{CuO}_2$   
                         Eg(p)~1500(1-p/p<sub>crit</sub>)(K)  
 optimal doped                       $E_c(0)$   
 (p<sub>opt</sub> ~0.16 holes/ $\text{CuO}_2$                        $\text{CuO}_2$                       x=0.17                      )  
 p > p<sub>crit</sub>                      ( $E_c(0)/\gamma_n T_c^2 \sim 0.52$ )                      BCS  
    . p<sub>opt</sub>  
    (NS)  
    .(Superconducting fluctuations)

### **Introduction:**

Despite the enormous amount of effort spent in the last decades in resolving the fundamental physics behind high temperature superconductivity (HTS) the mechanism is still puzzling (J.E.Hirsch, 2002). It is more or less generally accepted now that the conventional electron-phonon pairing mechanism cannot explain cuprate superconductivity, because as high a transition temperature as 164 K (the record  $T_c$  up to now (Q.Xiong, et al., 1994)) cannot be explained by energy scale of lattice vibrations without leading to lattice instability (J.E.Hirsch, 2002). It has also been shown recently that the c-axis transport of the hole-content to the doping site in the charge reservoir in the normal state (NS) of cuprate may be important in understanding HTS (S.Sachdev and J.Ye, 2000). HTS exhibits a common generic phase diagram in which the superconducting transition temperature  $T_c$  rises to a maximum at an optimal doping of approximately 0.16 holes per planar copper atom and then falls to zero on the overdoped side. In addition the underdoped normal state exhibits correlation, which introduces a gap in the density of states that strongly affects all physical properties. There is no phase transition associated with the opening of this gap and so it is called a pseudogap. Analysis of specific heat data, for example, suggests that the pseudogap energy decreases with doping and falls to zero at a critical doping of  $p_{crit} \sim 0.19$  holes/ $\text{CuO}_2$ , just beyond optimal doping (J.W.Loram, et al., 1998 and J.L.Tallon, et al. 2001), a behavior rather analogous to the quantum-critical heavy-fermions materials (N.D.Mathur, et al., 1998).

Many fundamental physical quantities such as the superconducting condensation energy (J.W.Loram, et al., 1998 and J.L.Tallon, et al. 2001), the superfluidity density (C.Panagopoulos, et al., 1999 and J.L.Tallon, et al., 2003a), and the quasiparticle weight (J.L.Tallon, et al. 2001), show abrupt changes as  $p \rightarrow p_{crit}$ .

The highly anomalous properties HTS cuprates are attributable in part to the presence of a normal state (NS) pseudogap extending across the underdoped region of the phase diagram (P.A.Lee, et al., 2004). The onset of the pseudogap opens up a gap in the density of states (DOS) which profoundly affects all physical properties in both the normal and superconducting (SC) states (J.L.Tallon, et al. 2003b). We believe that the evidence is firmly in support of the latter scenario and present here further supporting evidence, and the SC ground state is strongly perturbed by the pseudogap and major changes taken place at the critical doping state  $p_{crit}$  at which the pseudogap first appears.

To examine the so-called small pseudogap and the superconducting condensation energy  $E_c(0)$ , the electronic specific heat  $C_e$  was calculated from 4-160K for nine samples of  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  with planar hole density

$0.06 < p < 0.24 \equiv 0.03 < x < 0.24$ , where  $p$  was determined from the  $T_c$  values by using the approximate parabolic phase curve  $p = 0.16 \pm [(1 - T_c/T_{c,max})/82.6]^{1/2}$  (Hai-Hu Wen, et al., 2005). The used  $\gamma$  was reported by Loram and et al (1998). The condensation energy  $E_c(T)$  was calculated for these samples from the calculated electronic specific heat  $C_{es}$  and the entropy  $S_s$  of the superconducting phase for each doping rate using the relation (S.J.Fakher, 2005).

$$E_c(T) = \frac{1}{2} \int_0^{T_c} [C_{es}(T) - S_s(T)] dT \quad (1)$$

### **Results and Discussion:**

Figure (1) shows the electronic specific heat  $C_e$  for a series of (La:214) samples of different Sr doping rates ( $0.03 \leq x \leq 0.24$ ) versus temperature  $T$ , the relations which were used to calculate the superconducting and normal  $C_e$  phase is to be found in (S.J.Fakher, 2005 and S.J.Fakher, 2007). It is clear that  $C_e$  reaches a maximum peak at  $T_c$ . The maximum value of  $T_c \sim 38K$  is at  $p_{opt} \sim 0.16$  (holes/CuO<sub>2</sub>). The dotted curve in the figures denotes the optimal doping where  $x = 0.17$  and  $p_{opt} \sim 0.16$  (holes/CuO<sub>2</sub>), the bold curve denote the critical doping where  $x = 0.20$  and  $p_{crit} \sim 0.19$  (holes/CuO<sub>2</sub>).

Low temperature plot  $\gamma^{tot}$  vs  $T^2$  shows the doping independent value of the electronic specific heat coefficient  $\gamma(0) \sim 0$  [mJ/gat.K<sup>2</sup>] for all samples (see Figure 2), which shows also an initial phonon term that at the underdoped region (where  $x < 0.17$ ) decreases with increasing the Sr doping rate, whereas at the overdoped region (where  $x \geq 0.17$ )  $\gamma_s$  value start to increases.

The magnitude  $\Delta\gamma(T_c)$  of the specific heat jumps is presented in Figure(3).  $\Delta\gamma(T_c)$  increases with the increasing of the hole doping, whereas the value of  $T_c$  reaches it's maximum value at  $p_{opt}$  and at  $p > p_{opt}$  begin to decrease. This behavior matches with the  $T_c$  values in Literature, but differs for  $\Delta\gamma(T_c)$  values for BSCCO (J.W.Loram, et al., 2000), TlPb:1212 (C.Panagopoulos, et al., 1999) and YBCO [0,10,20% Ca] (T.Timusk and B.Statt, 1999), which stays almost independent of the doping in more heavily overdoped samples, and this is not the case for the studied samples where at  $p = p_{opt}$  there is a jump in  $\Delta\gamma(T_c) \sim 5$  mJ/gat.K<sup>2</sup>, and increases in the more heavily overdoped samples.

Plots of the entropy  $S(T)$  and  $S/T$  versus temperature  $T$  are presented in Fig.(4) for the samples, the following relation was used to calculate  $S(T)$  (T.Timusk and B.Statt, 1999):

$$S(T) = \int_0^T \gamma(T') dT' \quad (2)$$

The normal state (NS) entropy  $S_n(T)$  extrapolates to zero or positive values at  $T=0K$ , see Fig. (4a), as for a conventional metal with a constant NS DOS. Below  $p_{crit}$  and coincident with the increase of the specific heat jump,  $S_n$  extrapolates also to zero or positive values at  $T=0K$ .  $S/T$  develops a shoulder at  $T_s^*(p)$ , see Fig. (4b), revealing the presence of a pseudogap at EF. Since  $S_n$  and its temperature derivative  $\gamma_n$  are featureless above  $T_c$ , it is clear that  $T_s^*$  reflects the energy scale  $E_g \sim 3T_s^*$  of the pseudogap and not the onset of pseudogap correlations. The entropy curves do not converge at high temperature because of the permanent removal of the missing spectral weight at EF from the room temperature Fermi Window  $EF \pm 100$  meV, a property which is not compatible with a conventional states-conserving Fermi surface instability (J.W.Loram, et al., 2000).

Figure (5) include of accounts, it presents the energy gap  $E_g(p)$  which decreases linearly with  $p$  as  $E_g(p) \sim 1500(1-p/p_{crit})(K)$  (N.D.Mathur, et al., 1998) with  $p_{crit}=0.19$  holes/ $CuO_2$ .  $E_g$  values are close to those found for LSCO and doped YBCO (N.D.Mathur, et al., 1998), and Bi:2212 (S.J.Fakher, 2007). This universal behavior of the pseudogap energy  $E_g(p)$  contrasts sharply with the wide variation in  $T_{c,max}$  (P.A.Lee, et al., 2004), which is shown in Fig.(3).

Values of the condensation energy for zero temperature  $E_c(0)$  obtained by integrating specific heat and entropy data for superconductivity phase for each doping rate using equation (1) are plotted also in Figure (6), which shows that  $E_c(0)$  peaks sharply at  $p_{opt}$  rather than  $p_{crit}$ , and it is due to the approximate vanishing of the energy gap  $E_g$  at this region and the reaching of the transition temperature  $T_c$  reaches its maximum value ( $T_{c,max}$ ). The fact that  $E_c(0)$  passes through a sharp peak at  $p_{opt} \sim 0.16$  rather than  $p_{crit}$  further underscores the unique and sudden character of this optimal doping point. For  $p > p_{opt}$  the decrease in  $E_c(0)$  satisfies the relation  $E_c(0)/\gamma_n T_c^2 \sim 0.52$  (see Fig. 6) expected for a BCS SC with a flat DOS. The figure shows that  $E_c(0)$  decrease rapidly at underdoped region, where  $p < p_{opt}$ , and could be explained on a BCS model when account is taken of the reduced spectral weight in the region  $EF \pm \Delta(0)$  available for pairing (J.W.Loram, et al., 1994). The strongly reduced number of normal excitations below  $T_c$  and the associated weakening of the T-dependence of the free energy difference  $F_n - F_s$  explain the much weaker effect of the pseudogap on  $T_c$ .

For the underdoped region, where  $p < p_{opt}$  and  $x < 0.17$ , the  $E_c(0)$  values decreases because of the decrease of values of  $T_c$  and  $\gamma$  (where the  $E_c(0)$  depends on the  $T_c$  and  $\gamma$  values, pursuant to the equation (S.J.Fakher, 2005):

$$E_c = \frac{1}{2} \gamma T_c - \int_0^{T_c} S_s(T) dT \quad (3)$$

This decrease of  $E_c(0)$  values indicates that there is loss in energy resulting from the existence of the energy gap in this region, this is in agreement with (J.L.Luo, et al., 2003, P.A.Lee, et al., 2004 and S.J.Fakher, 2007) . Note that the  $E_g$  values are obtained from normal state data, but the  $E_c(0)$  values are T=0K properties reflecting the change in free energy in transforming from the normal state to the superconducting ground state, both identify the same critical doping point. Taken together, there is remarkable concurrence between normal state properties and the ground state superconducting properties, which indicates the existence a unique critical doping point in the phase diagram, where the properties changes suddenly and the pseudogap energy falls to zero and the superconductivity is almost robust. The effect of the pseudogap appears, when the values of  $E_g$  begin to increase in this region and the values of doping x% Sr decreases in the sample which leads to decrease the hole concentration in  $\text{CuO}_2$  planes (p), which effects directly and grievous the condensation energy (measure for condensate couples).

Figure (6) shows the calculated ratio  $E_c(0)/\gamma_n T_c^2$  against the hole concentration p holes/ $\text{CuO}_2$  plane , which is constant for  $p > p_{\text{crit}}$  but anomalous behavior for  $p < p_{\text{crit}}$ , further illustrating the abrupt change from conventional metallic to pseudogap behavior  $p < p_{\text{crit}}$ , this behavior is similar to studies of the Bi:2212 samples (S.J.Fakher, 2007). This confirms that the pseudogap (loss of low energy NS spectral weight) persists to  $T \ll T_c$ , suppressing  $E_c(0)$  and weakening the condensate, and is strong evidence against a precursor fluctuation interpretation of the pseudogap above  $T_c$ .

Comparing the results of this work for (La:214) with results of YBCO:123 (0, 20%Ca) (J.W.Loram, et al., 2000) and Bi:2212 (S.J.Fakher, 2007) with same techniques of the pseudogap temperature scale, the results of this study is much higher than for  $\text{YB}_2\text{Cu}_3\text{O}_{6+x}$ ,  $\text{Y}_{0.8}\text{Ca}_{0.2}\text{Ba}_2\text{Cu}_3\text{O}_{6+x}$  and  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  and the pseudogap state extends well into the overdoped region, and this results are in agreement with other techniques results published by T. Timusk and B. Statt (1999) and C. Panagopoulos and et al. (2003).

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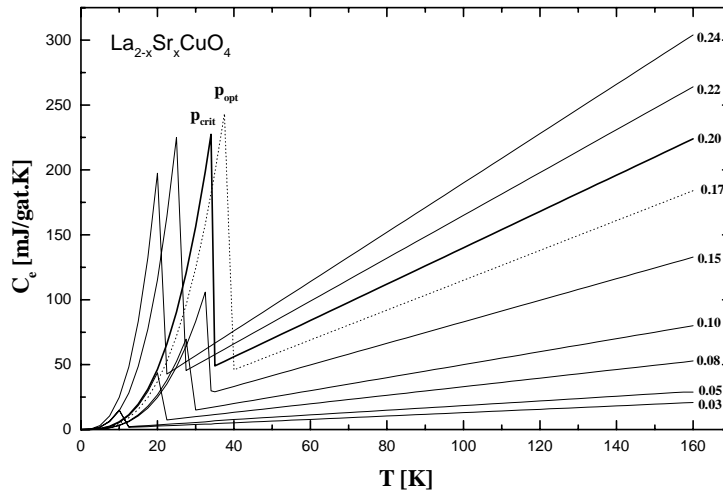
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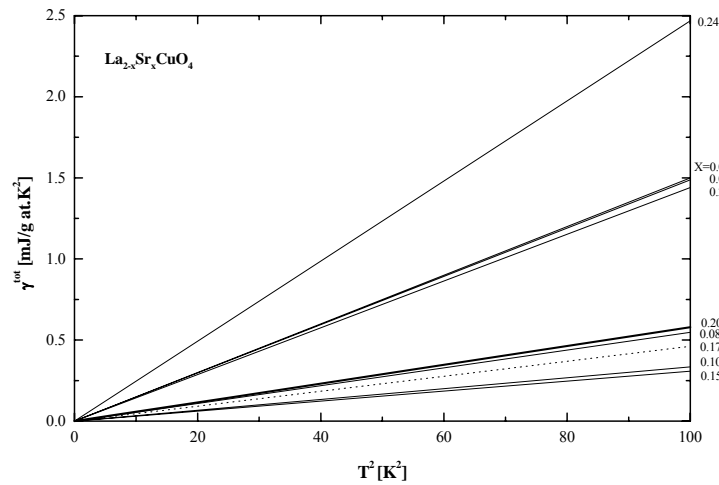
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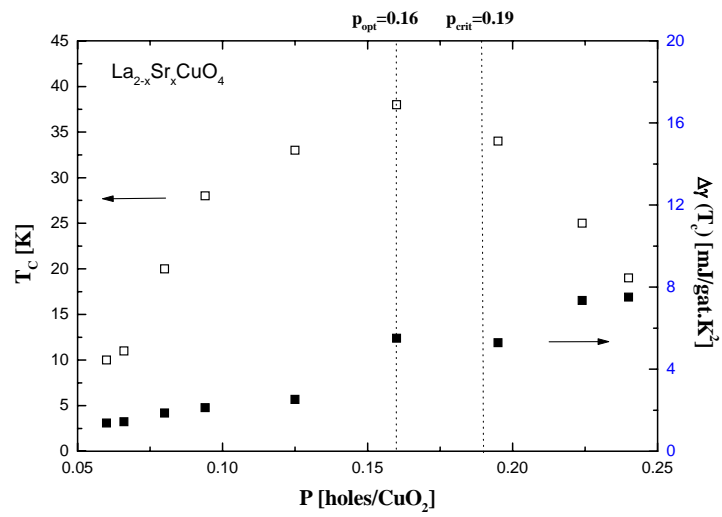




**Figure 1:** The electronic specific heat  $C_e$  versus temperature for a series of La:214 samples of different Sr doping rates ( $0.03 \leq x \leq 0.24$ ). Dotted line represents the optimal doping, bold line the critical doping.



**Figure 2:**  $\gamma^{\text{tot}}$  vs  $T^2$  at low temperature for La:214 samples.



**Figure 3:**  $T_c$  (□) and the anomaly step height  $\Delta\gamma(T_c)$  (■) dependence on doping.

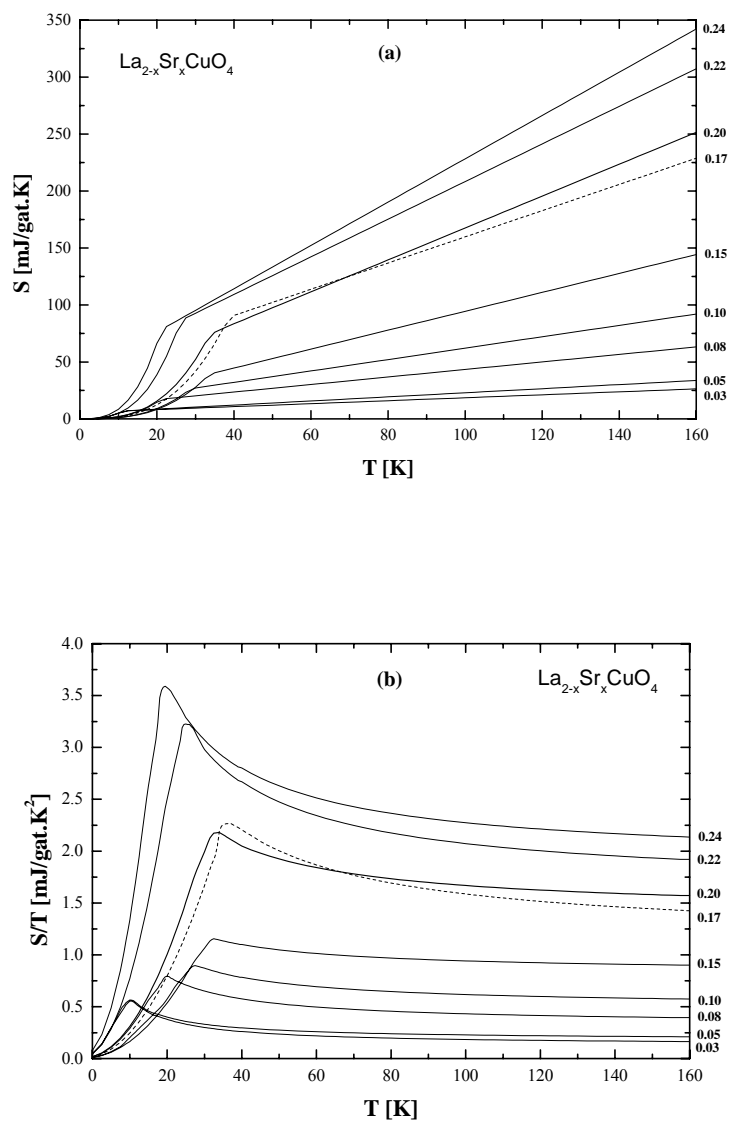
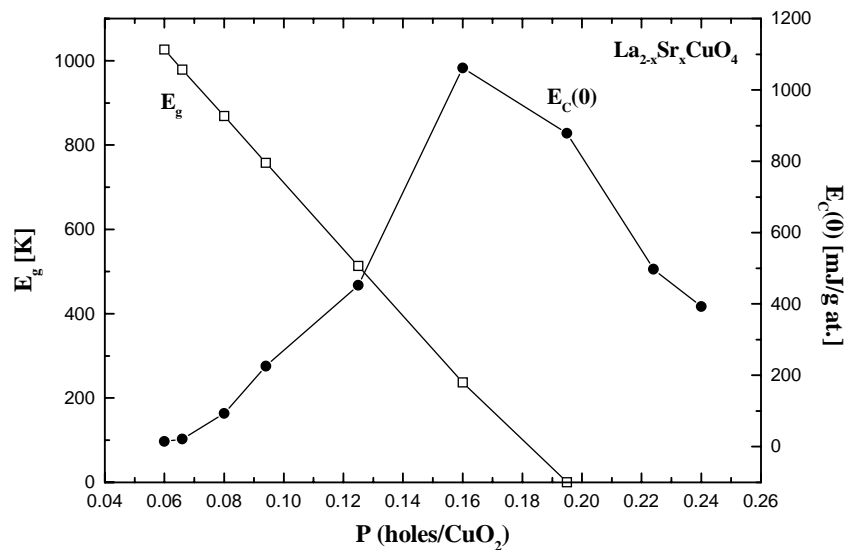
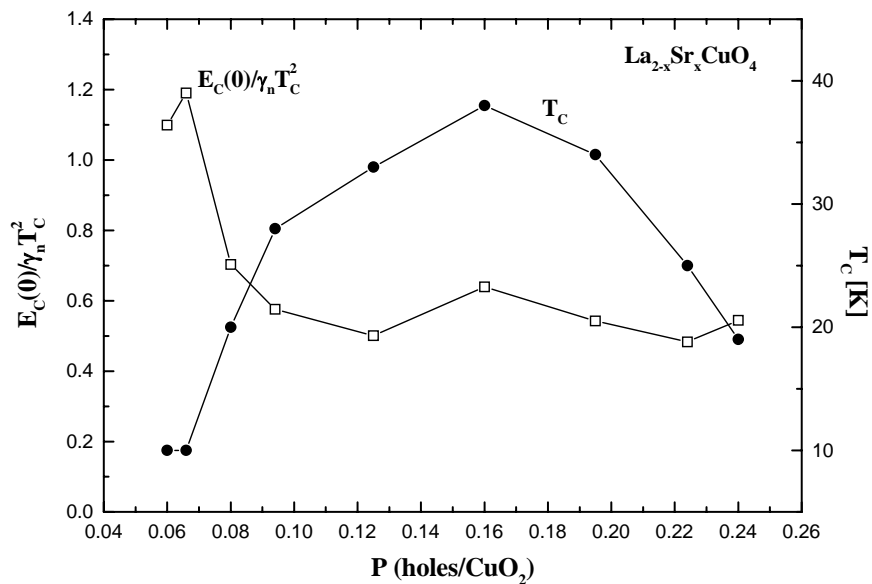


Figure 4: Entropy S (a) and S/T (b) for the La:214 Samples.



**Figure 5:**  $E_g$  and  $E_c(0)$  values vs hole concentration  $p$  for La:214 samples.



**Figure 6:** The  $T_c$  [6] and  $E_c(0)/\gamma_n T_c^2$  values vs hole concentration  $p$  for La:214 samples.