ELASTIC MAGNETIC FORM FACTORS FOR $^7$Li

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Abstract

Elastic magnetic electron scattering form factors have been calculated for the ground state $J^p = \frac{3}{2}^+$ of $^7$Li using the many-particle shell model. The calculations are based on the model space wave functions of Cohen-Kurath interaction. The results are compared with the available experimental data. The data are reasonably explained up to $q \approx 3.0 \text{ fm}^{-1}$ when the size parameter of the harmonic oscillator potential is reduced from that used to fit the root mean square charge radius.

Introduction

The scattering of electrons from nuclei gives the most precise information about nuclear size and charge distribution and it has provided important information about the electromagnetic currents inside the nuclei. One of the successes of the shell model picture has been the ability to describe the electromagnetic currents inside the nuclei. The multi-nucleon shell model can reproduce many observed properties of nuclei if the limited model space is used with effective operators rather than the free nucleon operators [1].

The cross section of electron scattering from the ground state of $^7$Li has been measured by Van Niffrig et al. [2]. The results agree very well with the results of lifetime measurements. The longitudinal and transverse form factors for the ground state doublet ($J^p = \frac{3}{2}^- \text{ g.s. and } J^p = \frac{1}{2}^+$, 0.478 MeV excitation) were measured up to momentum transfer $q \approx 4.2 \text{ fm}^{-1}$ by L. Lichtenstad et al. [3, 4]. Also, $^6$Li and $^7$Li nuclei have been described successfully in terms of clusters, as $\alpha^2d$ in the case of $^6$Li and $\alpha+t$ in the case of $^7$Li; K. Langanke [5] performed this description. The simple $0\hbar\omega$-shell model description of these nuclei automatically contains such clustering. The $1s$-shell inert core is the $\alpha$-particle, while the valence nucleons in the $1p$-shell naturally form the other cluster. Higher energy configurations have been studied for some $1p$-shell nuclei where $(0^+2)$ $\hbar\omega$ model space was used [6]. This extension of the model space improves the agreement with transverse form factors in the beginning of the $1p$-shell nuclei. Karataglidis et al. [7] have used $(0^+2+4)$ $\hbar\omega$ wave functions in the analysis of the elastic and inelastic electron scattering form factors in $^6$Li. Their results reproduce the data for $q$ above 1.0 fm$^{-1}$, and underestimate the data for $q < 1.0 \text{ fm}^{-1}$. This is associated with the underestimation in the B(C2) value of about a factor of 2. Radhi et al. [8] used the extended model space wave function to calculate the form factors for elastic magnetic electron scattering from $^{19}$F. The inclusion of a higher configuration with the model space configuration had produced a second maximum, which was found experimentally.

In the present work, almost same sort of analysis of the previous work [8] is adopted. The two-
body interaction of Cohen-Kurath [9] is used to
generate the 1p-shell wave functions. The single
particle wave functions of the harmonic
oscillator (HO) potential are used with the size
parameter (b) chosen so that the root mean
square (rms) charge radius is reproduced.
To describe the experimental data, the size
parameter is reduced from that to fit the
measured root mean square charge radius. Also
the effective g-factor is considered as a core-
polarization effect.

Theory

For many-particle system (N-valence
nucleons), the reduced matrix elements between
the initial state $|i\rangle$ and the final state $|f\rangle$
for
the one-body magnetic operator are expressed as
the sum of the product of elements of the one-
body density matrix (OBDM) times single-
particle matrix elements [1]:

$$\langle f|\mu_{ij},i,t_{ij}^{m},q|\rangle = \sum_{if} \text{OBDM}(M_{ij},i,t_{ij}^{m},q)|\langle f|\rangle$$

(1)

Where $t_{ij} = \frac{1}{2}$ for proton and $-\frac{1}{2}$ for neutron,
and the sum extends over all pairs of single-
particle states in model space $(|1p_{1/2}\rangle$ and $|1p_{3/2}\rangle$). The OBDM are obtained
from the work of Cohen-Kurath [9].
The transverse magnetic form factor of the
nucleus is defined as [10]:

$$F_{T}^{\mu}(J,q) = \frac{1}{\sqrt{2j+1}} \sum_{i_{z}} \langle f_{T}^{\mu}(T_{i_{z},q})|f_{\mu}(q)\rangle$$

(2)

Where $f_{\mu}(q)$ is the correction factor of the
center of mass motion when shell model wave
functions are used, and is given by [11]:

$$f_{\mu}(q) = \exp(q^{2}b^{2}/4A)$$

(3)

Where $b$ is the harmonic oscillator size
parameter, and $A$ is the total number of nucleons
in the nucleus. The function $f_{T}(q)$ is the
finite size form factor correction, which is the
same for the proton and the neutron and takes
the form [12]:

$$f_{T}(q) = \exp(-0.43q^{2}/4)$$

(4)

Results and discussion

The calculations have been performed with
the Cohen-Kurath interaction for the model
space contribution. According to the many-
particle shell model, the nucleus $^9$Li is
considered as a core of the $^4$He plus three
nucleons distributed over the $1p_{3/2}$ and $1p_{1/2}$ orbits.
The oscillator parameter $b$ is used to be
$b_{rms}=1.74\text{fm}$ [13]. Free nucleon g-factors are
used. The total magnetic $(M1+M3)$ form factor
for the state $3/2^+_1$ with these parameters is
shown in Fig.1 by solid curve. This form factor
has contributions from M1 and M3 components,
which are displayed by the dashed and dotted
curves respectively. The M3 component is
dominant over the diffraction minimum of M1
component. The data of Niftrik [2] (circles) and
Lichtenstad et al. [3] (squares) are well
described for momentum transfer values less
than 1.5 fm$^{-1}$. The inclusion of the effective g-
factor $(g_{\text{eff}}=0.9g_{\text{free}})$ allows the form factors to
be reduced at low $q$. Reducing the size
parameter $b$ from the standard value $b_{rms}$ to be,
b=1.65fm allows one to improve significantly
the agreement with the experiment at large $q$ as
shown in Fig.2 (solid curve). The calculated
magnetic moment with $g_{\text{eff}}$ is found to be
$\mu_{m} = 3.165\text{n.m}$ which is in excellent agreement
with the experimental value ($\mu_{exp} = 3.256$
\text{n.m})[12].

Figure (1): Elastic magnetic form factors
calculated with CK interaction, using 1p-
shell model space. The data are taken from
Ref. 2 (circles) and Ref. 3 (squares).
The calculations are quite successful and describe the data very well in both the magnetic moment and momentum transfer. We conclude that the calculated (M1+M3) form factors cannot explain the experimental data by carrying only the complete multi-nucleon configuration mixing of the ground state of $^5$Li. The high q-data are sensitive to the size parameter (b) of the HO potential and the data are better described when $b_{rms}$ is reduced by 5%. The reduction of $b_{rms}$ yielded enhancement on the form factor above q~3.5 fm$^{-1}$.

References