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Slip Effect on the Peristaltic Transport of MHD Fluid through a Porous Medium with Variable Viscosity

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Abstract

The present paper concerns with peristaltic analysis of MHD viscous fluid in a two dimensional channel with variable viscosity through a porous medium under the effect of slip condition. Along wave length and low Reynolds number assumption is used in the problem formulation. An analytic solution is presented for the case of hydrodynamic fluid while for magneto hydrodynamic fluid a series solution is obtained in the small power of viscosity parameter. The salient features of pumping and trapping phenomena are discussed in detail through a numerical integration. The features of the flow characteristics are analyzed by plotting graphs and discussed in detail. When $K \to \infty$.

Keywords: Peristaltic transport, Slip condition, Variable viscosity, MHD fluid, Porous medium.

تأثير الانزلاق على الانتقال التموجي لمائع مغناطيسي هايدروديناميكي خلال وسط مسامي مع لزوجة متغيره

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الخلاصة

هذا البحث يهتم بتحليل التموج لمائع مغناطيسي هايدروديناميكي لزج في قناة ثنائية مع لزوجة متغيره خلال وسط مسامي تحت تأثير شرط الانزلاق. فرضية طول الطول الموجي وعدد رينولد صغير قد استخدمت في هذه المسالة. قدم الحل التحليلي في حالة المائع الهايدروديناميكي بينما للمائع المغناطيسي الهايدروديناميكي قد حصلنا على سلسلة حلول لقوة صغيرة لعامل اللزوجة. السمات البارزه للضخ ومحاصرة الضواهر قد نوقشت بالتفاصيل من خلال تكامل عددي. حللنا ميزات خصائص الجريان بالرسوم ونوقشت بالتفاصيل. عندما $K \to \infty$

1. Introduction

Peristaltic pumping has been the object of scientific and engineering research in recent years. The word peristaltic comes from a Greek word "peristaltikos" which means clasping and compressing. The phenomenon of peristalsis is defined as expansion and contraction of an extensible tube in a fluid generate progressive waves which propagate along the length of the tube, mixing and transporting the fluid in the direction of wave propagation. The study of mechanism of peristalsis in both mechanical and physiological situations has recently become the object of scientific research. Several theoretical and experimental attempts have been made to understand peristaltic action in different situations. A review of much of the early literature is presented in an article by Shapero A. H., Jaffrin M. Y., and

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Weinberg S. L. [2]. A summary of most of the experimental and theoretical investigation reported with details of the geometry, fluid Reynolds number, wavelength parameter, wave amplitude parameter and wave shape have been studied by Srivastava LM, Srivastava VP [3]. The studies of peristaltic flows of Newtonian and non-Newtonian fluids have become important, not only because of biomedical and engineering sciences but also in view of the interesting mathematical features presented by the equations of governing the flow. The peristaltic flows and non-Newtonian fluids have been studied due to their application in urine transport from the kidney to bladder, swallowing food through the esophagus, chyme motion in the gastrointestinal tract, transport of spermatozoa, movement of ovum in the female fallopian tube and vasomotion of small blood vessels. The importance of such flow has also been recognized in transport of slurries, corrosive fluids and noxious fluids in the nuclear industry

Further, roller and finger pumps are widely operated under the mechanism. In most of the studies which deal with the peristaltic flows, the fluid viscosity is assumed to be constant. This assumptions is not valid everywhere. In general the coefficients of viscosity for real fluids are functions of space coordinates, temperature, and pressure. For many liquids such as water, oil, and blood, the variation of viscosity due to space coordinate and temperature change is more dominant than other effects. Therefore, it is highly desirable the include of variable viscosity instead of considering the viscosity of the fluid to be constant. Some important studies related to the variable viscosity are effect of variable viscosity on the peristaltic transport of Newtonian fluid in an a symmetric channel studied by Hayat T., Ali N. [4], peristaltic transport of a Jeffrey fluid with variable viscosity through a porous medium In an a symmetric channel studied by Afsar Khan A., Ellahi R., Vafai K. [5] and slip effect on the peristaltic transport of MHD fluid with variable viscosity studied by N. Ali, Q. Hussain, T. Hayat and S. Asghar [1].

Flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. Examples of natural porous media are beach sand, stand stone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. The first study of peristaltic flow through a porous medium is presented by Elshehawey E. F., Mekheimer Kh. S., Kalads S. F., Afifi N. A. S. [6], nonlinear peristaltic transport of MHD flow through a porous medium is presented by Mekheimer Kh. S, Al-Arabi T. H. [7], Effect of porous medium and magnetic field on peristaltic transport of a Jeffrey fluid is presented by Mahmoud S. R., Afifi N. A. S., Al-Isede H. M. [8], and peristaltic transport of conducting fluid through a porous medium in an asymmetric vertical channel is studied by Rami Reddy G., Venkataramana S. [9].

This paper discuss the peristaltic transport of MHD fluid with variable viscosity through a porous medium under the effect of slip condition. A regular perturbation method is used to so- . α -lve the problem, and the solutions are expanded in a power series of viscosity parameter The obtained expressions are utilized to discuss the influences of various emerging parameters.

2. The mathematical formulation of the problem

Consider the two-dimensional channel of uniform thickness 2a, which is filled with an incompressible viscous fluid with an incompressible viscous fluid with variable viscosity through porous medium. The walls of the channel are flexible and non-conducting. The sinusoidal wave trains propagate on the channel walls with constant speed c and propel the, the geometry of the wall surf- $(\bar{x},\bar{y})$ fluid along the walls. In rectangular coordinates system -ace is described by:

$$\bar{h}(\bar{X},\bar{t}) = a + bCos\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) \tag{1}$$

is the velocity of c is the wave length, λ is the wave amplitude, b In above equation is the direction of wave propagation. A uniform magnetic field $\bar{X}$ is the time and $\bar{t}$ propagation, is applied in the transverse direction to the flow. The electric field is taken zero, the mag- B_0 -netic Reynolds number is taken small so that the induced magnetic field is neglected.

In wave frame, the equations which govern the flow are ,

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{2}$$

$$\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} + 2\frac{\partial}{\partial\bar{x}}\left(\bar{\mu}(\bar{y})\frac{\partial\bar{u}}{\partial\bar{x}}\right) + \frac{\partial}{\partial\bar{y}}\left(\bar{\mu}(\bar{y})\left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\right)\right) - \sigma B_0^2(\bar{u}+c) - \frac{\nu}{\bar{K}}(\bar{u}+c) \tag{3}$$

$$\rho\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right)=-\frac{\partial\bar{p}}{\partial\bar{y}}+2\frac{\partial}{\partial\bar{y}}\left(\bar{\mu}(\bar{y})\frac{\partial\bar{v}}{\partial\bar{y}}\right)+\frac{\partial}{\partial\bar{x}}\left(\bar{\mu}(\bar{y})\left(\frac{\partial\bar{v}}{\partial\bar{x}}+\frac{\partial\bar{u}}{\partial\bar{y}}\right)\right) \tag{4}$$

$\bar{\mu}(\bar{y})$ is the pressure and $\bar{p}$ is the strength of the magnetic field, B_0 is the density, ρ where is the viscosity function.

The flow is unsteady. However, if observed in a coordinates $(\bar{X},\bar{Y})$ in laboratory frame it can be treated as steady. The coordinate $(\bar{x},\bar{y})$ system moving at wave speed c (wave frame) frame are related in the following:

$$\bar{p}(\bar{x})=\bar{p}(\bar{x},\bar{t}) \quad ,\bar{v}=\bar{V} \quad ,\bar{u}=\bar{U}-c \quad ,\bar{y}=\bar{Y} \quad ,\bar{x}=\bar{X}-c\bar{t} \tag{5}$$

are the respective velocity components in the corresponding $\bar{u},\bar{v}$ and $\bar{U},\bar{V}$ In which coordinates system.

To make the equations (2), (3), (4) non-dimensional, it is convenient to introduce the following non-dimensional variable and parameters [1]:

$$h=\frac{\bar{h}}{a} \quad ,t=\frac{c\bar{t}}{\lambda} \quad ,S=\frac{a\bar{S}}{\mu_0 c} \quad ,p=\frac{a^2\bar{p}}{c\lambda\mu_0} \quad ,v=\frac{\bar{v}}{c\delta} \quad ,u=\frac{\bar{u}}{c} \quad ,y=\frac{\bar{y}}{a} \quad ,x=\frac{\bar{x}}{\lambda} \ ,$$

$$.M=\sqrt{\frac{e}{\mu}}aP_0 \quad ,\mathrm{Re}=\frac{ca\rho}{\mu_0} \quad ,\bar{K}=\frac{\kappa}{a^2} \quad ,\delta=\frac{a}{\lambda} \quad ,\mu(y)=\frac{\bar{\mu}(\bar{y})}{\mu_0} \quad ,\phi=\frac{b}{a} \tag{6}$$

In which M is the Hartman number, Re, is the Reynolds number, σ is the electrical conductivity, δ ,is the wave number and μ_0 is the constant viscosity.

Substituting eq. (6) into Eqs. (2) - (4) can be simplified into the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+\frac{1}{\delta}v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+2\delta^2\left(\mu(y)\frac{\partial^2 u}{\partial x^2}\right)+\frac{\partial}{\partial y}\left(\mu(y)\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right)-\frac{v}{\mu_0\bar{K}}(u+1)$$
$$-M^2(u+1) \tag{8}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+2\delta^2\left(\mu(y)\frac{\partial^2 v}{\partial y^2}\right)+\delta^2\frac{\partial}{\partial x}\left(\mu(y)\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right) \tag{9}$$

In above equations and under the $\left(u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}\right)$ Introducing the stream function as, we have: $\frac{1}{\kappa}=\frac{v}{\mu_0\bar{K}}$ assumptions of long wavelength $(\delta\ll 1)$ and low Reynnolds number, let

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right)-\frac{1}{\kappa}\left(\frac{\partial\psi}{\partial y}+1\right)-M^2\left(\frac{\partial\psi}{\partial y}+1\right) \tag{10}$$

$$0=-\frac{\partial p}{\partial y} \tag{11}$$

3. Rate of volume flow and boundary conditions

Coordinates system is given $(\bar{X},\bar{Y})$ the instantaneous volume rate of flow in the fixed by

$$Q=\int_0^{\bar{h}(\bar{X},t)}\bar{U}(\bar{X},\bar{Y},t)d\bar{Y}\ , \tag{12}$$

where $\bar{t}$ and $\bar{X}$ is the function of $\bar{h}$

The above expression in the wave frame equation (12) is given by

$$q = \int_0^{\bar{h}} \bar{u}(\bar{x}, \bar{y}) d\bar{y} \quad (13)$$

where $\bar{h}$ is a function of $\bar{x}$ alone. Employing equations (5),(12) and (13), the two volume flow rates can be related through the following relation

$$Q = q + ch. \quad (14)$$

can be written as $\bar{X}$ at a fixed position $T = \frac{\lambda}{c}$ The time mean flow over a period

$$\bar{Q} = \frac{1}{T}\int_0^T Q dt \ . \quad (15)$$

Invoking eq. (14) into eq. (15) and integrating it, we arrive at

$$\bar{Q} = q + ac. \quad (16)$$

In the wave frame F in the fixed frame and θ defining the dimensionless time-mean flow one has

$$F = \frac{q}{ac} \ , \quad \theta = \frac{\bar{Q}}{ac} \ , \quad (17)$$

the eq. (16) may be rewritten as $\theta = F + 1$ (18)

where

$$F = \int_0^h \frac{\partial \psi}{\partial y} dy = \psi(h) - \psi(0). \quad (19)$$

$(y = 0)$If we select the zero value of the stream line at the center line, $\psi(0) = 0$ is a stream line of value $y = h$ then the wall at $\psi(h) = F$

The boundary conditions in dimensionless stream function will now take the following form

$$y = 0 \text{ at} \quad , \frac{\partial^2 \psi}{\partial y^2} = 0 \quad , \psi = 0 \ , \quad (20a)$$

$$y = h = 1 + \phi Cos(2\pi x) \text{at} \quad , \frac{\partial \psi}{\partial y} + \beta\mu(y)\frac{\partial^2 \psi}{\partial y^2} = -1 \quad , \psi = F \ . \quad (20b)$$

Is $\left(\phi = \frac{b}{a} \langle 1\right)$ is the dimensionless viscosity function, $\mu(y)$ is the slip parameter, β where the amplitude ratio and h is the dimensionless form of the peristaltic wall.

In the forthcoming analysis, we will use $\alpha \langle\langle 1$ for $\mu(y) = 1 - \alpha y$ or $\mu(y) = e^{-\alpha y}$ is the viscosity parameter. α Where Solution of the problem Case (1) $\boldsymbol{M = 0}$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2 \psi}{\partial y^2}\right) - \frac{1}{\kappa}\left(\frac{\partial \psi}{\partial y} + 1\right) \quad (21)$$

$$0 = -\frac{\partial p}{\partial y} \quad (22)$$

By differentiating equation (21) with respect to y, the resulting equation is:

$$0 = \frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2 \psi}{\partial y^2}\right) - \frac{1}{\kappa}\frac{\partial^2 \psi}{\partial y^2} \quad (23)$$

For small parameter α ,we can write

$$\psi = \psi_0 + \alpha\psi_1 + O(\alpha^2) \quad (24)$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + \alpha\frac{dp_1}{dx} + O(\alpha^2) \quad (25)$$

$$F = F_0 + \alpha F_1 + O(\alpha^2) \quad (26)$$

Using Eqs. (24) – (26) and (20 a), (20 b) and then comparing the coefficient of like powers of α, we have: The zero order system with its boundary conditions are

$$0=\frac{\partial^4\psi_0}{\partial y^4}-\frac{1}{\kappa}\frac{\partial^2\psi_0}{\partial y^2} \tag{27}$$

$$\frac{dp_0}{dx}=\frac{\partial^3\psi_0}{\partial y^3}-\frac{1}{\kappa}\left(\frac{\partial\psi_0}{\partial y}+1\right), y=0\text{ a } , \tag{28}$$

$$\frac{\partial^2\psi_0}{\partial y^2}=0 \quad ,\psi_0=0 \tag{29a}$$

$$y=h\text{ at} \quad ,\frac{\partial\psi_0}{\partial y}+\beta\frac{\partial^2\psi_0}{\partial y^2}=-1 \quad ,\psi_0=F_0\ . \tag{29b}$$

The first order system with its boundary conditions are

$$0=\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\psi_1}{\partial y^2}-y\frac{\partial^2\psi_0}{\partial y^2}\right)-\frac{1}{\kappa}\frac{\partial^2\psi_1}{\partial y^2} \tag{30}$$

$$\frac{dp_1}{dx}=\frac{\partial}{\partial y}\left(\frac{\partial^2\psi_1}{\partial y^2}-y\frac{\partial^2\psi_0}{\partial y^2}\right)-\frac{1}{\kappa}\frac{\partial\psi_1}{\partial y} \tag{31}$$

$$\psi_1=0\,,\frac{\partial^2\psi_1}{\partial y^2}=0\,,\ y=0\text{ at}\,, \tag{32a}$$

$$y=h\text{ at} \quad ,\frac{\partial\psi_1}{\partial y}+\beta\left(\frac{\partial^2\psi_1}{\partial y^2}-y\frac{\partial^2\psi_0}{\partial y^2}\right)=0 \quad ,\psi_1=F_1\ . \tag{32b}$$

It is found that solution of zero order system is given by

$$\psi_0=\left(F_0\sqrt{K}\,yCosh\left[\frac{h}{\sqrt{K}}\right]+y(K+F_0\beta)Sinh\left[\frac{h}{\sqrt{K}}\right]-(F_0+h)KSinh\left[\frac{y}{\sqrt{K}}\right]\right)/$$
$$\left(h\sqrt{K}Cosh\left[\frac{h}{\sqrt{K}}\right]+(-K+h\beta)Sinh\left[\frac{h}{\sqrt{K}}\right]\right) \tag{33}$$

$$\frac{dp_0}{dx}=(F_0+h)\left(\sqrt{K}Cosh\left[\frac{h}{\sqrt{K}}\right]+\beta Sinh\left[\frac{h}{\sqrt{K}}\right]\right)/-hK^{3/2}Cosh\left[\frac{h}{\sqrt{K}}\right]+K$$
$$(K-h\beta)Sinh\left[\frac{h}{\sqrt{K}}\right] \tag{34}$$

And the solution of first order system is found to be of the form

$$\psi_1=\left(-h(F_0+h)\sqrt{K}\,y^2\beta Cosh\left[\frac{y}{\sqrt{K}}\right]+4F_1y(K+\beta)\left(h\sqrt{K}Cosh\left[\frac{h}{\sqrt{K}}\right]+(-K+h\beta)\right.\right.$$
$$\left.Sinh\left[\frac{h}{\sqrt{K}}\right]\right)+h\sqrt{K}\left(\sqrt{K}\left(-(F_0+h)(h-y)\beta+4F_1(K-h\beta)\right)+h\left(-4F_1K+h\right.\right.$$
$$\left.\left.\left.(F_0+h)\beta\right)Coth\left[\frac{h}{\sqrt{K}}\right]\right)Sinh\left[\frac{y}{\sqrt{K}}\right]\right)/\left(4h\beta\left(h\sqrt{K}Cosh\left[\frac{h}{\sqrt{K}}\right]+(-K+h\beta)\,Sinh\left[\frac{h}{\sqrt{K}}\right]\right)\right) \tag{35}$$

$$\frac{dp_1}{dx}=-F(K+\beta)/hK\beta \tag{36}$$

Summarizing the perturbation results, the expression of stream function and longitudinal pressure up to order α are:

$$\psi = \psi_0 + \alpha\psi_1$$
$$= \left(h\sqrt{K}y\left(4(F_0\beta + F_1\alpha(K+\beta))Cosh\left[\frac{h}{\sqrt{K}}\right] - (F_0+h)y\alpha\beta Cosh\left[\frac{y}{\sqrt{K}}\right]\right) + 4y(h\beta\right.$$
$$(K+F_0\beta) - F_1\alpha(K+\beta)(K-h\beta))Sinh\left[\frac{h}{\sqrt{K}}\right] + h\sqrt{K}\left(\sqrt{K}(-(F_0+h)(4+h\alpha-y\alpha)\right.$$
$$\beta + 4F_1\alpha(K-h\beta)) + h\alpha(-4F_1K + h(F_0+h)\beta)Coth\left[\frac{h}{\sqrt{K}}\right]\Big)Sinh\left[\frac{y}{\sqrt{K}}\right]\Big) / (4h\beta$$
$$\left(h\sqrt{K}Cosh\left[\frac{h}{\sqrt{K}}\right] + (-K+h\beta)Sinh\left[\frac{h}{\sqrt{K}}\right]\right)\Big) \tag{37}$$

And

$$\frac{dp}{dx} = \frac{dp_0}{dx} + \alpha\frac{dp_1}{dx}$$
$$= -\frac{F_0 + h + F_1\alpha}{K} - \frac{F_1\alpha}{\beta} + \frac{F_0 + h}{K - h\beta - h\sqrt{K}Coth\left[\frac{h}{\sqrt{K}}\right]} / h \tag{38}$$

Case (2) $(\boldsymbol{M \neq 0})$

Again we start from equation (10)

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - \frac{1}{\kappa}\left(\frac{\partial\psi}{\partial y} + 1\right) - M^2\left(\frac{\partial\psi}{\partial y} + 1\right) \tag{39}$$

$$0 = \frac{\partial p}{\partial y} \tag{40}$$

By differentiating equation (39) with respect to y, the resulting equation is:

$$0 = \frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - \frac{1}{\kappa}\frac{\partial^2\psi}{\partial y^2} - M^2\frac{\partial^2\psi}{\partial y^2} \tag{41}$$

$N^2 = M^2 + \frac{1}{\kappa}$ Let

Then equation (41) becomes

$$0 = \frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - N^2\frac{\partial^2\psi}{\partial y^2} \tag{42}$$

For small parameter α, we can write

$$\psi = \psi_0 + \alpha\psi_1 + O(\alpha^2) \tag{43}$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + \alpha\frac{dp_1}{dx} + O(\alpha^2) \tag{44}$$

$$F = F_0 + \alpha F_1 + O(\alpha^2) \tag{45}$$

Using eqs. (43) – (45) and (20 a), (20 b) and then comparing the coefficient of like powers of α, we have:

The zero order system with its boundary conditions are

$$0 = \frac{\partial^4\psi_0}{\partial y^4} - N^2\frac{\partial^2\psi_0}{\partial y^2} \tag{46}$$

$$\frac{dp_0}{dx} = \frac{\partial^3 \psi_0}{\partial y^3} - N^2 \left(\frac{\partial \psi_0}{\partial y} + 1 \right) \tag{47}$$

$$y = 0 \text{ at } \quad , \frac{\partial^2 \psi_0}{\partial y^2} = 0 \quad , \psi_0 = 0 \ , \tag{48a}$$

$$y = h \text{ at } \quad , \frac{\partial \psi_0}{\partial y} + \beta \frac{\partial^2 \psi_0}{\partial y^2} = -1 \quad , \psi_0 = F_0 \ . \tag{48b}$$

The first order system with its boundary conditions are

$$0 = \frac{\partial^2}{\partial y^2} \left(\frac{\partial^2 \psi_1}{\partial y^2} - y \frac{\partial^2 \psi_0}{\partial y^2} \right) - N^2 \frac{\partial^2 \psi_1}{\partial y^2} \tag{49}$$

$$\frac{dp_1}{dx} = \frac{\partial}{\partial y} \left(\frac{\partial^2 \psi_1}{\partial y^2} - y \frac{\partial^2 \psi_0}{\partial y^2} \right) - N^2 \frac{\partial \psi_1}{\partial y} \tag{50}$$

$$y = 0 \text{ at } \quad , \frac{\partial^2 \psi_1}{\partial y^2} = 0 \quad , \psi_1 = 0 \ , \tag{51a}$$

$$y = h \text{ at } \quad , \frac{\partial \psi_1}{\partial y} + \beta \left(\frac{\partial^2 \psi_1}{\partial y^2} - y \frac{\partial^2 \psi_0}{\partial y^2} \right) = 0 \quad , \psi_1 = F_1 \ . \tag{51b}$$

It is found that the solution of zero order system is given by

$$\psi_0 = \left(F_0 N y Cosh[hN] + y\left(1 + F_0 N^2 \beta\right) Sinh[hN] - (F_0 + h) Sinh[Ny] \right) / \left(hNCosh[hN] + \left(-1 + hN^2 \beta\right) Sinh[hN] \right) \tag{52}$$

$$\frac{dp_0}{dx} = -(F_0 + h) N^3 \left(Cosh[hN] + N\beta Sinh[hN] \right) / \ hNCosh[hN] + \left(-1 + hN^2 \beta\right) Sinh[hN] \tag{53}$$

And the solution of first order system is found to be of the form

$$\psi_1 = \Big(y\Big(h\left(-1 + 4F_1 N^2\right) + F_0\left(-1 + 2h^2 N^2\right) + 2N^2\left(h^3 + 2F_1\beta - 2F_1 hN^2 \beta^2\right)\Big) + y\Big(F_0 h + 4F_1 hN^2 - 4F_1 N^2 \beta + 4F_1 hN^4 \beta^2\Big) Cosh[2hN] - 2Ny\Big(h(F_0 + h) + F_1\left(2 - 4hN^2\beta\right)\Big) Sinh[2hN] + 2hNCosh[hN]\Big(-(F_0 + h)Ny^2 Cosh[Ny] + \Big(-4F_1 + (F_0 + h)\left(y + h^2 N^2 \beta\right)\Big) Sinh[Ny]\Big) + 2Sinh[hN]\Big(-(F_0 + h)Ny^2\left(-1 + hN^2\beta\right) Cosh[Ny] + \Big(F_1\left(4 - 4hN^2\beta\right) + (F_0 + h)\left(-y + hN^2\left(h^2 - h\beta + y\beta\right)\right)\Big) Sinh[Ny]\Big)\Big) / 8\Big(hNCosh[hN] + \left(-1 + hN^2\beta\right) Sinh[hN]\Big)^2 \tag{54}$$

$$\frac{dp_1}{dx} = \Big(N^2 \Big(F_0 + h - 4F_1 hN^2 - 2F_0 h^2 N^2 - 2h^3 N^2 - 4F_1 N^2 \beta + 4F_1 hN^4 \beta^2 - \big(F_0 + h + 4F_1 hN^2 - 4F_1 N^2 \beta + 4F_1 hN^4 \beta^2\big) Cosh[2hN] + 2N\big(2F_1 + h(F_0 + h) - 4F_1 h N^2 \beta\big) Sinh[2hN] \Big)\Big) / \Big(8\left(hNCosh[hN] + \left(-1 + hN^2\beta\right) Sinh[hN] \right)^2 \Big) \tag{55}$$

Summarizing the perturbation results, the expression of stream function and longitudinal pressure up to order α are: $\psi = \psi_0 + \alpha \psi_1$

$$= \Big(8\Big(hNCosh[hN] + \left(-1 + hN^2\beta\right) Sinh[hN]\Big)\Big(F_0 N y Cosh[hN] + y\left(1 + F_0 N^2 \beta\right) Sinh[hN] - (F_0 + h) Sinh[Ny]\Big) + \alpha\Big(y\Big(h\left(-1 + 4F_1 N^2\right) + F_0\left(-1 + 2h^2 N^2\right) + 2N^2\left(h^3 + 2F_1\beta - 2F_1 hN^2\beta^2\right)\Big) + y\Big(F_0 + h + 4F_1 hN^2 + 4F_1 N^2\beta + 4F_1 hN^4\beta^2\Big) Cosh[2hN] - 2Ny\Big(h(F_0 + h) + F_1\left(2 - 4hN^2\beta\right)\Big) Sinh[2hN] + 2hNCosh[hN]\Big(-(F_0 + h)Ny^2 Cosh[Ny] + \Big(-4F_1 + (F_0 + h)\left(y + h^2N^2\beta\right)\Big) Sinh[Ny]\Big) + 2Sinh[hN]\Big(-(F_0 + h)Ny^2\left(-1 + hN^2\beta\right) Cosh[Ny] + \Big(F_1\left(4 - 4hN^2\beta\right) + (F_0 + h)\left(-y + hN^2\left(h^2 - h\beta + y\beta\right)\right)\Big) Sinh[Ny]\Big)\Big)\Big) /$$

$$\left(8\left(hNCosh[hN]+\left(-1+hN^2\beta\right)Sinh[hN]\right)^2\right) \quad (56)$$

And

$$\frac{dp}{dx}=\frac{dp_0}{dx}+\alpha\frac{dp_1}{dx}$$
$$=\left(N^2\left(-8\left(F_0+h\right)N\left(Cosh[hN]+N\beta Sinh[hN]\right)\left(hNCosh[hN]+\left(-1+hN^2\beta\right)Sinh[hN]\right)\right.\right.$$
$$+\alpha\left(F_0+h-4F_1hN^2-2F_0h^2N^2-2h^3N^2-4F_1N^2\beta+4F_1hN^4\beta^2-\left(F_0+h+\right.\right.$$
$$\left.4F_1hN^2-4F_1N^2\beta+4F_1hN^4\beta^2\right)Cosh[2hN]+2N\left(2F_1+h\left(F_0+h\right)-4F_1hN^2\beta\right)$$
$$Sinh[2hN])))/\left(8\left(hNCosh[hN]+\left(-1+hN^2\beta\right)Sinh[hN]\right)^2\right) \quad (57)$$

The pressure rise and friction force

$$\Delta P_\lambda=\int_0^h\frac{dp}{dx}dx \quad (58)$$

$$F_\lambda=\int_0^h -h\frac{dp}{dx}dx. \quad (59)$$

where $\frac{dp}{dx}$ is defined in equation(57) and $F=\theta-1$.

4. Results and discussion

A. stream line

The formation of an internally circulating bolus of fluid by closed stream lines is called trapping and this trapped bolus is pushed ahead along with the peristaltic wave, the effect of $\beta, M, \phi, \alpha, \mathrm{K}$ on trapping can be seen through figures. We observe that an increase in slip parameter β decreases the size of trapped bolus. This observation is true for the effect of M when increase M decrease the trapped bolus. To see the effect of ϕ on trapping we note that increase in the amplitude ratio increases the size of trapped bolus. The effect of viscosity parameter α when increase α increase the size of trapped bolus and the effect of K we note that when increase K does not effect on the size of trapped bolus.

B. variation of pressure

The variation of $\frac{dp}{dx}$ versus x is shown for a different values of $\beta, M, \phi, \alpha, \mathrm{K}$ by keeping the other parameter at a fixed values. It is noticed with increase β decrease $\frac{dp}{dx}$, where it resists the flow, with increase M increase $\frac{dp}{dx}$, with increase ϕ increase $\frac{dp}{dx}$ and meets the maxima in the interval $x\in[0.3,0.7]$, with increase K decrease $\frac{dp}{dx}$, and with increase α increase $\frac{dp}{dx}$ and meets the maxima in the interval $x\in[0.2,0.8]$.

C. pressure rise and friction force

The pressure rise has been plotted in figs. (11-15). Here the upper right-hand quadrant (1) denotes the region of peristalsis pumping, where $\theta>0$ (positive pumping) and $\Delta p>0$ (adverse pressure gradient). Quadrant (2), where $\Delta p<0$ (favorable pressure gradient) and $\theta>0$ (positive pumping), is designated as augmented flow (copumping region). Quadrant (3), such that $\Delta p>0$ (adverse pressure gradient) and $\theta<0$, is called retro-grade or backward pumping. In figure-11 it is observed the pressure rise increase in quadrant (3) and decrease in quadrant (2),in figure- 12 the pressure rise decrease in quadrant (3) and increase in quadrant (2), in fig. 13 the pressure rise decrease in quadrant (3) and increase in quadrant (2), in fig. 14 the pressure rise decrease in quadrant (3) and increase in quadrant (2) and in fig. 15 the pressure rise decrease in quadrant (3) and increase in quadrant (2). To discuss the behavior of friction force F_λ with θ for various values of $, M, \phi, \alpha, \mathrm{K}, \beta$, we have plotted in figures (16-20), figure - 16 illustrate variation of F_λ with θ for different values of M. It reveals that friction force increase when increase$M$, fig. 17 illustrate variation F_λ with θ for different values of α when increase α decrease F_λ, fig. 18 illustrate variation F_λ with θ for different values of β when increase β decrease F_λ, fig. 19 illustrate variation F_λ with θ for different values of K when increase K decrease F_λ and fig. 20 illustrate variation F_λ with θ for different values of ϕ when increase ϕ increase F_λ.

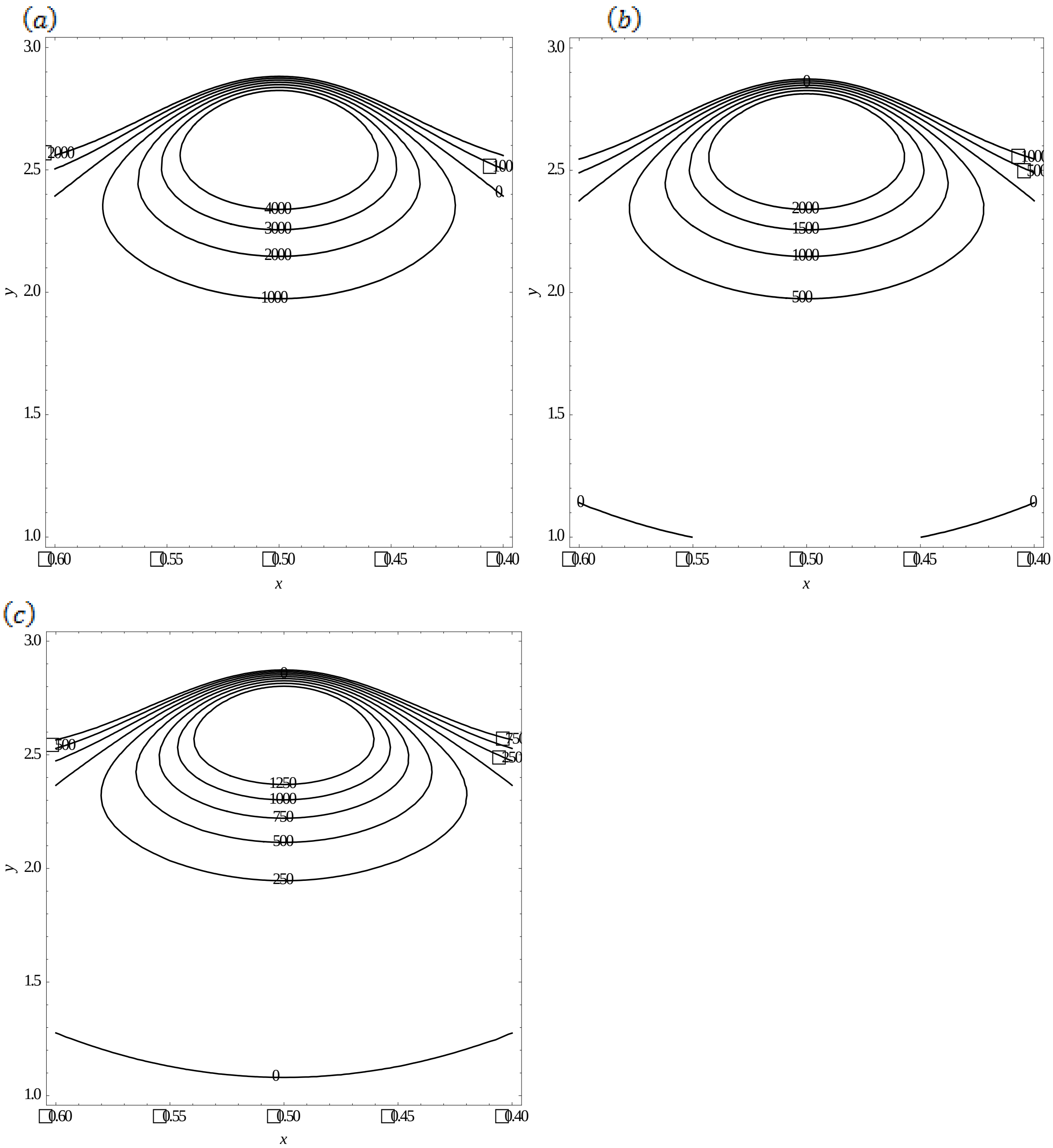

Figure 1- Stream lines for different values of β $(a)\beta = 0.1, (b)\beta = 0.3, (c)\beta = 0.6$ and the other parameters$M = 5, \alpha = 0.5, k = 10, \phi = 0.6$.

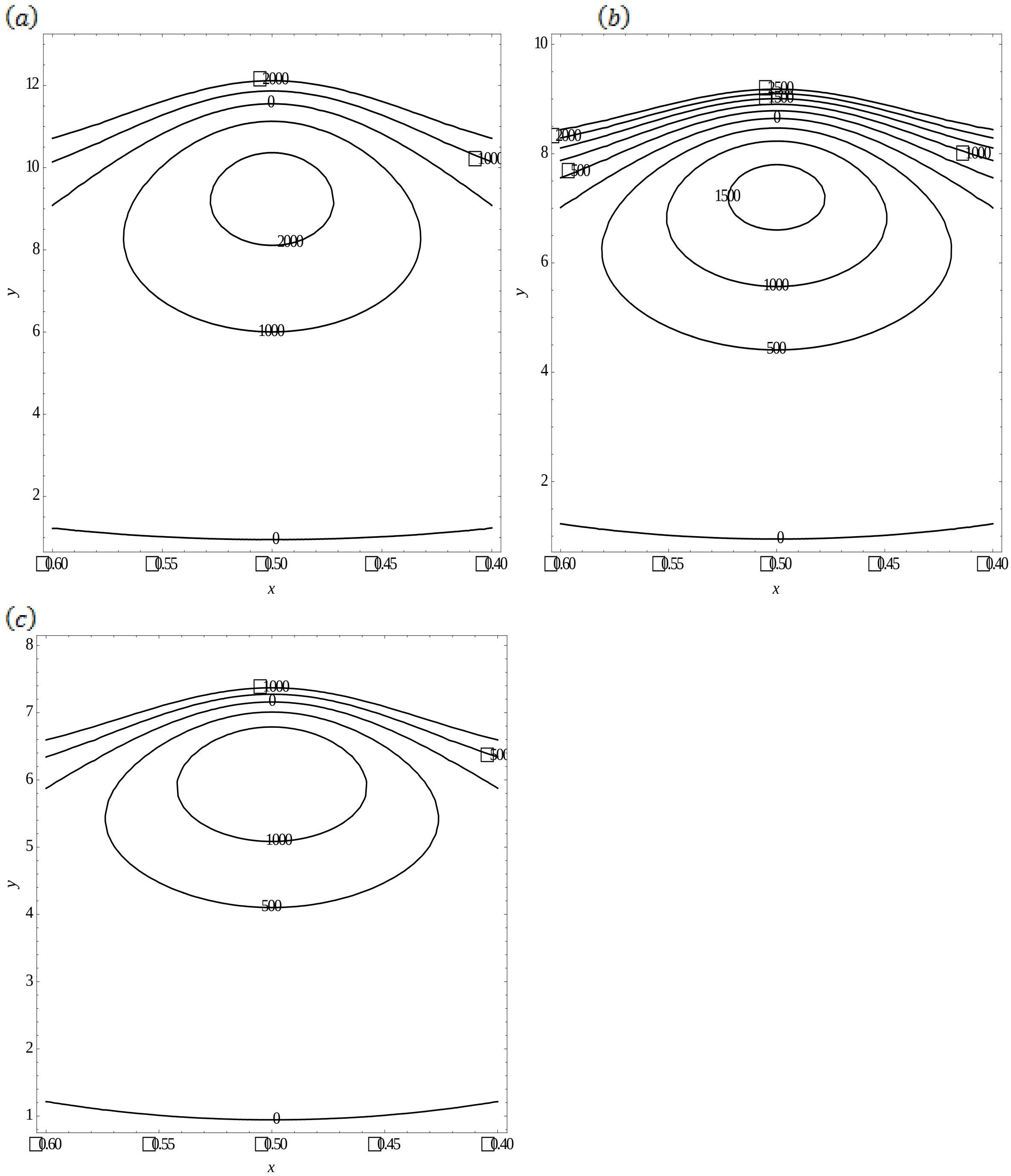

Figure 2- Stream lines for different values of M $(a)M = 0, (b)M = 0.5, (c)M = 0.8$ and the other parameters $\alpha = 0.5, \beta = 0.1, k = 10, \phi = 0.6$.

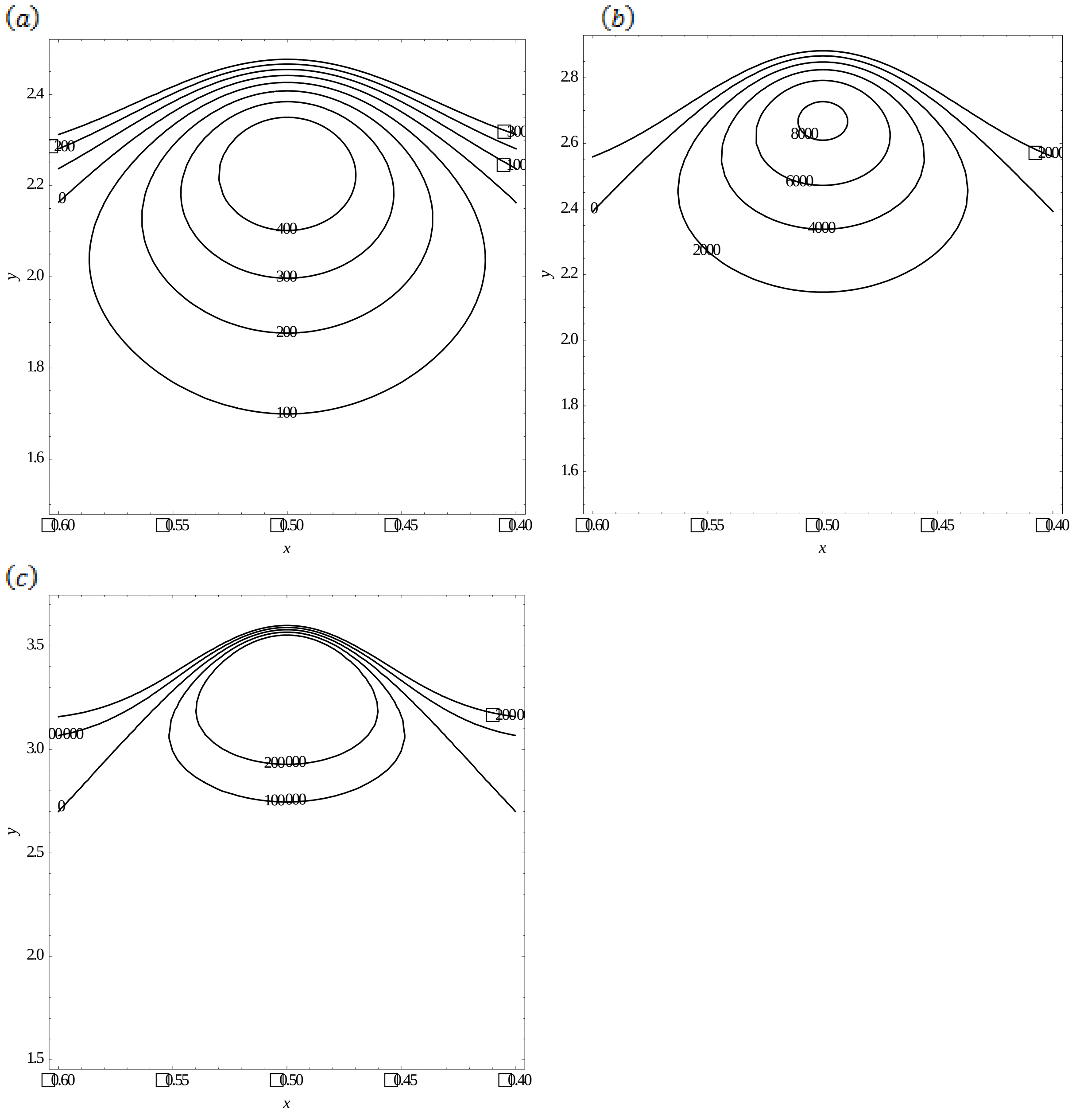

Figure 3- Stream lines for different values of ϕ $(a)\phi = 0.5, (b)\alpha = 0.6, (c)\alpha = 0.7$ and the other parameters $M = 5, \alpha = 0.5, \beta = 0.1, k = 10$.

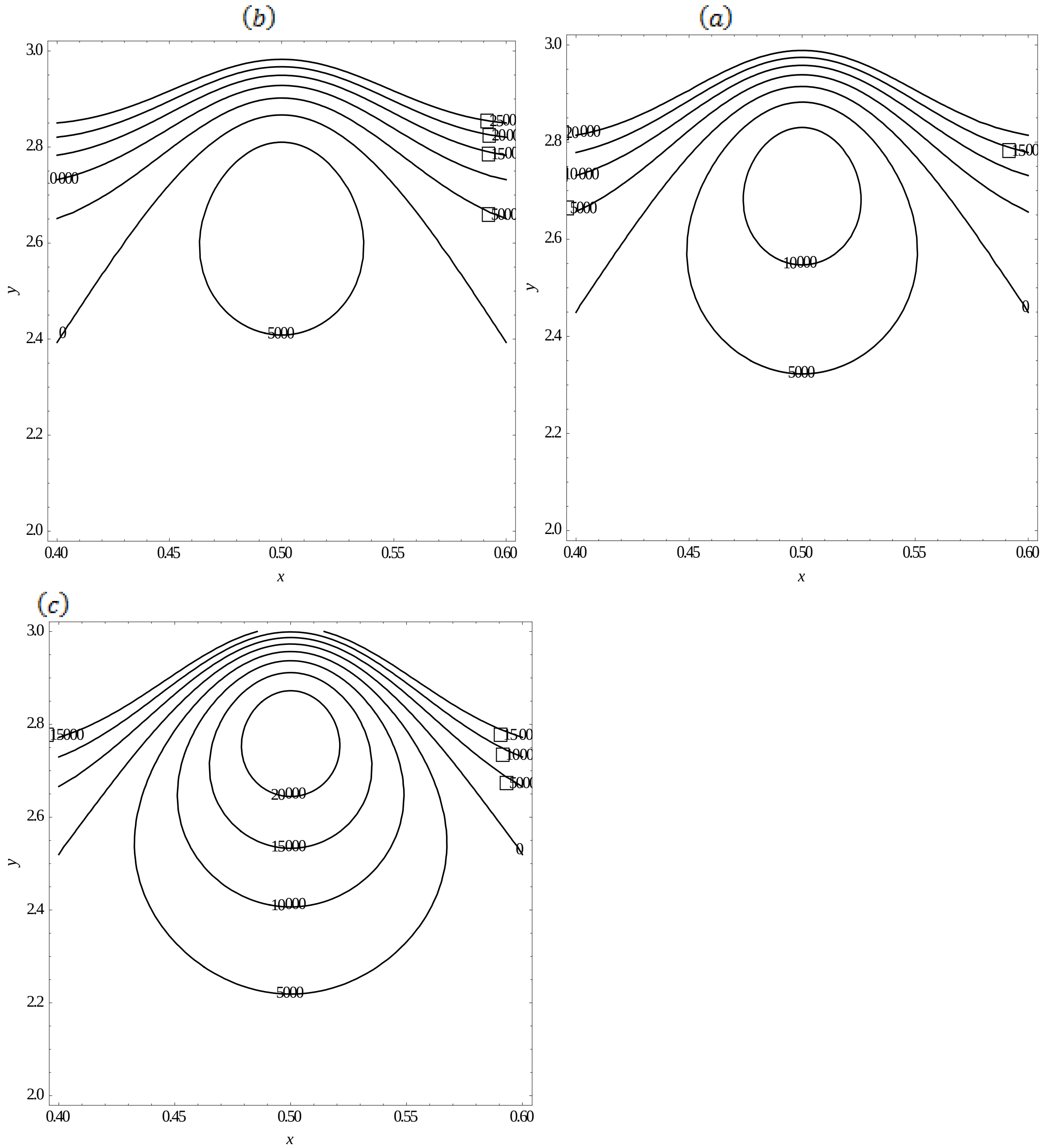

Figure 4- Stream lines for different values of α $(a)\alpha = 0.5, (b)\alpha = 0.6, (c)\alpha = 0.8$ and the other parameters $M = 5, \beta = 0.1, k = 10, \phi = 0.6$.

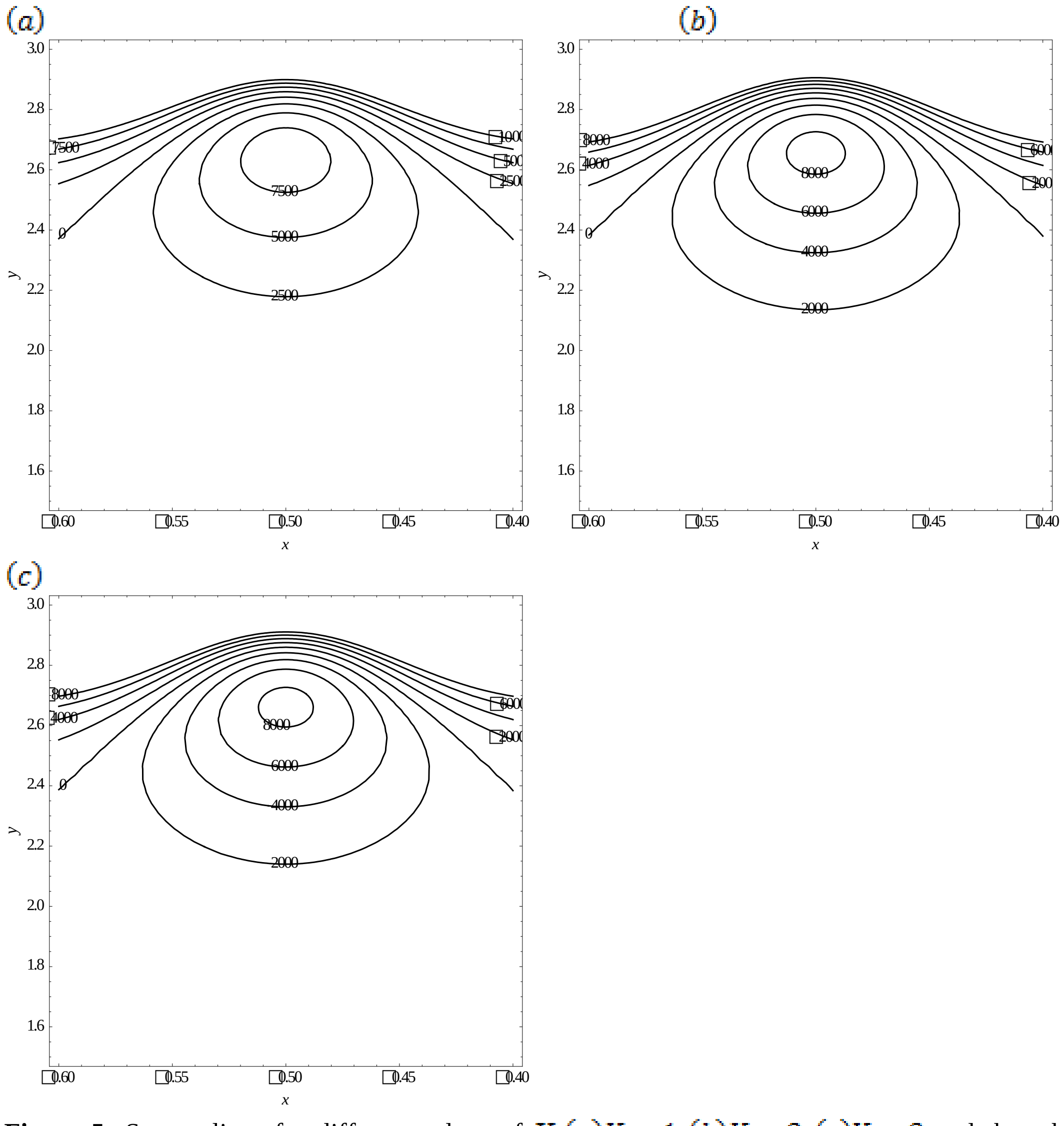

Figure 5- Stream lines for different values of K $(a)\mathrm{K}=1, (b)\mathrm{K}=2, (c)\mathrm{K}=3$ and the other parameters $M=5, \alpha=0.5, \beta=0.1, \phi=0.6$.

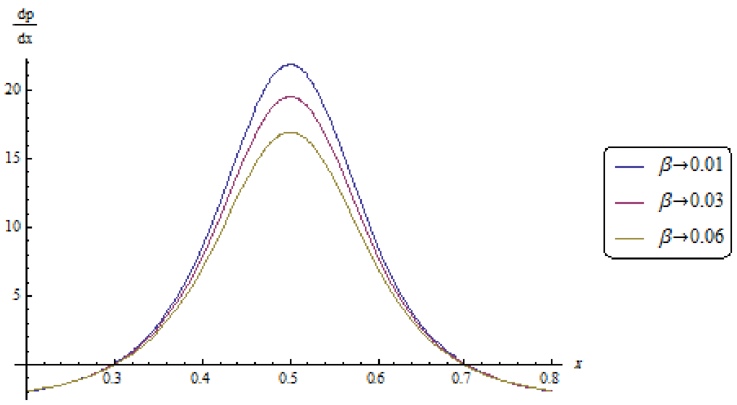

Figure 6- Variation of dp/dx with x for different values of β at K=10, $M = 2, \alpha = 0.2\ , \phi = 0.6$.

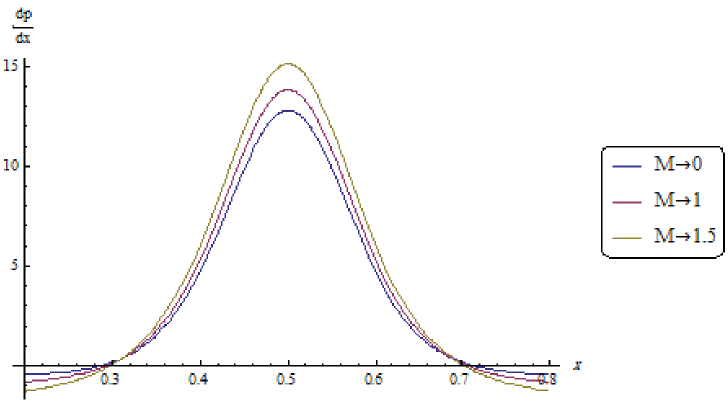

Figure 7- Variation of dp/dx with x for different values of M at $\alpha = 0.2, \beta = 0.06, k = 10, \phi = 0.6$.

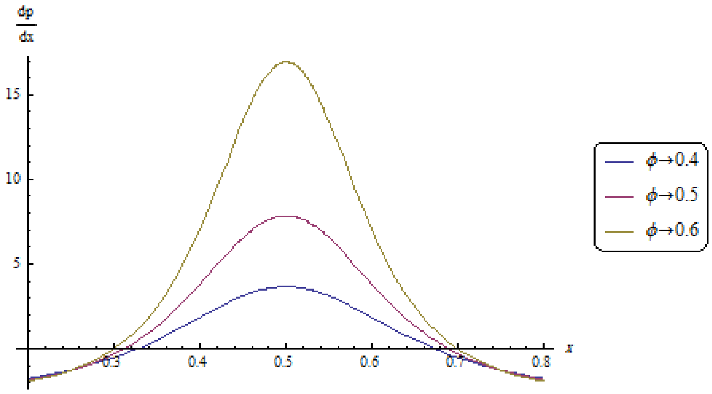

Figure 8- Variation of dp/dx with x for different values of ϕ at $M = 2, \alpha = 0.2, \beta = 0.06, k = 10$.

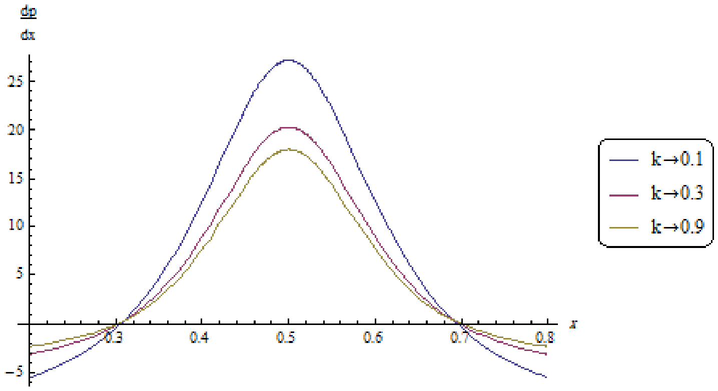

Figure 9- Variation of dp/dx with x for different values of k at $M = 2, \alpha = 0.2, \beta = 0.06, \phi = 0$.

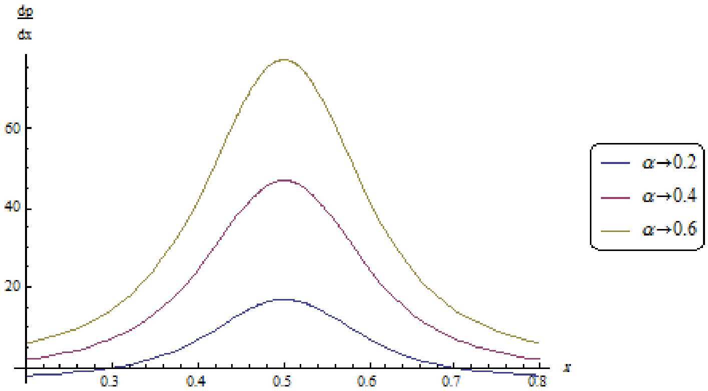

Figure 10- Variation of dp/dx with x for different values of α at $M = 2, \beta = 0.06, k = 10, \phi = 0.6$.

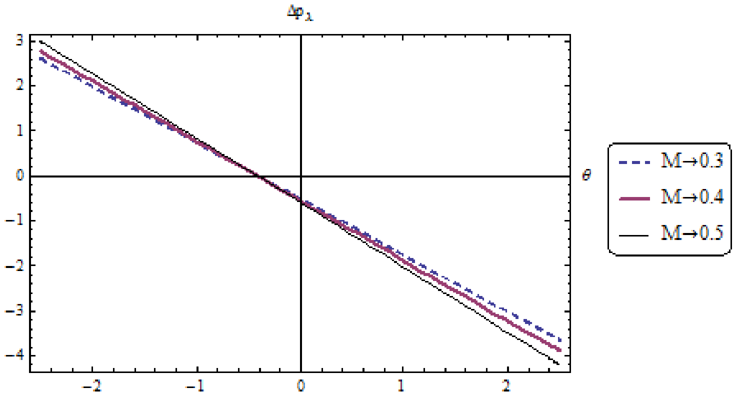

Figure 11- The pressure rise versus the flow rate for $\alpha = 0.2, \beta = 0.02, k = 10, \phi = 0.6$.

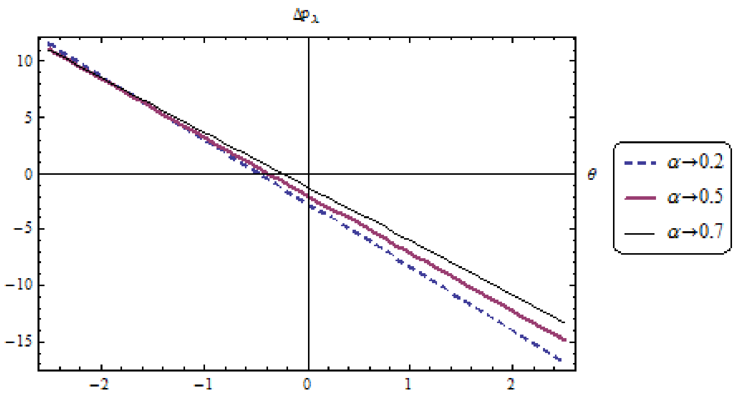

Figure 12- The pressure rise versus the flow rate for $M = 2, \beta 0.0 = 2, k = 10, \phi = 0.6$.

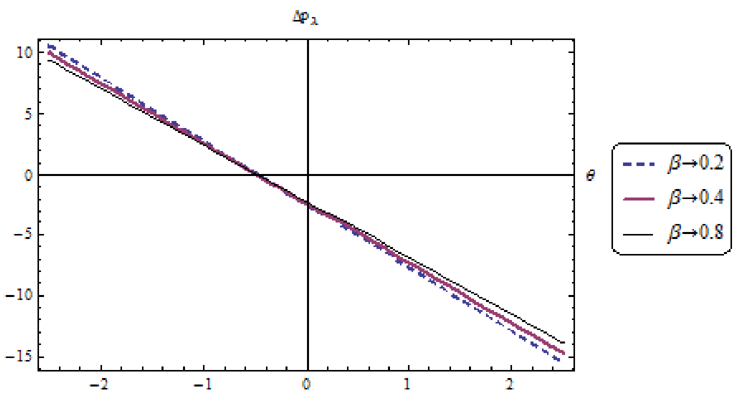

Figure 13- The pressure rise versus the flow rate for $M = 2, \alpha = 0.2, k = 10, \phi = 0.6$.

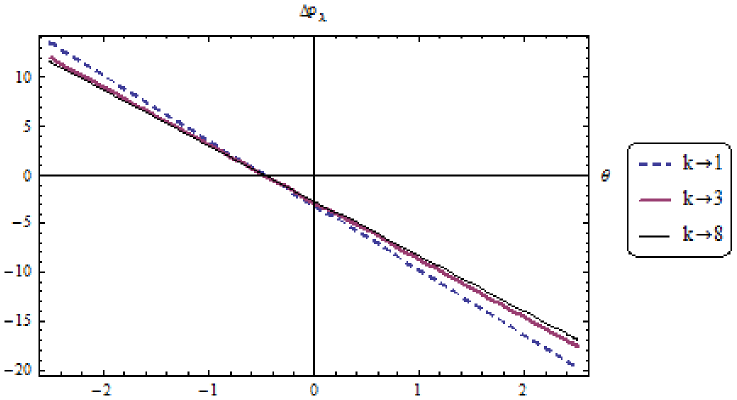

Figure 14- The pressure rise versus the flow rate for $M = 2, \alpha = 0.2, \beta = 0.02, \phi = 0.6$.

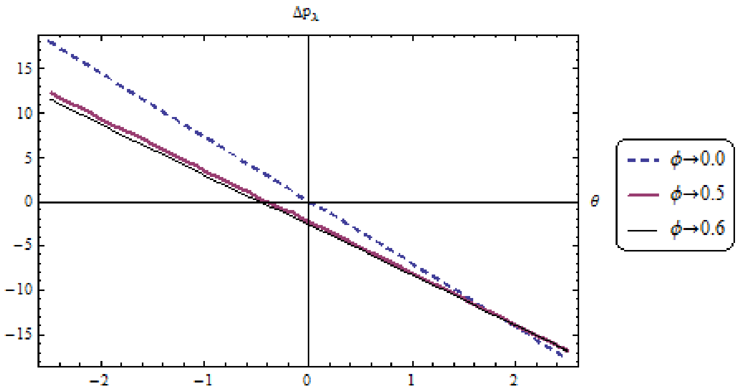

Figure 15- The pressure rise versus the flow rate for $M = 2, \alpha = 0.2, \beta = 0.02, k = 10$.

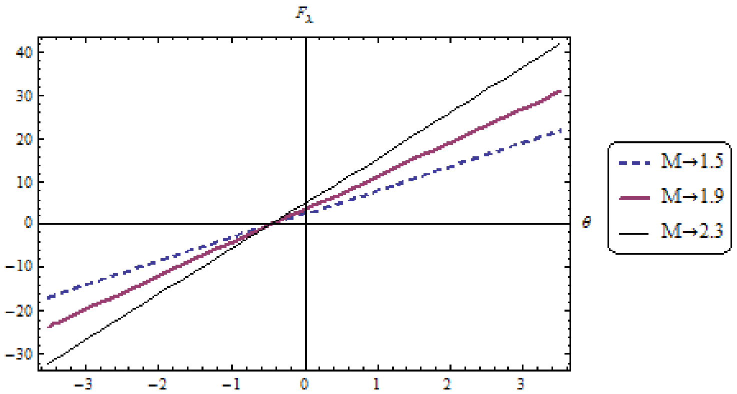

Figure 16- Friction force at the wall verses the flow rate for $\alpha = 0.2, \beta = 0.02, k = 10, \phi = 0.6$.

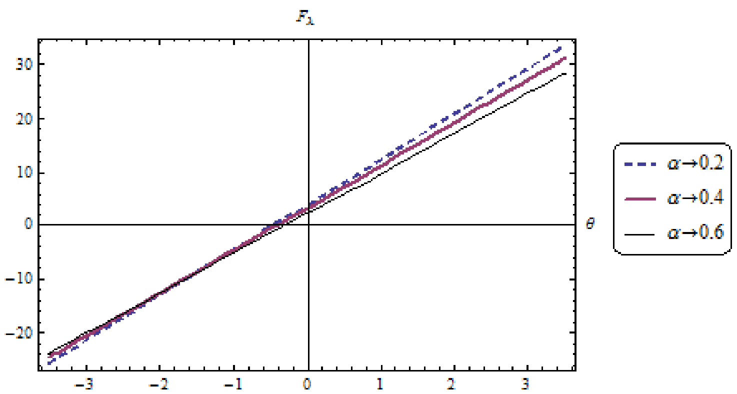

Figure 17- Friction force at the wall verses the flow rate for $M = 2, \beta = 0.02, k = 10, \phi = 0.6$.

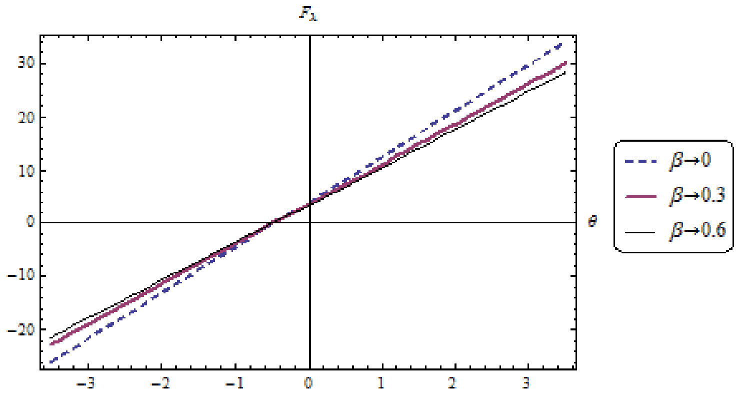

Figure 18- Friction force at the wall verses the flow rate for $M = 2, \alpha = 0.2, k = 10, \phi = 0.6$.

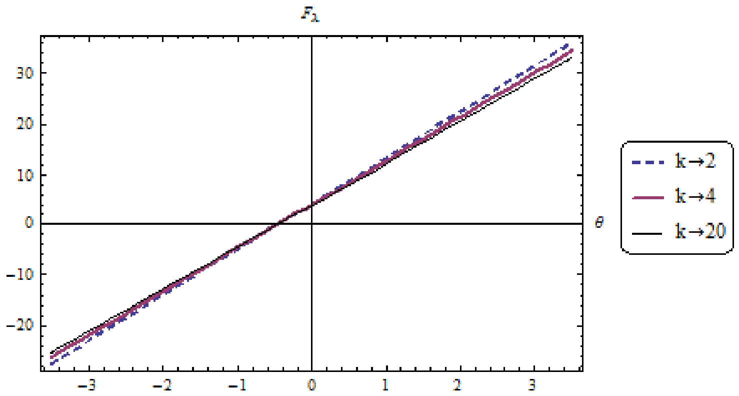

Figure 19- Friction force at the wall verses the flow rate for $M = 2, \alpha = 0.2, \beta = 0.02, \phi = 0.6$.

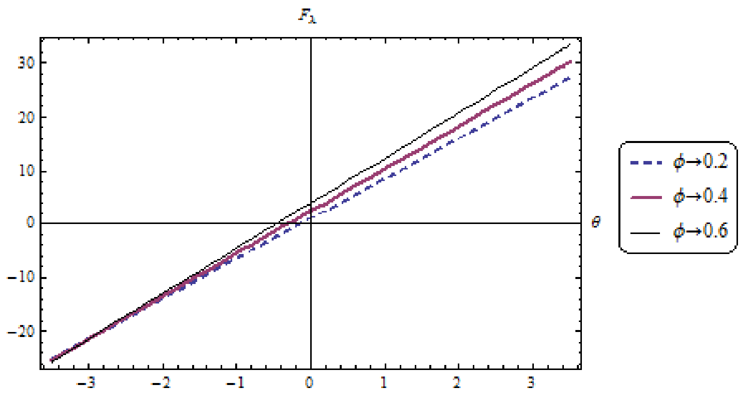

Figure 20- Friction force at the wall verses the flow rate for $M = 2, \alpha = 0.2, \beta = 0.02, k = 10$

5. Conclusion

The influence of the slip condition on the peristaltic transport of MHD fluid through a porous medium with variable viscosity has been analyzed. The analytical expressions are constructed for the stream function, pressure gradient, pressure rise and fractional force. The main findings of the present study are given in following points:

1- Decrease the size of the trapped bolus when increase in slip parameter β and M.

2- Increase the size of the trapped bolus when increase in the amplitude ratio ϕ and the viscosity parameter α.

3- The size of trapped bolus does not effect when change in value of K.

4- Decrease the pressure gradient when increase β, K.

5- Increase the pressure gradient when increase M, ϕ and K.

6- Increase the pressure rise in backward pumping and decrease in co pumping region when increase M.

7- The pressure rise decrease in backward pumping and increase in co pumping region when increase $\alpha, \beta,$ K and ϕ.

8- Increase the fractional force when increase M and ϕ.

9- Decrease the fractional force when increase α, β and K.

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