

Analysis of Composite Bridge Superstructures Using Modified Grillage Method

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Abstract

The analysis principle was used to analyze anisotropic plates (having different elastic properties and geometries in different directions), the model consists of four side beams with flexural rigidity and torsional rigidity and two diagonal beams with only flexural rigidity .

The substitute grid framework is analyzed to give the same deformations and deflections of the orthotropic plate elements of the modeled bridge. Applicability of the suggested procedure in the analysis of actual bridge decks is investigated using STAAD Pro.2006 program. The results show that the suggested procedure is an acceptable procedure which can be adopted to analyze this type of bridge deck. It is found that the modified grillage method gives simpler method and adequate results when compared with the Finite Element Method or orthotropic plate theory solved using Finite Difference Method for this type of bridges.

Keywords: composite bridges, superstructures, grillage, orthotropic.

تحليل التراكيب العلوية للجسور المركبة بطريقة المشبكات المعدلة

الخلاصة

مبدأ التحليل يستخدم لوحات متباينة الخواص (أي ذات خواص مرونة مختلفة وشكل هندسي مختلف في كلا الاتجاهين) النموذج يشمل أربعة عتبات جانبية بجساءة انحنائية وبجساءة التوائية وعتبتان قطريتان بجساءة انحنائية فقط.

إن الشبكة المكافئة تم تحليلها لإعطاء نفس التشوهات والانحناء للعناصر الصفيحية الأورثوتروبية للجسور الممنمجة. النتائج أظهرت إن الدراسة الحالية مقبولة ويمكن تبنيها عند تحليل أي جسر من ذلك النوع باستخدام برنامج STAAD Pro.2006 . وجد أن طريقة المشبكات المعدلة تعطي نتائج جيدة مقارنة بطريقة العناصر المحددة و من نظرية الألواح الأورثوتروبية باستخدام طريقة الفروقات المحددة في تحليل الجسور المعنية بالدراسة.

Introduction

After the end of World War II, a new method based on the analogy between a grid system and an orthotropic plate was developed. The fundamentals of this approach were established by Huber [1] in the twenties of the last century. The most difficult problem was establishing a solution to the biharmonic equation governing the plate problem. Guyon [2], in 1946, gave a solution of orthotropic plates of negligible torsional rigidity. He showed

that any variation in the loading can be handled if the coefficients of lateral distribution are employed. Later on Massonnet [3] used the principles given by Guyon to generalize a solution that includes the effect of torsion.

An orthotropic plate is defined as one which has different specified elastic properties in two orthogonal directions. In practice two forms of orthotropy may be identified [4]; material orthotropy and shape

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orthotropy. Most bridge decks are generally [5] orthotropic due to geometric (shape) rather than material differences in two orthogonal directions. More rarely, however, there exists a combination of material and geometric orthotropy.

The concept of considering the bridge as an orthotropic plate for the purpose of determining the distribution of the stress is well established. It was firstly used by Huber in (1914) to analyze reinforced concrete slabs [4]. This was followed by Guyon in (1946), who used the method to analyze a torsionless deck.

Massonnet in (1950) [3] extends Guyon analysis to include the effect of torsional rigidity. He introduces a torsional parameter (f), in order with the original flexural parameter (θ) defined by Guyon as follows:

$$f = \frac{D_{xy} + D_{yx}}{2(D_x D_y)^{0.5}} \quad \text{(Torsional parameter)..... (1)}$$

in which

D_{xy} = torsional rigidity per unit width of the deck.

D_{yx} = torsional rigidity per unit length of the deck.

$$q = \frac{b}{L} \left(\frac{D_x}{D_y} \right)^{0.25} \quad \text{(Flexural parameter) (2)}$$

in which

b = half width of the deck.

L = span of the deck.

Massonnet analysis is limited to decks with torsional parameter (f) ranging from "0" to "1" which represent the limits of no torsional decks and isotropic decks respectively. Rowe in (1955) extended Massonnet method to include the effect of Poisson's ratio. He

made a review on the previous methods and presented applications and extensions of them in his book [6], at a stage before the widespread availability of the electronic digital computer. After Massonnet, Cusens and Pama 1975 [4] rederived the basic equation of orthotropic plate theory and presented new design curves for the longitudinal and transverse moments using nine terms of the series expression. They also derived a solution to the case of batch loading, statically indeterminate, curved, and skew bridge decks. The general treatment of orthotropic plate element is based on the classical Poisson – Kirchhoff assumptions which are specified as follows [7]:

1. The material is perfectly elastic and homogeneous.
2. The thickness is uniform and small as compared to the other dimensions of the plate.
3. The normal strain in the direction transverse to the plane of the plate is negligible and the plate thickness does not undergo any deformation during bending.
4. Points of the plate lying on a normal to middle plane remain on the normal to the middle surface after bending.
5. Deflection of the plate is small compared to the thickness.
6. Body forces are either disregarded or assumed as a part of external loads.
7. External forces are assumed to act perpendicular to the plane of the plate.

Grillage Analysis

A grillage is a plane structure consisting of orthogonally or obliquely rigidly connected beams with three degrees of freedom at each connection

node. The most common method of analysis is the stiffness (or displacement) method based on member stiffness. Alternatively, the grillage may be solved by the flexibility (or force) method, which is usually not applicable to large structures. The total stiffness matrix may be constructed, expressing the nodal forces Q , M_x and M_y in terms of the corresponding nodal displacements w , θ_x and θ_y , respectively, using member stiffnesses.

The grillage is assumed to be a two-dimensional plane structure and the displacements in the plane of the grillage are ignored. As a result, the rotation (θ_z) about the axis normal to the grillage plane is also ignored.

Formulation of Rectangular Element

The rectangular grid-framework model consists of side and diagonal beams as shown in Figure (1a). The cross-sectional properties of the beams are obtained by equating the rotation of the nodes of the grid model with those of a plate element of equal size as shown in Figure (1b), when both are subjected to statically equivalent moments and torsion. A rectangular grid model with five cross-sectional properties will define uniquely a rectangular element of a plate. These properties are chosen to be the flexural and torsional rigidities of the side beams and the flexural rigidity of the diagonals.

Evaluation of the Cross-Sectional Properties

For the grid model to simulate the behavior of the plate element, corresponding rotation must be equal for the two systems (Figure 1). Thus

$$\theta_1 = \theta_6 \dots \dots \dots (3a)$$

$$\theta_2 = \theta_7 \dots \dots \dots (3b)$$

$$\theta_3 = \theta_8 \dots \dots \dots (3c)$$

$$\theta_4 = \theta_9 \dots \dots \dots (3d)$$

$$\theta_5 = \theta_{10} \dots \dots \dots (3e)$$

In this case h_1 and h_2 are not identical in the Eqs. (3b) and (3d).

A- Case 1 $h_1 > h_2$

Taking Eqs. (3a), (3b), and (3c) will provide the flexural rigidity of the grid beams as [8]

$$(EI)_x = \frac{I(E_x h_1^3 - k^2 m E_y h_2^3)}{(E_x h_1^3 - m^2 E_y h_2^3)} \frac{E_x h_1^3}{24} \dots (4)$$

$$(EI)_y = \frac{I E_x h_1^3 (k^2 - m)}{k(E_x h_1^3 - E_y h_2^3 m^2)} \frac{E_y h_2^3}{24} \dots (5)$$

$$(EI)_d = \frac{I m r^3 E_y h_2^3}{k(E_x h_1^3 - m^2 E_y h_2^3)} \frac{E_x h_1^3}{24} \dots (6)$$

The last Eqs. of (3) gives the torsional rigidity as

$$(GJ)_x = \frac{k E I h^3}{24(1+m)} - \frac{2k}{r^3} (EI)_d \dots (7)$$

The torsional rigidity of the side beams of length λ may be found as [8, 9]

$$(GJ)_y = \frac{E I h^3}{24(1+m)} - \frac{2k}{r^3} (EI)_d \dots (8)$$

B- Case 2 $h_1 < h_2$

Taking Eqs. (3a), (3d), and (3c) will provide the flexural rigidity of the grid beams as

$$(EI)_x = \frac{I E_y h_2^3 (1 - k^2 m)}{(E_y h_2^3 - m^2 E_x h_1^3)} \frac{E_x h_1^3}{24} \dots (9)$$

$$(EI)_y = \frac{I (k^2 E_y h_2^3 - m E_x h_1^3)}{k(E_y h_2^3 - E_x h_1^3 m^2)} \frac{E_y h_2^3}{24} \dots (10)$$

$$(EI)_d = \frac{I m r^3 E_y h_2^3}{k(E_y h_2^3 - m^2 E_x h_1^3)} \frac{E_x h_1^3}{24} \dots(11)$$

The last Eqs. of (3.34) gives the torsional rigidity as

$$(GJ)_x = \frac{k E I h^3}{24(1+m)} - \frac{2k}{r^3} (EI)_d \dots(12)$$

The torsional rigidity of the side beams of length λ may be found as [8, 9]

$$(GJ)_y = \frac{E I h^3}{24(1+m)} - \frac{2k}{r^3} (EI)_d \dots(13)$$

A square grid pattern is, in most cases, preferable to a rectangular grid pattern as the former will provide better results. Where rectangular grid patterns are needed (for instance, to fit the geometry of the boundaries), the values of k should be in the range $1/2 \leq k \leq 2$.

For a plate with $h_1=h_2$, and $E_x=E_y$ the expressions (4) to (13) in two cases reduce to equations [8, 9]

$$I_y = \frac{(k^2 - m) I h^3}{2k(1 - m^2) 12} \dots\dots\dots (14)$$

$$I_x = \frac{(1 - m k^2) I h^3}{2(1 - m^2) 12} \dots\dots\dots (15)$$

$$I_d = \frac{m r^3 I h^3}{2k(1 - m^2) 12} \dots\dots\dots (16)$$

$$\frac{GJ_x}{E} = \frac{(1 - 3m) I h^3}{2(1 - m^2) 12} \dots\dots\dots (17)$$

$$\frac{GJ_y}{E} = \frac{(1 - 3m) k I h^3}{2(1 - m^2) 12} \dots\dots\dots (18)$$

Application

The analytical result curves for the bridges have been obtained using

the procedure presented in sections 3 and 4 and checked by STAAD Pro.2006 Program. These curves have been compared with the experimental results and predicted analytical result curves which are obtained (using the orthotropic plate theory) by finite difference model [10].

Equivalent Plate Rigidities of Composite Slab-on-Beam Deck

There are different methods to calculate the values of equivalent plate rigidities D_x , D_y , D_{xy} , and D_{yx} for different types of bridge decks.

Typical cross section of composite slab-on-beam deck is shown in Figure (2a). Figure (2b) shows a typical section of concrete T-beam. The equivalent plate rigidities of such decks can be obtained as follows [11, 12]:

$$D_x = \frac{E_g}{P_y} I_g \dots\dots\dots (19)$$

$$D_y = \frac{E_c t^3}{12(1 - m_c^2)} = \frac{E_c t^3}{12} \dots\dots\dots (20)$$

$$D_{xy} = \frac{G_s J}{P_y} + \frac{G_c t^3}{6} \dots\dots\dots (21)$$

$$D_{yx} = \frac{G_c t^3}{6} \dots\dots\dots (22)$$

where " I_g " is the combined second moment of beam area and associated portion of deck slab in units of beam material. The subscript "g" applied to "E" and "G" refers to the material of the beam. $G = E/2(1+\mu)$, $n = E_s/E_c$ where subscript "s" and "c" refer to steel and concrete respectively, and "J" is the torsional constant.

Equivalent plate rigidities corresponding to reinforced concrete are calculated by ignoring the steel reinforcement and by assuming that, the concrete is uncracked. It is used to

ignore the reduction of equivalent plate rigidities due to cracking. The torsional constant of a beam section is often calculated by dividing the section into a number of rectangles. The torsional constant of a rectangle of sides b_w and d , where d is the smaller side, is given as [11,12]

$$J = kb_w d^3 \quad (23)$$

where "k" can be obtained from Table (1)

Application: Analysis of bridge with steel girders

To study the reliability of the method and its applicability in actual analysis, a small bridge deck sealed with high molecular weight methacrylate (slab and overlay) is analyzed for four concentrated loads. In an effort to stop or slow down the corrosion process in existing bridge decks, the bridge deck may be sealed by a high molecular weight methacrylate or gravity fed epoxy resin prior to the application of portland cement overlay. In addition to sealing the existing cracks, the sealer stops the intrusion of water and chlorides if the overlay cracks [14].

This test was done by Cole et al [14]. The presence of a sealer at the deck – overlay interface is expected to reduce the available bond strength. This test was carried out to investigate the performance of the overlays placed over sealed bridge decks (to examine the level of bond strength). Test results indicate that the sealer reduces the available bond strength by as much as 50 %. Up to 85 % of the bond strength can be restored if sand is broadcast over the sealer while it is curing or if dried sealed surface is lightly sanded.

In calculating bridge deck rigidities, the bond strength is assumed

to be equal to 100%. From beam tests, bond strength between overlay and bridge deck becomes more critical when the overlay is subjected to positive moments, therefore, a simply supported specimen was selected [14].

Geometry and Structure

The test deck is a right simply supported deck, of 6.096 m span and 2.438 m width. The clear span is 5.486 m. Plan and cross-sectional dimensions are shown in Figure (3). The deck was constructed from three W12x19 steel beams and L1x1x1/8 cross braces. The compressive strength of deck was 61 MPa at the time of testing and the compressive strength of overlay was 52 MPa when the specimen was loaded. The Poisson's ratio of concrete μ was taken as 0.18 [14].

The flexural and torsional rigidities of the equivalent orthotropic plate (D_x , D_y , D_{xy} and D_{yx}) needed for analysis are calculated using formulas which were suggested by Al-Dawar [12] and Flaih [10]. Details of dimensions, material properties, flexural and torsional rigidities for this bridge deck are tabulated in Table (2) [10]. The deck is assumed to act composite with longitudinal beams and with horizontal cross braces, while the effect of diagonal cross braces is neglected. The moduli of elasticity of the longitudinal beams and cross braces are 205 GPa and 215 GPa respectively.

The equivalent rigidities of the test deck ($(EI)_x$, $(EI)_y$, $(GJ)_x$, $(GJ)_y$ and $(EI)_d$ needed for the analysis are calculated using Eqs.(4) through (8). Details of dimensions ($\lambda = 406\text{mm}$, $k\lambda = 499\text{mm}$), material properties, flexural and torsional rigidities for this bridge deck are tabulated in Table (3).

Load and Analysis

The specimen was loaded statically up to 48.9 kN. This load was applied to the slab through a transfer beam positioned on four 152 mm x 152mm x 12.7mm steel plates placed at the intended load points as seen in Figure (3) [14]. Concentrated loads are linearly distributed to adjacent nodes based on the location of the loading as presented in Figure (4) [13]. The modified grillage mesh which is adopted for the analysis consists of "84" nodes represented by "7" transverse nodes in "12" longitudinal rows equally spaced along the span of the bridge deck model as shown in Figure (5).

By using modified grillage technique, the values of deflections (w) are obtained. The distributed load and maximum deflection of the bridge are as shown in Figures (6) and (7) respectively. These results are compared with those found from the test of bridge deck and finite difference method. The deflection profile along the length of the middle girder is shown in Figure (8).

Discussion of Results

For modified grillage method, it can be seen that acceptable values are obtained for the deflections of the middle girder if compared with those found during field test and finite difference analysis. The obtained results are compared in Table (4).

The examination of the applicability, limitation, accuracy and economy of the modified grillage analysis has been the main concern of the present work. This was achieved through a comparison with other commonly used but rather sophisticated analytical techniques namely, the finite

difference and the finite element methods.

The analysis of bridge decks can be made by using different approaches; the modified grillage theory is one of these approaches, the effect of Poisson's ratio is taken into consideration.

Conclusions

The following conclusions based upon the findings of this study:

1. The modified grillage technique is easy to use and gives an acceptable accuracy for the elastic analysis of simply supported right bridge decks.
2. The proposed technique for the analysis of slab-on-beam bridge deck type including support edge deflections given in this work has shown slight difference compared with those neglecting support edge deflections.
3. Right bridges with steel girders with cross braces may be successfully analyzed using modified grillage method with good accuracy.

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Table (1) Torsional Coefficient K for Rectangular Sections [12]

b_w/d	A	1.2	1.5	1.75	2	2.25	2.5	3	4	5	∞
k	0.141	0.166	0.196	0.213	0.229	0.24	0.249	0.263	0.281	0.291	0.333

Table (2) Values of Flexural and Torsional Rigidities for Test Specimen [10]

Flexural rigidities	D_x	4.73×10^{10} N.mm
	D_y	6×10^9 N.mm
Torsional rigidities	D_{xy}	1.221×10^9 N.mm
	D_{yx}	1.217×10^9 N.mm

Table (3) Main Properties of Beams by Equivalent Plate for Bridge

Type of Rigidities	Value (N.mm ²)
$(EI)_x$	$9.308975704 \times 10^{12}$
$(EI)_y$	$1.324062172 \times 10^{12}$
$(EI)_d$	$7.125244264 \times 10^{11}$
$(GJ)_x$	$4.60899615 \times 10^{12}$
$(GJ)_y$	$3.667946914 \times 10^{12}$

Table (4) Middle Girder Deflections in mm

Node No.	M1[10] [D1=D2= μ (smallest value of Dx and Dy)]	M2 [10] [D1=D2= $\mu(Dx.Dy)^{0.5}$]	M3 [10] [D1= μ Dx and D2= μ Dy]	Exp.[14]	Modified Grillage
38	0.326	0.322	0.314	0.303	0.320
39	0.634	0.626	0.61	0.66	0.618
40	0.904	0.891	0.87	0.9	0.877
41	1.12	1.1	1.07	1.05	1.075
42	1.24	1.22	1.19	1.2	1.192
43	1.26	1.24	1.21	1.17	1.207
44	1.18	1.16	1.13	1.08	1.125
45	0.997	0.979	0.953	0.93	0.951

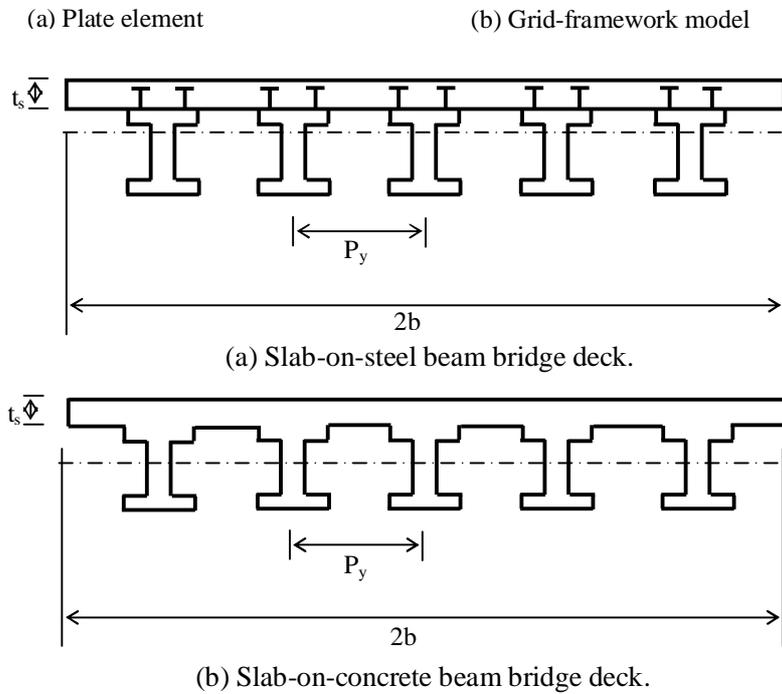


Figure (2) Cross Section of Composite Slab-On-Beam Deck with Concrete Deck Slab

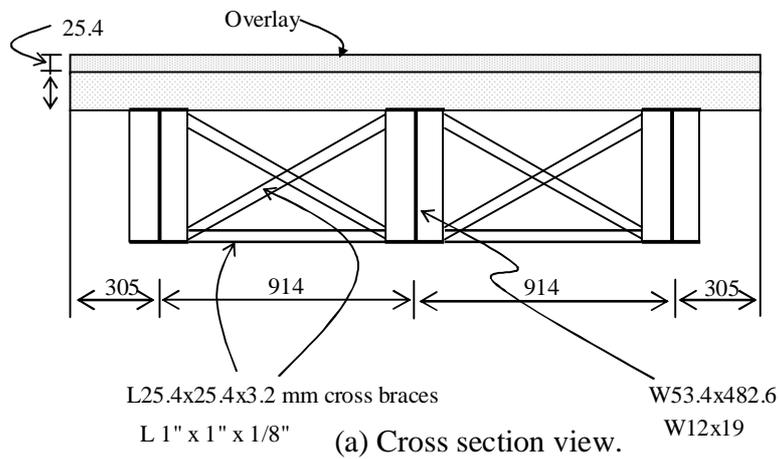
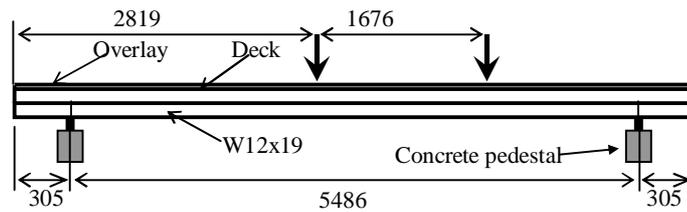
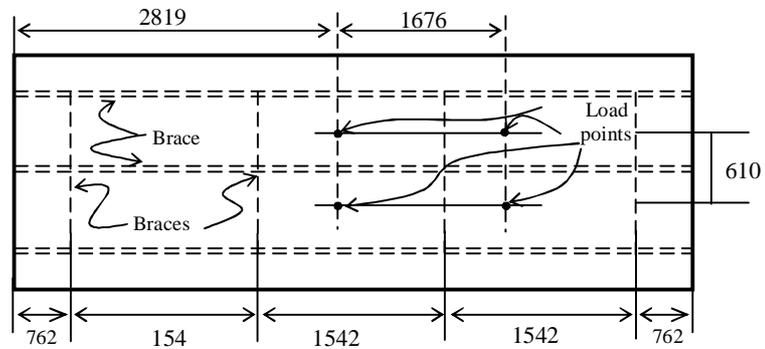


Figure (2) Cross Section of Composite Slab-On-Beam Deck with Concrete Deck Slab



(c) Loading arrangement.

Figure (3) Plan and Cross-Section of the Composite Bridge [14].
(All Dimensions are in mm.)

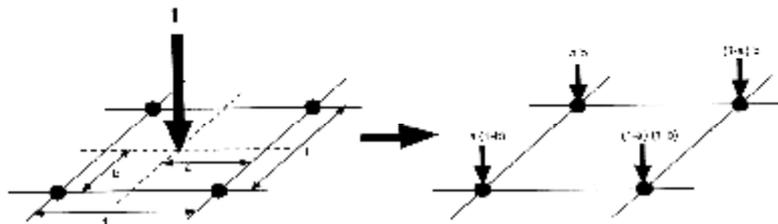


Figure (4) Distribution of Concentrated Load.

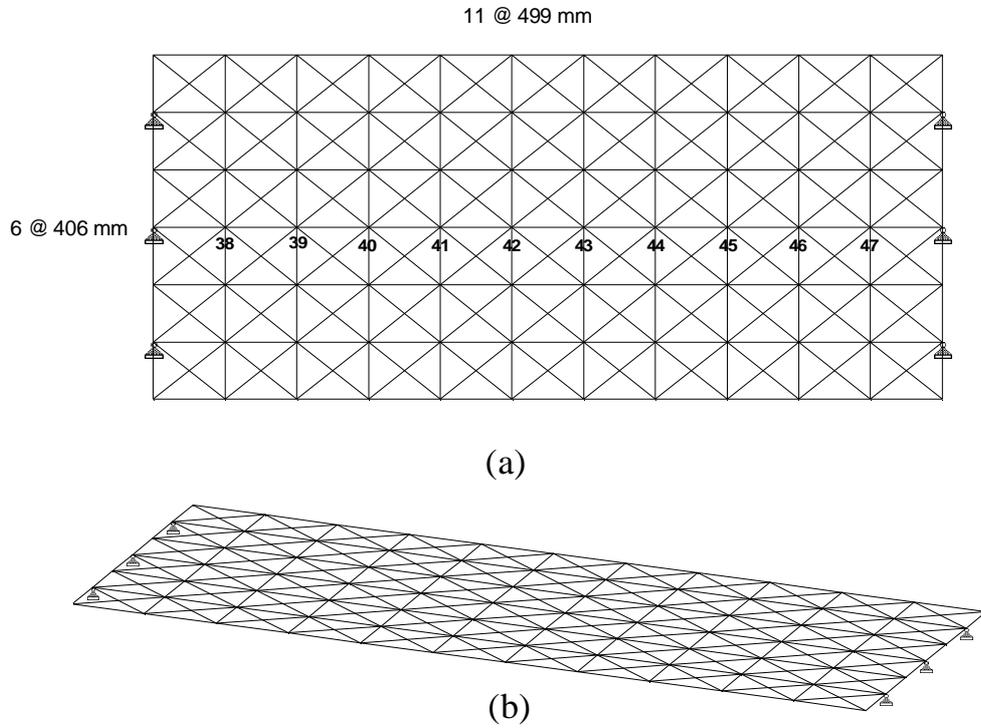


Figure (5) Bridge Deck Model by Modified Grillage.

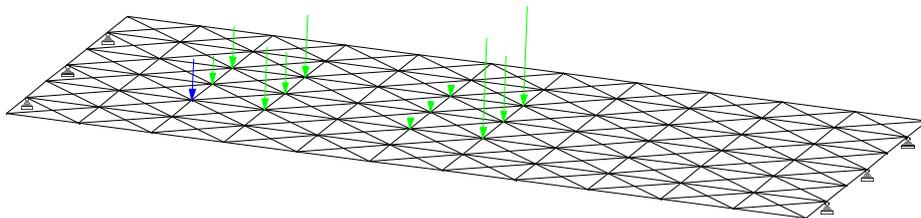


Figure (6) Distribution of Concentrated Load for Bridge Deck Model.

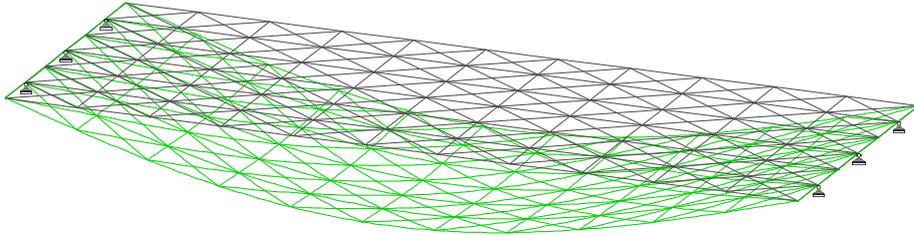


Figure (7) Maximum Deflection in Bridge Deck Model.

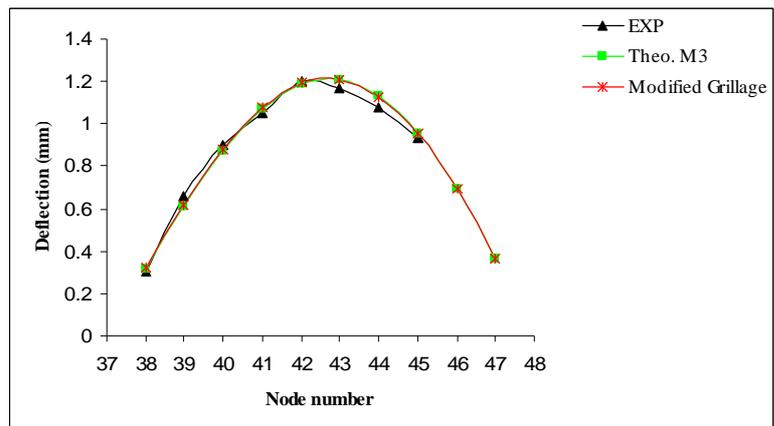


Figure (8) Deflection Profile for Middle Girder.