The Predator-Prey Model Simulation

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Abstract

Many of the most interesting dynamics in nature have to do with interactions between organisms. These interactions are often subtle , indirect and difficult to detect. Interactions in which one organism consumes all or part of another. This includes predator-prey, herbivore-plant, and parasite-host interactions. These linkages are the prime movers of energy through food chains. They are an important factor in the ecology of populations, determining mortality of prey and birth of new predators.

Mathematical models and logic suggests that a coupled system of predator and prey should cycle: predators increase when prey are abundant, prey are driven to low numbers by predation, the predators decline, and the prey recover, ad infinitum. One of such models that simulates predator-prey interactions is the Lotka-Volterra Model.

Keywords: Lotka-Volterra Model, Predator-prey interaction, Numerical solution, MATLAB

Introduction

A predator is an organism that eats another organism. The prey is the organism which the predator eats. Some examples of predator and prey are lion and zebra, bear and fish, and fox and rabbit. The words "predator" and "prey" are almost always used to mean only animals that eat animals, but the same concept also applies to plants: Bear and berry, rabbit and lettuce, grasshopper and leaf.

Predator and prey evolve together. The prey is part of the predator's environment, and the predator dies if it does not get food, so it evolves whatever is necessary in order to eat the prey: speed, stealth, camouflage (to hide while approaching the prey), a good sense of smell, sight, or hearing (to find the prey), immunity to the prey's poison, poison (to kill the prey) the right kind of mouth parts or digestive system, etc. Likewise, the predator is part of the prey's environment, and the prey dies if it is eaten by the predator, so it evolves whatever is necessary to avoid being eaten: speed, camouflage (to hide from the predator), a good sense of smell, sight, or hearing (to detect the predator), thorns, poison (to spray when approached or bitten), etc..

Predation is an important evolutionary force: natural selection favors more effective predators and more evasive prey. "Arms races" have been recorded in some snails, which over time become more heavily armored prey, and their predators, crabs, which over time develop more massive claws with greater crushing power. Predation is widespread and easy to observe. Neither its existence nor its importance is in doubt.

Mathematical models of predation are amongst the oldest in ecology. The Italian mathematician Volterra is said to have developed his ideas about predation from watching the rise and fall of Adriatic fishing fleets. When fishing was good, the number of fishermen increased, drawn by the success of others. After a time, the fish declined, perhaps due to over-harvest, and then the number of fishermen also declined. After some time, the cycle repeated. A main purpose of modeling population interactions is to understand what causes such fluctuations. Indeed, the very first Lotka-Volterra system is the result of such an effort.

Predator Prey Model

The Lotka-Volterra equations describe an ecological predator-prey (or parasite-host) model which assumes that, for a set of fixed positive constants **a** (the growth rate of prey), **b** (the rate at which predators destroy prey), **r** (the death rate of predators), and **c** (the rate at which predators increase by consuming prey). The following conditions will be hold for our computer simulation model.

Let the prey population at time **t** be given by $\mathbf{y_1(t)}$, and the predator population by $\mathbf{y_2(t)}$. Assume that, in the absence of predators, the prey will grow exponentially according to $\mathbf{y_1' = a y_1}$ for a certain $\mathbf{a > 0}$. We also assume that the death rate of the prey due to interaction is proportional to $\mathbf{y_1(t)\ y_2(t)}$, with a positive proportionality constant. So:

$$\mathbf{y_1'(t) = a\, y_1(t) - b\, y_1(t)\, y_2(t)} \qquad (1)$$

Without prey, predators will die exponentially according to $\mathbf{y_2'(t) = -ry_2\,dt}$ for a certain **r>0**. Their birth strongly depends on both population sizes, so we finally find for a certain **c>0**:

$$\mathbf{y_2'(t) = -r\,y_2(t) + c\,y_1(t)\,y_2(t)} \qquad (2)$$

These equations (1) and (2), lead to the following system differential equations:

$$\begin{cases} \mathbf{y_1'(t) = a\,y_1(t) - b\,y_1(t)\,y_2(t)} \\ \mathbf{y_2'(t) = -r\,y_2(t) + c\,y_1(t)\,y_2(t)} \end{cases}$$

We see that both $(e^{at}, 0)$ and $(0, e^{-ct})$ are solutions of $\mathbf{(y_1(t),\ y_2(t))}$. From this system we find that for every solution we must have

$$\mathbf{y_1'\left(\frac{r}{y_1} - c\right) + y_2'\left(\frac{a}{y_2} - b\right) = 0}$$

Integrating both sides gives us, for more details see [1].

$$\mathbf{r\log y_1(t) - cy_1(t) + a\log y_2(t) - by_2(t) = constant} \quad (3)$$

The Lotka-Volterra equations are a pair of first order, non-linear, differential equations that describe the dynamics of biological systems in which two species interact. Earliest predator-prey model based on sound mathematical principles forms the basis of many models used today in the analysis of population dynamics original form has problems.

Numerical Methods for Ordinary Differential Equation (ODE) in MATLAB

MATLAB is a technical computing environment for high-performance numeric (and not typically symbolic) computation and visualization. It is a proprietary software used by researchers, educators, and students in industry and academia. GNU Octave* is an open source high-level language, primarily intended for numerical computations that is mostly compatible with MATLAB. For the purpose of these examples, all of the code and commands can be used in MATLAB or Octave and nearly identical results would be produced.

In addition to writing script files, we can create user-defined functions using m-files (also text files with the .m extension). MATLAB and Octave have an extensive library of mathematical functions built in, but there is often a need for a user to create their own functions. New functions may be added to the software "vocabulary" as function files.

MATLAB has a number of tools for numerically solving ordinary differential equations. ODE23 and ODE45 are functions for the numerical solution of ordinary differential equations provided in MATLAB. They employ variable step size Runge-Kutta integration methods. ode23 uses a simple 2nd and 3rd order pair of formulas for medium accuracy and ode45 uses a 4th and 5th order pair for higher accuracy. We will focus on one of its most rudimentary solvers, ode45, which implements a version of the Runge–Kutta 4th order algorithm.

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* Octave is a high-level interactive language, primarily intended for numerical computations that is mostly compatible with Matlab. Octave uses the GNU readline library to handle reading and editing input.

In this study we will use the Matlab command ode45 to solve our systems of differential equation. This command is a robust implementation for systems of differential equations, which uses a variable step size method and the fourth and fifth order Runge-Kutta method.

Simulation of Predator-Prey in MATLAB

Continuous simulation concerns the modeling over time of a system by a representation in which state variables change continuously with respect to time. Typically, we use differential equations, that give relationships for the rates of change of the state variables with time.

In biological differential equations models, it is more common to have multiple dependent variables, and hence a system of two or more interrelated differential equations.

Solving a system of ODE in MATLAB is quite similar to solving a single equation, though since a system of equations cannot be defined as an inline function, we must define it as an M-file. We implemented this technique to solve the Lotka–Volterra predator–prey system:

$$\frac{dy_1}{dt} = a\, y_1 - b\, y_1\, y_2 \, ; \qquad y_1(0) = y_1^0$$

$$\frac{dy_2}{dt} = -\, r\, y_2 + c\, y_1\, y_2 \, ; \qquad y_2(0) = y_2^0$$

with a = .5471, b = .0281, r = .8439, c = .0266, and y_1^0 = 30, y_2^0 = 4. (These values correspond with data collected by the Hudson Bay Company [1] between 1900 and 1920.)

The M-file lv.m:

```
function yprime = lv(t,y)
%LV: Contains Lotka-Volterra equations
a = .5471;b = .0281;c = .0266;r = .8439;
yprime = [a*y(1)-b*y(1)*y(2);-r*y(2)+c*y(1)*y(2)];
```

We can now solve the equations as follows:

```
>>[t,y]=ode45(@lv,[0 20],[30;4])
```

The output from this command consists of a column of times and a matrix of populations. The first column of the matrix corresponds with y1 (prey in this case) and the second corresponds with y2 (predators). Suppose we would like to plot the prey population as a function of time. In general, y(n,m) corresponds with the entry of y at the intersection of

row n and column m, so we can refer to the entire first column of y with y(:, 1), where the colon means all entries. In this way, Figure 2 can be created with the command

```
>>plot(t,y(:,1), 'red',t,y(:,2),'.');
   xlabel('Time, t');
   ylabel('Population Sizes');
   title('Our Predator Prey Example, Solutions Over Time');
   legend('Prey','Predator')
```

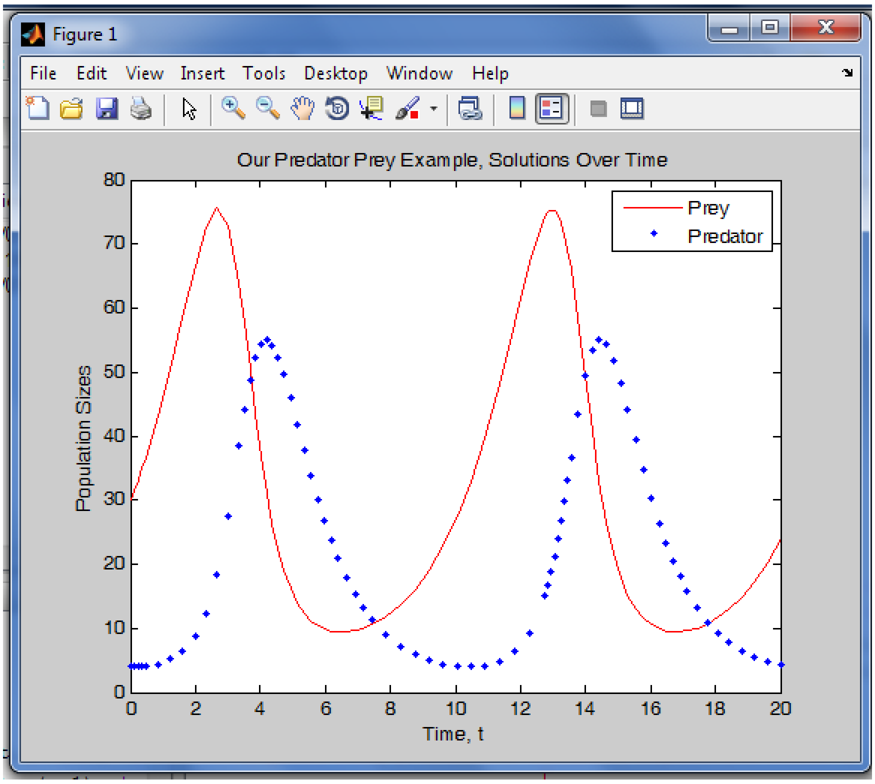

Figure 1: Plot of predator and prey populations for the Lotka–Volterra model

The functions y1 and y2 measure the sizes of the prey and predator populations respectively. The quadratic cross term accounts for the interactions between the species. Note that the prey population increases when there are no predators, but the predator population decreases when there are no prey.

To simulate a system, create a function that returns a column vector of state derivatives, given state and time values. For this example, we've created a file called LOTKA.M.

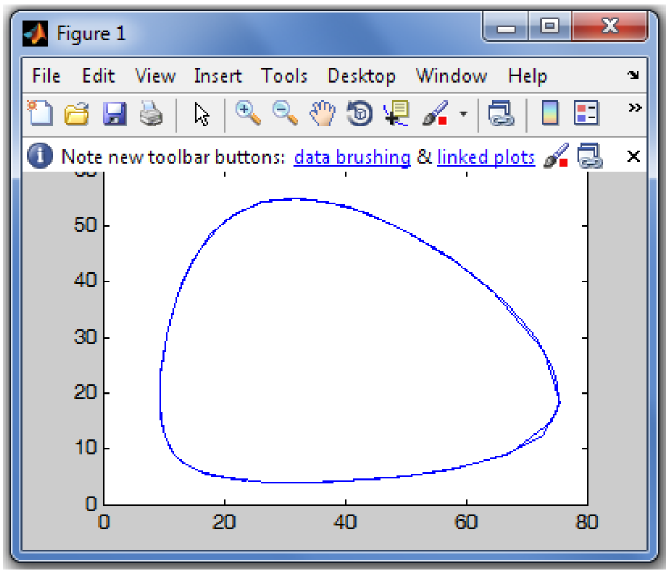

Figure 2, we can plot an integral curve in the phase plane with>>plot(y(:,1),y(:,2))

which creates Figure 4. In this case, the integral curve is closed, and we see that the populations are cyclic. (Such an integral curve is called a cycle.)

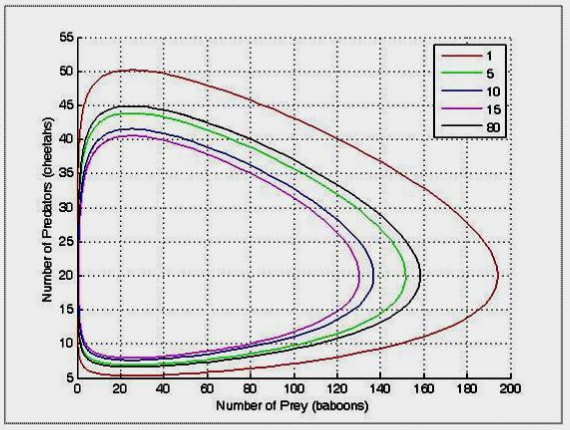

Figure 3, Prey-Predator dynamics as described by the level curves of a conserved quantity.

```
>> r = 1.3; s = .5; u = .7; v = 1.6; a = 1; K = 3;
h = .01;
>> P = .7; Q = 3; y1 = [P Q]; for t = 1:10000;
dP = (r*(1-P/K)-(s*Q)/(a+P))*P*h;
dQ = (-u+(v*P)/(a+P))*Q*h; P = P+dP; Q =
Q+dQ; y1 = [y1; P Q]; end;
>> P = 2; Q = 6; y2 = [P Q]; for t = 1:10000;
dP = (r*(1-P/K)-(s*Q)/(a+P))*P*h;
dQ = (-u+(v*P)/(a+P))*Q*h; P = P+dP; Q =
Q+dQ; y2 = [y2; P Q]; end;
>> P = 0:.1:3; nullP = (r/s)*(1-P/K).*(a+P);
>>
plot(y1(:,1),y1(:,2),y2(:,1),y2(:,2),P,nullP,[a*
u/(v-u), a*u/(v-u)],[0 8])
>> % phase diagram with two trajectories
```

Discussion

We have discussed continuous systems whose process of evolution depends on differential equations given as a nonlinear differential equations system. Such a system contains a number of parameters that must be estimated accordingly to some exist studies. In this field, (for instance, the a, b, c, d > 0 in the predator-prey model). Usually point estimates are calculated and used in the model. These estimates typically have uncertainty associated with them.

Solutions of which are plotted figure 1 where prey are shown in red (sold), and predators in blue (dashed). In this sort of model, the prey curve always lead the predator curve. Figure 1 shows a periodic activity generated by the Predator-Prey model.

The level curves of the figure 2, are closed so that the solution is periodic. It is desirable that the numerical solution of (3) is also periodic, but this is not always the case. Note that the prey population increases when there are no predators, but the predator population decreases when there are no prey

Conclusion and Further Work

Consider an environment consisting of two populations, predators and prey. We are interested in both the predator and prey population size. However, these populations interact. The Lotka-Volterra equations describe an ecological predator-prey (or parasite-host) model. The Lotka—Volterra model is sensitive to modifications in the differential equations that describe the dynamics of the nonlinear system.

We can incorporate uncertainty in our differential equations. This is done by using fuzzy numbers as estimates of the unknown parameters. Finite-difference algorithms for studying the dynamics of spatially extended predator–prey interactions would be very excited and effective.

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الخلاصة

النماذج الديناميكية الأكثر إثارة للاهتمام في الطبيعة هي النماذج التي لها علاقات تفاعلية بين الكائنات الحية. هذه التفاعلات غالبا ما تكون خفية وغير المباشرة ويصعب اكتشافها. في مثل هذه العلاقات فان احد كائنات يستهلك كل أو جزء من الكائن الاخر. ويشمل هذا المفترس والفريسة ، الحيوانات المقتاته على الاعشاب، وكذلك علاقة الطفيلي و المضيف. هذه الروابط هي المحرك الرئيسي للطاقة من خلال السلاسل الغذائية. فهي تشكل عاملا مهما في البيئة الطبيعية للسكان، وتحديد معدل وفيات المواليد وفريسة جديدة من الحيوانات المفترسة.

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