Some Fixed Point Theorems For Various Types Of Expanding Self Mappings In Partial Cone Metric Space

Tamara. Sh. Ahmed

Department of Mathematics , College Of Education For Pure Science /Ibn Al Haitham, University of Baghdad

tamaraalshareef@yahoo.com

Abstract

In this paper , We shall prove the existence of fixed point for expanding self mapping in the setting of complete partial cone metric space with assumption that the cone is normal and present some results in this setting of generalized cone metric space .Our results are generalizations of other results.

Key words: fixed point, partial cone metric space ,expanding mapping, normal cone.

1. Introduction :

Metric fixed point theory is a branch of fixed point theory which finds its primary application in functional analysis . It is a sub – branch of the functional analytic theory in which geometric conditions on the mapping and underlying space play a crucial role . Although it has a purely metric facet , it is also a major branch of non linear functional analysis with close ties to branch space geometry (1).

Historically ; the basic idea of metric fixed point principle firstly appeared in explicit from Banach's thesis 1922 (2,p.5) , Where it was used to establish the existence of solution to an integral equation . This principle - Banach contraction mapping principle is remarkable in its simplicity contraction, it is perhaps the most widely applied fixed point theorem in all of analysis .This is because the contractive condition on the mapping is simple and easy to test because :

i. It requires only complete metric space for its setting.
ii. It provides a contractive algorithm (iterative method)

iii. It finds almost Conociale applications in the theory of differential and integral equations specially the existence solution , Uniqueness solution .

All these properties motivate authers to study this principle and there appeared many types of contraction mapping on matric space .

Recently; Huang and Zhang (3) introduced the concept of Cone metric Space and generalized the concept of metric Space , In this space they replaced the set of real numbers of metric by an ordered Banach space and they established some fixed point theorems for certain types of contraction mapping in normal cone metric space . sub sequently some other authers for example (4,5,6,7,8,9) studied the existence of fixed points of self mappings satisfying many types of contraction condition, therefore many papers on cone metric space have been appeared and main topological properties of such spaces have been obtained . Now ,Ayse Sonmez (10) was generalized the setting of cone metric space and introduced the concept of partial cone metric space and proved some fixed point theorems of contractive mapping in this generalized setting .In partial cone metric space the self distance for any point need not to be equal to zero , specially from the point of sequences , convergent sequence need not have unique limit , but the usuall metric which is defined on cone metric space suggest that the self distance for any point is zero and for any convergent sequence has unique limit. The concept of partial cone metric play avery important role not only in mathematic but also in other branches of science and applied science involving mathematics especially in computer science , information science and biological science (10) . sub sequently other authers for example (11,12) generalized the results of (10) and obtained some fixed point theorems in the setting of normal partial cone metric space .

In 1984 , Wang et.al (13) introduced the concept of expanding mappings and proved some fixed point theorems in complete metric space .Recently, Xianjiu et .al (14) were generalize the definition of expanding mapping in metric space to the setting of cone metric space and proved some fixed point in complete cone metric space .subsequently , some other authers for example (15) were proved some fixed point theorems for expansion mapping in cone metric space .

Now ,in this paper we generalize the results of (14,15) and use the definition of expanding mapping and prove some fixed point theorems in the setting of normal partial cone metric space.

2- Preliminaries :

Definition (2.1)(10): let E be a real Banach space and w be a subset of E. The set w is called a cone if the following conditions are satisfied :

(c1) w is closed , non empty and $w \neq \{0\}$.

(c2) $a,b \in R$, $a,b \geq 0$,$x,y \in w \rightarrow ax+by \in w$

(c3) $x \in w$ and $-x \in w \rightarrow x=0$

Given a cone $w \subset E$, We can define a partial ordering $\leq$ with respect to w by $x \leq y$ if and only if $y-x \in w$. We write $x < y$ to indicate that $x \leq y$ but $x \neq y$, While $x << y$ will stand for $y-x \in int(w)$, int(w) denotes the interior of w.

The cone W is called normal if there is a constant number $k>o$ such that for all $x,y \in E$,$0 \leq x \leq y$ implies $\| x\| \leq k \|y\|$. The least positive number satisfying the above inequality is called the normal constant of w .

Definition (2.2)(10): Let X be non empty set ,E be a real Banach space .Suppose the mapping $d: X \times X \rightarrow E$ satisfies for all $x,y,z \in X$:

(cM_1) $d(x,y) = 0$ if and only if $x=y$

(cM_2) $d(x,y) = d(y,x)$

(cM_3) $d(x,y) \leq d(x,z)+d(z,y)$

Then d is called a cone metric on X and (x,d) is called a cone metric space .In this paper, We always suppose that E is a Banach space , w is a cone in E with $int(w) \neq \emptyset$ and " $\leq$ " is a partial ordering with respect to w .

Definition (2.3)(10):

A partial cone metric on non – empty set X is a function $P:X\times X\rightarrow E$ such that for all $x,y,z\in X$:

(p_1) x=y if and only if $p(x,x)=p(x,y)=p(y,y)$

(p_2) $0\leq p(x,x)\leq p(x,y)$

(p_3) $p(x,y)=p(y,x)$

(p_4) $p(x,y)\leq p(x,z)+p(z,y)-p(z,z)$

A partial cone metric space is a pair (x,p) such that X is non empty set and p is a partial cone metric on X. It is clear that , if $p(x,y) = 0$ then from (p_1) and (p_2) x=y. But if x=y , p(x,y) may not be 0.

A Cone metric space is a partial cone metric space But there are partial cone metric spaces which are not cone metric spaces , for examples we refer to (10).

Definition (2.4)(10): Let (x,p) be a partial cone metric space , let (X_n) be a sequence in X and $x\in X$. if for every $c\in \text{int}(w)$ there is N such that for all n> N , $p(x_n,x)\ll c+p(x,x)$, then (x_n) is said to be convergent and (x_n) converges to x , and x is the limit of (x_n) . We denote this by $\lim_{n\to\infty} x_n = x$ Or $x_n\rightarrow x$ as $n\rightarrow\infty$

Theorem (2.5)(10):

Let (x,p) be a partial cone metric space , w be a normal cone with normal constant k. Let (x_n) be a sequence in X . Then (x_n) converges to x if and only if $p(x_n ,x)\rightarrow p(x,x)$ (as $n\rightarrow\infty$) . For proof see (10).

We note that if (x,p) be a partial cone metric space ,W be a normal cone with normal constant k , if $p(x_n ,x)\rightarrow p(x,x)$ (as $n\rightarrow\infty$) then $p(x_n ,x_n)\rightarrow p(x,x)$ (as $n\rightarrow\infty$) (10).

Lemma(2.6) (10):

Let (x_n) be a sequence in partial cone metric space (x,p) . If a point x is the limit of (x_n) and $p(y,y)=p(y,x)$ then y is a limit point of (x_n). for proof see (10).

This lemma showed that the limit point of any convergent sequence in (x,p) need not be unique.

Definition (2.7)(10): Let (x,p) be a partial cone metric space , (x_n) be a sequence in X , (x_n) is a Cauchy sequence if there is $a\in w$ such that for every $\epsilon >0$

there is N such that for all n,m ∈ N, $\| p(x_x,x_m) - a\| < \epsilon$.

We call a partial cone metric space complete if every Cauchy sequence is convergent (10).

For more basic results we refer (10) .Recently, Xianjiu et .al (14) were generalize the definition of expanding mapping in metric space to the setting of cone metric space as follows :

Definition (2.8) (14): Let (x,d) be a cone metric space and $T:X\rightarrow X$, then T is called an expanding mapping ,if for every $x,y\in X$ there exist a number k>1 such that : $d(T_x,T_y) \geq k\, d(x,y)$.Now in this paper we generalize the definition of expanding mapping into the setting of partial cone metric space .

3- Main result :

Before we give our results , it is important to remark that we use a similar way of the proof of main result of Xianjiu et .al (14) in the setting of cone metric space in our main results in the setting of partial cone metric space .

Theorem (3.1) : Let (x,p) be a complete partial cone metric space , w be a normal cone with normal constant h. Suppose the mapping $T:X\rightarrow X$ be onto ,one to one and expanding mapping .(i.e), $p(T_xT_y)\geq kp(x,y)$, for all $x,y \in X$, k>1 …..(3.1.1) Then T has a unique fixed point in X .

Proof : From the assumption that T is onto , one to one so T has inverse , let T^{-1} be the inverse mapping of T.

Choose $x_0\in X$. Define the sequence $x_1=T^{-1}(x_0)$, $x_2=T^{-1}(x_1) =T^{-1}(T^{-1}(x_0)) = (T^{-1})^2(x_0)$ ………… $x_{n+1}= T^{-1}(x_n)= (T^{-1})^{n+1}(x_0)$.

First we shall prove that (x_n) is a Cauchy sequence .So from the condition (3.1.1) we have : $p(x_{n-1},x_n) = p(TT^{-1}x_{n-1},TT^{-1}x_n) \geq kp(T^{-1}x_{n-1},T^{-1}x_n) = kp(x_n ,x_{n+1})$

So , that implies $p(x_n ,x_{n+1}) \leq \frac{1}{k}\, p(x_{n-1},x_n)$,

Let $\frac{1}{k} = \lambda < 1$

That is , $p(x_n ,x_{n+1}) \leq \lambda p(x_{n-1},x_n) \leq \lambda^2 p(x_{n-2},x_{n-1}) \leq \ldots\leq \lambda^n p(x_0,x_1)$

Hence for m> n

$p(x_n, x_m) \leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + \ldots\ldots\ldots + p(x_{m-1}, x_m) - [p(x_{n+1}, x_{n+1}) + p(x_{n+2}, x_{n+2}) + \ldots\ldots + p(x_{m-1}, x_{m-1})]$

$\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + \ldots\ldots + p(x_{m-1}, x_m)$

$\leq \lambda^n p(x_0, x_1) + \lambda^{n+1} p(x_0, x_1) + \ldots.. + \lambda^{m-1} p(x_0, x_1)$

$\leq \lambda^n [1 + \lambda + \lambda^2 + \ldots..] p(x_0, x_1)$

$\rightarrow p(x_n, x_m) \leq \frac{\lambda^n}{1-\lambda} p(x_0, x_1)$. Hence by normality of the cone p we have :

$\| p(x_n, x_m) \| \leq \frac{h\lambda^n}{1-\lambda} \| p(x_0, x_1) \| \rightarrow 0$ as $n \rightarrow \infty$

Therefore , (x_n) is a Cauchy sequence in X , since (x ,p) is a complete , the sequence (x_n) converges to z in X . (i.e.) $\lim_{n\to\infty} x_n = z$

Therefore ,

$\lim_{n\to\infty} p(x_n, z) = p(z, z) = \lim_{n\to\infty} p(x_n, x_n) = 0$

Now ,since T is onto mapping , so there exist u∈X such that $Z = T_u$.

Consider , $p(x_n, z) = p(TT^{-1}x_n, Tu) \geq kp(T^{-1}x_n, u) = kp(x_{n+1}, u)$

(i.e.) $p(x_{n+1}, u) \leq \frac{1}{k} p(x_n, z)$. Hence by normality of a cone we have :

$\| p(x_{n+1}, u) \| \leq \frac{h}{k} \| p(x_n, z) \|$, but any norm is continues function .So by taking limit to both sides as $n \rightarrow \infty$ we get $\| p(z, u) \| \leq 0$ that implies $\| p(z, u) \| = 0$ and so , $p(z, u) = 0$ Or Z=u. Therefore , z is a fixed point of T.

To prove the uniqueness of fixed point of T . Let y be another fixed point of T (i.e. $T_y = y$) ,so we have: $p(z, y) = p(T_z, T_y) \geq kp(z, y)$

(i.e.), $kp(z, y) \leq p(z, y) \rightarrow (k-1)p(z, y) \leq 0$ that implies $p(z, y) = 0$ and z=y .So ,T has a unique fixed point .

<u>Corollary(3.2):</u> Let (x, p) be a complete partial cone metric space , w be a normal cone with normal constant h .Suppose the mapping T:X→X be onto ,one to one and there exists a positive integer N and a real number k >1 such that :

$P(T^N_x, T^N_y) \geq kp(x,y)$,
for all x, y ∈ X…………..(3.1.2)

Then T has a unique fixed point in X .

Proof : By theorem (3.1), we conclude that T^N has a unique fixed point say x.

(i.e.), $T^N(x) = x$, for some positive integer N .Now , $T^{N+1}(x)= T^N(T(x))=T(x)$

So, T(x) is also fixed point of T^N,but T^N has a unique fixed point which is x ,therefore T(x) = x so , T has a fixed point which is x .

Now , since the fixed point of T is also fixed point of T^N , the fixed point of T is unique .

Remark(3.3): In the following theorems ,T may have more than one fixed point .

Theorem (3.4): Let (x ,p) be a complete partial cone metric space , w be a normal cone with normal constant h .Suppose that the mapping T:X→X be onto ,one to one and satisfy the contractive condition :

$P(T_x ,T_y) \geq k [p(x, T_x) + p(y, T_y)]$ …….(3.1.3) for all x ,y ∈ X ,where

$\frac{1}{2} < k < 1$ is a constant . Then T has a fixed point in X .

Proof : From the assumption that T is onto and one to one , so T has inverse .

Let T^{-1} be the inverse mapping of T .

Choose $x_0 \in X$. Define the sequence $x_1=T^{-1}(x_0)$, $x_2=T^{-1}(x_1) =T^{-1}(T^{-1}(x_0)) = (T^{-1})^2(x_0)$ …………$x_{n+1}= T^{-1}(x_n)= (T^{-1})^{n+1}(x_0)$.

First we shall prove that (x_n) is a Cauchy sequence .Taking $x= T^{-1}x_{n-1}$,$y=T^{-1}x_n$ in condition (3.1.3) we have :

$p(TT^{-1}x_{n-1},TT^{-1}x_n) \geq$

$k [pT^{-1}x_{n-1},TT^{-1}x_{n-1}) + p(T^{-1}x_n , TT^{-1}x_n)]$

$p(x_{n-1},x_n) \geq k [p(x_n,x_{n-1})+p(x_{n+1},x_n)]$

$(1-k)\, p(x_{n-1},x_n)\geq k\, p(x_n,x_{n+1}) \rightarrow p(x_n,x_{n+1}) \leq \frac{1-k}{k} p(x_{n-1},x_n)$.

Therefore ; $p(x_n,x_{n+1}) \leq \lambda\, p(x_{n-1},x_n)$,where $\lambda = \frac{1-k}{k} < 1$. That is , $p(x_n,x_{n+1}) \leq \lambda\, p(x_{n-1},x_n) \leq \lambda^2 p(x_{n-2} ,x_{n-1}) \leq \ldots\ldots.\leq \lambda^n p(x_0 , x_1)$

Hence for m> n we have :

$p(x_n ,x_m) \leq p(x_n ,x_{n+1}) +p(x_{n+1} ,x_{n+2}) +\ldots\ldots\ldots..+ p(x_{m-1},x_m) -[p(x_{n+1} ,x_{n+1}) + p(x_{n+2} ,x_{n+2}) +\ldots\ldots\ldots+p(x_{m-1},x_{m-1})]$

$\leq p(x_n ,x_{n+1}) + p(x_{n+1} ,x_{n+2}) +\ldots\ldots+ p(x_{m-1},x_m)$

$\leq \lambda^n p(x_0,x_1) + \lambda^{n+1} p(x_0,x_1) +\ldots..+ \lambda^{m-1} p(x_0,x_1)$

$\leq \lambda^n [1+ \lambda+\lambda^2+\ldots..] p(x_0,x_1)$

$\rightarrow p(x_n ,x_m) \leq \frac{\lambda^n}{1-\lambda} p(x_0,x_1)$. by normality of the cone we have :

$\| p(x_n ,x_m) \| \leq \frac{h\lambda^n}{1-\lambda} \| p(x_0,x_1) \| \rightarrow 0$ as $n\rightarrow\infty$

Therefore , (x_n) is a Cauchy sequence in X , since (x,p) is a complete , the sequence (x_n) converges to z in X . (i.e) $\lim_{n\to\infty} x_n = z$

Therefore ,

$\lim_{n\to\infty} p(x_n, z) = p(z,z) = \lim_{n\to\infty}(x_n, x_n) = 0$

Now ,since T is onto mapping , so there exist u∈X such that $Z=T_u$.

Consider , $p(x_n ,z) = p(TT^{-1}x_n, Tu) \geq$ k $[p(T^{-1}x_n, TT^{-1}x_n + p(u,T_u)]$

$\rightarrow p(x_n , z)\geq k [p(x_{n+1},x_n) +p(u,T_u)]$

(i.e) , $k [p(x_{n+1},x_n) + p(u,T_u)] \leq p(x_n , z)$. Hence by normality of cone we have :

$\| K [p(x_{n+1},x_n) + p(u,T_u)] \| \leq h \| p(x_n , z) \|$. Now by taking limit to both sides as $n\rightarrow\infty$ we get $\| kp(u,z) \| \leq 0$ that implies $\| kp(u,z) \| =0$ .Therefore , we have u=z , so , z is a fixed point of T.

Theorem (3.5): Let (x,p) be a complete partial cone metric space , w be a normal cone with normal constant h .Suppose that the mapping T:X→X be onto ,one to one and satisfy the contractive condition :

$P(T_x ,T_y) \geq a_1 p(x,y) +a_2 p(x, T_x) + a_3 p(y, T_y)]$ …….(3.1.4) for all x,y ∈ X ,where $a_1 ,a_2,a_3 \geq 0$ are constant with $a_1 +a_2+a_3 > 1$.Then T has a fixed point.

Proof: From the assumption that T is onto and one to one , so T has inverse .

Let T^{-1} be the inverse mapping of T .

Choose $x_0\in X$. Define the sequence $x_1=T^{-1}(x_0)$, $x_2=T^{-1}(x_1) =T^{-1}(T^{-1}(x_0)) = (T^{-1})^2(x_0)$ …………$x_{n+1}= T^{-1}(x_n)= (T^{-1})^{n+1}(x_0)$.

First we shall prove that (x_n) is a Cauchy sequence .Taking $x= T^{-1}x_{n-1}$,$y=T^{-1}x_n$ in condition (3.1.4) we have :

p$(TT^{-1}x_{n-1}, TT^{-1}x_n) \geq a_1$ p$(T^{-1}x_{n-1}, T^{-1}x_n)$ $+a_2$ p$(T^{-1}x_{n-1}, TT^{-1}x_{n-1})$ + a_3 p$(T^{-1}x_n, TT^{-1}x_n)$

p$(x_{n-1},x_n) \geq a_1$ p$(x_n,x_{n+1}) + a_2$ p$(x_n,x_{n-1}) + a_3$ p(x_{n+1},x_n)

$(1-a_2)$ p$(x_{n-1},x_n) \geq (a_1+a_3)$ p(x_n,x_{n+1})

If $a_1+a_3 = 0$ then $a_2 > 1$. The above inequality implies that a negative number is greater than or equal to zero . That is impossible , so $a_1+a_3 \neq 0$ and $1-a_2 > 0$

(i.e) , p$(x_n,x_{n+1}) \leq \frac{1-a_2}{a_1+a_3}$ p(x_{n-1},x_n) . But

$a_1+a_2+a_3 > 1 \rightarrow a_1+a_3 > 1-a_2$

$\rightarrow 0 < \frac{1-a_2}{a_1+a_3} < 1$. Therefore ;

p$(x_n,x_{n+1}) \leq \lambda$ p(x_{n-1},x_n) ,where

$\lambda = \frac{1-a_2}{a_1+a_3} < 1$

That is , p$(x_n,x_{n+1}) \leq \lambda$ p$(x_{n-1},x_n) \leq$ λ^2 p$(x_{n-2},x_{n-1}) \leq \ldots\ldots \leq \lambda^n$ p(x_0, x_1)

Hence for m> n we have :

p$(x_n,x_m) \leq$ p(x_n,x_{n+1}) +p(x_{n+1},x_{n+2}) +……….+ p(x_{m-1},x_m) –[p(x_{n+1},x_{n+1}) + p(x_{n+2},x_{n+2}) +………+p(x_{m-1},x_{m-1})]
$\leq$ p(x_n,x_{n+1}) + p(x_{n+1},x_{n+2}) +……+ p(x_{m-1},x_m)
$\leq \lambda^n$ p(x_0,x_1) + λ^{n+1} p(x_0,x_1) +…..+ λ^{m-1} p(x_0,x_1)

$\leq \lambda^n$ $[1+ \lambda + \lambda^2 + \ldots..]$ p(x_0,x_1)

$\rightarrow$p$(x_n,x_m) \leq \frac{\lambda^n}{1-\lambda} p(x_0, x_1)$. by normality of the cone we have :

$\| p(x_n,x_m) \| \leq \frac{h\lambda^n}{1-\lambda} \| p(x_0,x_1) \| \rightarrow 0$ as $n \rightarrow \infty$

Therefore , (x_n) is a Cauchy sequence in X , since (x,p) is a complete , the sequence (x_n) converges to z in X . (i.e) $\lim_{n\to\infty} x_n = z$

Therefore ,

$\lim_{n\to\infty} p(x_n, z) = p(z,z) = \lim_{n\to\infty}(x_n, x_n) = 0$

Now ,since T is onto mapping , so there exist u∈X such that $Z=T_u$.

Consider , p(x_n,z) = p$(TT^{-1}x_n, Tu) \geq$ a_1 p$(T^{-1}x_n, u)$ $+a_2$ p$(T^{-1}x_n, TT^{-1}x_n)$ + a_3 p(u, T_u) .

So , p$(x_n,z) \geq a_1$ p(x_{n+1}, u) $+a_2$ p(x_{n+1},x_n) + a_3 p(u , z) .

Now , by normality of the cone we have :

$\| a_1 p(x_{n+1}, u) + a_2 p(x_{n+1}, x_n) +$
$a_3 p(u, z) \| \leq h\, p(x_n, z)$.

By taking limit to both sides as $n \to \infty$

$\| a_1 p(z, u) + a_2 p(z, z) + a_3 p(u, z) \| \leq 0$
.

$\to \| (a_1 + a_3) p(u, z) \| = 0$ that implies $p(u, z) = 0$ Or $Z = u$.

Therefore ; Z is a fixed point of T .

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بعض مبرهنات النقطة الصامدة لأنواع متعددة من تطبيقات ذاتية في فضاء متري شبه قرصي

تمارا شهاب احمد

قسم الرياضيات –كلية التربية للعلوم الصرفة ابن الهيثم –جامعة بغداد

الخلاصة

في هذا البحث تم برهان وجود النقطة الصامدة لتطبيقات ذاتية موسعة لفضاء متري شبه قرصي كامل بأفتراض ان القرص المستخدم هو طبيعي وقد قدمنا بعض النتائج لأستخدام هذا الفضاء. نتائجنا هي تعميمات لنتائج اخرى.