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A Proposed Equation for the Evaluation of the Nominal Ultimate Bending Moment Capacity of Rectangular Singly Reinforced RPC Sections

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Abstract

Based on compressive stress – compressive strain curves of Reactive Powder Concrete (RPC) which have been established recently in a Ph.D thesis(1), an equivalent bi-linear compressive stress block for RPC sections under pure bending moment is proposed and used to derive an equation for calculating the nominal ultimate bending moment capacity (Mn) of rectangular singly reinforced RPC sections. The accuracy of the derived equation of Mn is examined by comparison with the results of existing experimental tests.

Keywords: Reactive Powder Concrete, Compressive stress block, bending moment capacity.

معادلة مقترحة لاحتساب سعة العزم القصوى لمقطع مستطيل من خرسانة المساحيق الفعالة مفرد التسليح

الخلاصة

بالاعتماد على منحنيات إجهاد الضغط – انفعال الضغط لخرسانة المساحيق الفعالة، والمقدمة في أطروحة دكتوراه حديثة(1) ، تم اقتراح شكل نموذجي لكتلة الضغط التي تحصل في مقطع من هذه الخرسانة عند تعرضه لعزم انحناء صافي، واستخدمت كتلة الضغط النموذجية هذه لاشتقاق معادلة رياضية يتم بواسطتها احتساب عزم الانحناء الأقصى الاسمي (Mn) لمقاطع مستطيلة من هذه الخرسانة مفردة التسليح. تم تدقيق صحة المعادلة Mn المقترحة بمقارنتها مع نظيراتها المستحصلة من تجارب عملية.

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Introduction

The concrete industry has recently succeeded in producing new cement – based materials that have not only higher strength limits but also good durability[2]. The product of such effort is known as Reactive Powder Concrete (RPC) whose compressive strength can reach up or becomes much higher than 200 MPa[3]. It is a fiber reinforced, superplasticized, Silica Fume – cement mixture with very low water - cement ratio characterized by the presence of very fine quartz sand (0.15-0.40mm) instead of ordinary aggregate[4,5,6].

The use of this type of concrete in construction requires a good knowledge of its mechanical properties. Recent developments of the subject have concentrated on the behavior of RPC under direct compression or tension and little have been done so far concerning its behavior in flexure, shear and torsion. Therefore it is the aim of this paper to make use of existing compressive stress - compressive strain relationships which have recently been established[1] for different mixes of RPC and try to assess an equivalent compressive block to be used in deriving a theoretical equation for predicting the nominal ultimate bending moment capacity of rectangular singly reinforced RPC sections.

Idealized fc– εc Relationship of RPC

Figure (1) shows a typical stress – strain curve of reactive powder concrete in compression. This curve is representative to the experimental curves obtained for 18 different RPC Mixes in reference (1) and has theoretically been expressed by equations in the same reference as listed in Tables (1) and (2).

The ascending part of this curve, covering values of strain ε_c ranging between zero and $\varepsilon_\circ$ or implicitly values of stress f_c ranging between zero and f_c' , is in fact a slightly-nonlinear curve, very close to the shape of a straight line. The descending part is a double curvature curve representing the stage of concrete softening which ends when the concrete crushes at an ultimate strain ε_{cu}.

In order to make use of $f_c - \varepsilon_c$ this relationship in the design and analysis of flexural RPC members, it is proposed in this research to simplify such behavior and convert it into an equivalent bi- linear relationship which is plotted as dashed line in the same figure (1). This dashed line consists of an ascending straight line (from zero stress to the peak stress f_c') and a descending straight line (from the peak stress f_c' to a stress level $\alpha f_c'$). The descending straight line is terminated at a value of strain equal to twice the value $\varepsilon_\circ$ that corresponds to the peak stress f_c'. It is proposed that such strain value can be considered as the ultimate strain of RPC such that;

$$\varepsilon_{cu} = 2\varepsilon_\circ$$

For any particular RPC mix, the value of α can be calculated by substituting $\alpha f_c'$ for f_c and $2\varepsilon_\circ$ for ε_c in the appropriate equation of the descending part given in Tables (1) and (2).

It is important to note that the proposed equivalent bi-linear stress-strain relationship of RPC in compression is based on the following two facts;

i- The area under the dashed line is nearly equal to the area under the actual curve.

ii- The centriod of the area under the dashed line is very close to the centriod of the area under the actual curve.

Analysis of RPC Section Under Pure Bending

Fig. (2) shows a rectangular RPC section of width b and total depth *h*, reinforced in the tension zone with steel bars of area A_s located at an effective depth *d*. The section is subjected to a positive bending moment *M* such that at ultimate stage the strain and actual stress distributions are as shown in the figure. The figure also shows a conversion of the actual compressive stress block to an equivalent bi-linear stress block as proposed in this research.

The analysis will be carried out based on the following assumptions;

1- The RPC rectangular section to be analyzed under pure bending moment is under-reinforced such that at ultimate stage it fails according to the tensile type of failure.

2- The strain distribution across the depth of the RPC section is linear with maximum compressive strain $\varepsilon_{cu} = 2\varepsilon_{\circ}$.

3- The compression zone is defined by an equivalent bi-linear stress block.

4- The steel or polypropylene fibers in the compression zone are ineffective.

5- Strain hardening of steel bars is neglected.

6- The steel or polypropylene fibers in the tension zone do not rupture but they exhibit an average pullout stress f_t at values of tensile strain exceeding ε_t . The average pullout fiber stress f_t can be defined as the total longitudinal pullout force carried by the randomly oriented fibers in the tension zone divided by the effective tensile area of the RPC section which extends from the tension face of the section to the fiber exercising a minimum tensile strain ε_t.

7- The tensile contribution of the steel or polypropylene fibers is represented by a rectangular stress block of intensity f_t covering the effective tensile area of the RPC section.

Referring to the strain and equivalent stress distributions of Fig.(2),

From strain compatibility:

$$\frac{\varepsilon_t}{y_t} = \frac{\varepsilon_\circ}{\frac{c}{2}}$$

$$\therefore y_t = \frac{\varepsilon_t}{\varepsilon_\circ} \cdot \frac{c}{2} \qquad \text{.....(1)}$$

The depth h_f of the effective tensile area is given by

$$h_f = h - c - y_t \qquad \text{.........(2)}$$

Substitution of Equation (1) into Equation (2) gives

$$h_f = h - c\left(1 + \frac{\varepsilon_t}{2\varepsilon_\circ}\right) \text{......(3)}$$

the values of the compressive forces C_1, C_2 and C_3 and the tensile forces T_1 and T_2 are listed below;

$$C_1 = \left(\alpha\, f'_c\, \frac{c}{2}\right)(b) = \frac{\alpha}{2}\, b\, c\, f'_c \quad \text{...(4)}$$

$$C_2 = \left[(1-\alpha)\, f'_c \cdot \frac{c}{2} \cdot \frac{1}{2}\right] \cdot (b) = \frac{(1-\alpha)}{4}\, b\, c\, f'_c \quad \text{..(4}a\text{)}$$

$$C_2 = \left[(1-\alpha)\, f'_c \cdot \frac{c}{2} \cdot \frac{1}{2}\right] \cdot (b) = \frac{(1-\alpha)}{4}\, b\, c\, f'_c \text{......(4}b\text{)}$$

$$C_3 = \left(f'_c \cdot \frac{c}{2} \cdot \frac{1}{2} \right) \cdot (b) = \frac{b\,c}{4}\, f'_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4c)$$

$$T_1 = A_s f_y \quad \ldots\ldots (4d)$$

$$T_2 = (f_t\, h_f) \cdot (b) = f_t \left[h - c \left(1 + \frac{\varepsilon_t}{2\,\varepsilon_\circ} \right) \right] \cdot (b)$$

$$= f_t\, b\, h - f_t\, b\, c \left(1 + \frac{\varepsilon_t}{2\,\varepsilon_\circ} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4e)$$

The locations of such forces from the neutral axis $Y_{c_1} = \frac{3}{4}\, c$ ……………………(5a)

$$Y_{c_2} = \frac{2}{3}\, c \quad \ldots\ldots\ldots\ \ldots\ldots\ldots (5b)$$

; $Y_{c_3} = \frac{1}{3}\, c$

For C_1;

For C_2 ; $Y_{T_1} = d - c$

For C_3 ; $Y_{T_2} = h - c - \frac{h_f}{2} = h - c - \frac{1}{2}\left[h - c \left(1 + \frac{\varepsilon_t}{2\varepsilon_\circ} \right) \right]$ ……(5c)

For T_1 ; $= \frac{h}{2} - \frac{c}{2}\left(1 - \frac{\varepsilon_t}{2\,\varepsilon_\circ} \right)$ …..(5d)

For T_2 ;

The total compressive force on the RPC section can therefore be calculated;

$$C = C_1 + C_2 + C_3$$

$$= \frac{\alpha}{2}\, b\, c\, f'_c + \frac{(1-\alpha)}{4}\, b\, c\, f'_c + \frac{bc}{4}\, f'_c \quad \ldots\ldots (5e)$$

$$= \frac{1}{2}\left(1 + \frac{\alpha}{2}\right) b\, c\, f'_c \quad \ldots\ldots (6)$$

The total tensile forces on the section is;

$$T = T_1 + T_2$$

$$= A_s\, f_y + f_t\, b\, h - f_t\, b\, c \left(1 + \frac{\varepsilon_t}{2\,\varepsilon_\circ} \right) \quad \ldots (7)$$

In pure bending;

$$C = T$$

$$\therefore \frac{1}{2}\left(1 + \frac{\alpha}{2} \right) b\, c\, f'_c = A_s\, f_y + f_t\, b\, h - f_t\, b\, c \left(1 + \frac{\varepsilon_t}{2\varepsilon_\circ} \right) \quad \ldots (8)$$

A simplification of Equation (8) gives;

$$\frac{c}{d} = \frac{\left[\rho \frac{f_y}{f'_c} + \frac{f_t}{f'_c} \left(\frac{h}{d} \right) \right]}{\left[\frac{f_t}{f'_c} \left(1 + \frac{\varepsilon_t}{2\varepsilon_\circ} \right) + \frac{1}{2} \left(1 + \frac{\alpha}{2} \right) \right]} \quad \ldots\ldots\ldots (9)$$

where $\rho = \frac{A_s}{b\,d}$

Equation (9) defines the location of the neutral axis as a ratio of the effective depth *d*. In this equation the concrete properties f'_c and $\varepsilon_\circ$ can be determined by testing RPC cylinders under uni-axial compression, while f_y is determined from the tension test of the steel bars. The value of α, as mentioned earlier, can be determined from the equations listed in Tables (1) and (2). The only unknown terms in Equation (9) are f_t and ε_t. For the sake of carrying out numerical analysis of a specific RPC mix in the present investigation, the values of f_t and ε_t will be assumed but it is recommended that a comprehensive future study should be directed towards finding the actual values of these terms through testing numerous RPC prisms under uni-axial tension. Therefore this subject can be considered as a promising field for future research on reactive powder concrete.

The nominal ultimate bending moment capacity of the rectangular singly reinforced RPC section under study can now be determined by summing up the moments around the neutral axis caused by all the compressive and tensile forces on the section such that;

$$M_n = C_1 Y_{c_1} + C_2 Y_{c_2} + C_3 Y_{c_3} + T_1 Y_{T_1} + T_2 Y_{T_2} \quad \ldots\ldots (10)$$

Substituting into this equation the values of the compressive forces $C_{1,2,3}$ and the tensile forces $T_{1,2}$ as given by Equations (4) together with their distances from the neutral axis as given by Equations (5) will lead to the following final equation after simplification;

$$M_n = \frac{1}{4}\left(\frac{5}{6}\alpha + 1\right)\left(\frac{c}{d}\right)^2 f_c' b\, d^2 + A_s f_y d\left(1 - \frac{c}{d}\right) + \left[\frac{1}{2}\left(\frac{h}{d}\right)^2 - \left(\frac{h}{d}\right)\cdot\left(\frac{c}{d}\right) + \frac{1}{2}\left(1 - \frac{\varepsilon_t^2}{4\varepsilon_\circ^2}\right)\left(\frac{c}{d}\right)^2\right] f_t\, b\, d^2 \quad \ldots. (11)$$

Where $\frac{c}{d}$ is as given by Equation (9).

As can be seen from Equation (11) the nominal ultimate bending moment capacity M_n of a singly reinforced RPC section is produced by the combined action of three essential elements in the section which are in their respective order appearing in the three terms of the equation, the concrete in the compression zone represented by f_c', the steel bars represented by $A_s f_y$ and the steel or polypropylene fibers in the tension zone represented by f_t. The contribution of the latter term to the value of M_n depends largely on the area of steel bars used in the section. In heavily reinforced RPC sections, the neutral axis shifts towards the tension face of the section reducing the area of the effective tension zone and this results in a lesser contribution of the steel or polypropylene fibers to the value of M_n.

Application

The accuracy of Equation (11) can be examined through comparisons with the results of the tests conducted in ref. (1) on moderately reinforced RPC beams; namely (RBS9) and (RBP9). Both beams have rectangular section of width b = 200mm and total depth h = 50mm, reinforced with four steel bars of diameter 10mm located at an effective depth d = 39mm and having a yield stress f_y = 400MPa. Each beam was simply supported with a clear span of 600mm and tested up to failure under the action of two point loads each of magnitude P/2 located at distance 200mm from the nearer support. By neglecting the self weight of the beam, the maximum applied moment in the beam at failure is therefore;

$$M_{max} = \frac{P_u}{2} * 200 = 100\, P_u \quad \ldots\ldots.(12)$$

where Pu and M_{max} are in units of N (Newton) and N.mm respectively. Pu is the total failure load.

A) Comparison Between Theoretical Predictions and Experimental Results for RPC Beam RBS9:

The properties of RBS9 given in ref.(1) are as listed below;

Type of pozzolanic material = Silica Fume (SF)

Type of fibers = Steel fibers

% of fibers by volume = 1.25%

f_c' =184 MPa and $\varepsilon_\circ = 0.0043$ (as given in Table 1)

The equation describing the descending part of the compressive stress-compressive strain relationship for RPC mix S9 as given in Table (1) is;

$$f_c = \frac{-69.021\times10^3\varepsilon_c + 16.509\times10^6\varepsilon_c^{\,2}}{1-0.8402\times10^3\varepsilon_c + 0.1438\times10^6\varepsilon_c^{\,2}}$$

Substituting into this equation;

$$\varepsilon_c = 2\varepsilon_\circ = 2*0.0043 = 0.0086$$
$$f_c = \alpha\, f_c' = 184\alpha$$

$$\therefore 184\alpha =$$

$$\frac{-69.021\times10^3(0.0086)+16.509\times10^6(0.0086)^2}{1-0.8402\times10^3(0.0086)+0.1438\times10^6(0.0086)^2}$$

$$\therefore \alpha = 0.773$$

For b = 200mm, h = 50mm, d = 39mm, f_y = 400MPa;

$$A_s = 4\times\frac{\pi}{4}(10)^2 = 314.16mm^2$$

$$\therefore \rho = \frac{A_s}{bd} = \frac{314.16}{200\times39} = 0.040277$$

Assuming that $f_t = 12$MPa , $\frac{\varepsilon_t}{\varepsilon_\circ} = 0.1$

(to be verified experimentally in future research), then from Equation (9); $f_c = \alpha\, f_c' = 172\,\alpha$

$$\frac{c}{d} = 0.2247$$

Substituting this value of c/d into Eq. (11); gives;

$$M_n = 7001626 \ N.mm$$

which when used in Eq. (12) one gives;

$$P_u = 70016.26\ N$$

Thus the expected theoretical ultimate load of RBS9 is $P_u)_{theo.}$=*70.016 kN* which when compared with the experimental collapse load $P_u)_{exp..}$=*72 kN* gives the following ratio;

$$\frac{P_u)_{theo.}}{P_u)_{\exp.}} = \frac{70.016}{72} = 0.97$$

B) Comparison Between Theoretical Predictions and Experimental Results for RPC Beam RBP9:

The properties of RBP9 as given in ref. (2) are as listed below;

Type of pozzolanic material = Silica Fume (SF)

Type of fibers = Polypropylene fibers% of fibers by volume = 1.25%

f_c' = 172 *MPa* $\varepsilon_\circ = 0.004$

and (as given in Table 2)

The equation describing the descending part of the compressive stress-compressive strain relationship for RPC mix P9 as given in Table (2) is;

$$f_c = \frac{-80.152\times10^3\varepsilon_c + 20.6615\times10^6\varepsilon_c^{\,2}}{1-0.966\times10^3\varepsilon_c + 0.182625\times10^6\varepsilon_c^{\,2}}$$

A substitute of $\varepsilon_c = 2\varepsilon_\circ = 0.008$ and into this equation gives;

$$\alpha = 0.798$$

Using *b = 200mm , h = 50mm , d = 39mm , f_y = 400mm , ρ = 0.040277* and assuming

= 8 f_t *MPa* $\frac{\varepsilon_t}{\varepsilon_\circ} = 0.1$,

then

From Equation (9), $\frac{c}{d} = 0.2048$

From Equation (11), $M_n = 6222823\ N.mm$

From Equation (12),

$$P_u)_{theo.} = 62228.23\ N\ = 62.228\ kN$$

The experimental collapse load of this beam is $P_u)_{exp.} = 65\ kN$ giving the following ratio;

$$M_n = \rho\, f_y\, b\, d^2 \left(1 - 0.59\,\rho\,\frac{f_y}{f_c'}\right) \quad \text{.......(13)}$$

Both equations estimate the nominal ultimate bending moment capacity of rectangular singly reinforced concrete sections based on tensile failure type.

However, to ensure a correct comparison to be made, a rectangular section of specific dimensions and reinforced with a specific amount of tension reinforcement A_s that has a certain value of yield stress f_y is to be used for both types of concrete. Obviously f_c' is different for both

concretes. Therefore the following properties are considered for such comparison;

- Ordinary RC Section

Using $f_c' = 25\ MPa$, $f_y = 400\ MPa$

$\rho = 0.02032$, $b = 400mm$,
$h = 700mm$, $d = 610mm$

from Equation (13) ,

$M_n = 977.71\ kN.m$

- RPC Section

Assuming RPC mix of the type S9

$f_c' = 184\ MPa$, $\varepsilon_\circ = 0.0043$, $\alpha = 0.773$

Using $f_t = 12\ MPa$, $\dfrac{\varepsilon_t}{\varepsilon_\circ} = 0.1$,

$b = 400mm$, $h = 700mm$,

$d = 610mm$, $\rho = 0.02032$;

$$\therefore A_s = \rho\, b\, d = 4958 mm^2$$

From Equation (9) ,

$$\frac{c}{d} = 0.1562$$

From Equation (11) ,

$$M_n = 2173.03\ kN.m$$

Thus the ratio between $M_n)_{RPC}$ and $M_n)_{ORC}$ is;

$$\frac{M_n)_{RPC}}{M_n)_{ORC}} = \frac{2173.03}{977.71} = 2.22$$

which indicates an enhancement in M_n above that of ordinary concrete of the order 122% .

Conclusions

Depending on the results of the present research, the following conclusions can be drawn;

1- The derived equation for the nominal ultimate bending moment capacity Mn of rectangular singly reinforced rectangular RPC sections (which is based on an idealized bi-linear compressive stress block proposed in this research) is found to compare favorably with available flexural test results(1) conducted on moderately reinforced RPC beams.

2- A comparison between the derived equation of M_n for RPC and a corresponding equation of M_n for ordinary reinforced concrete (as suggested by the ACI-05 Code) indicates a considerable enhancement in the ultimate bending moment capacity of RPC sections above that of ordinary concrete sections of the order 122% for the selected example of comparison. This reflects an important practical significance in esign and analysis of flexural members.

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Table (1): Analytical Stress – Strain Relationships for RPC Specimens In Compression (Groups G1, G2, G3 – using Steel Fibers) – Taken from Ref. (1)

Group No. *	RPC Mix **	f_c' (MPa)	ε_o	Ascending Part $(0 \le \varepsilon_c \le \varepsilon_o)$	Descending Part $(\varepsilon_c \ge \varepsilon_o)$
G1 (Mk)	S1	138	0.0036	$f_c = \frac{43.43\times10^3\times\varepsilon_c - 10.31\times10^6\times\varepsilon_c^2}{1-0.241\times10^3\times\varepsilon_c + 0.00247\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-2.108\times10^3\times\varepsilon_c + 1.959\times10^6\times\varepsilon_c^2}{1-0.571\times10^3\times\varepsilon_c + 0.0914\times10^6\times\varepsilon_c^2}$
	S2	141	0.0038	$f_c = \frac{40.927\times10^3\times\varepsilon_c - 9.579\times10^6\times\varepsilon_c^2}{1-0.236\times10^3\times\varepsilon_c + 0.001316\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-35.027\times10^3\times\varepsilon_c + 9.627\times10^6\times\varepsilon_c^2}{1-0.7747\times10^3\times\varepsilon_c + 0.1375\times10^6\times\varepsilon_c^2}$
	S3	144	0.0042	$f_c = \frac{40.491\times10^3\times\varepsilon_c - 7.673\times10^6\times\varepsilon_c^2}{1-0.195\times10^3\times\varepsilon_c + 0.0034\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-34.389\times10^3\times\varepsilon_c + 9.4204\times10^6\times\varepsilon_c^2}{1-0.715\times10^3\times\varepsilon_c + 0.1221\times10^6\times\varepsilon_c^2}$
G2 (MS)	S4	155	0.0036	$f_c = \frac{48.782\times10^3\times\varepsilon_c - 11.577\times10^6\times\varepsilon_c^2}{1-0.240\times10^3\times\varepsilon_c + 0.00247\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-6.028\times10^3\times\varepsilon_c + 3.803\times10^6\times\varepsilon_c^2}{1-0.5944\times10^3\times\varepsilon_c + 0.1017\times10^6\times\varepsilon_c^2}$
	S5	158	0.0039	$f_c = \frac{47.4\times10^3\times\varepsilon_c - 9.837\times10^6\times\varepsilon_c^2}{1-0.2128\times10^3\times\varepsilon_c + 0.00348\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-9.277\times10^3\times\varepsilon_c + 1.392\times10^6\times\varepsilon_c^2}{1-0.4541\times10^3\times\varepsilon_c + 0.07456\times10^6\times\varepsilon_c^2}$
	S6	163	0.0041	$f_c = \frac{45.282\times10^3\times\varepsilon_c - 9.357\times10^6\times\varepsilon_c^2}{1-0.21\times10^3\times\varepsilon_c + 0.002082\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-21.11\times10^3\times\varepsilon_c + 7.573\times10^6\times\varepsilon_c^2}{1-0.6137\times10^3\times\varepsilon_c + 0.10595\times10^6\times\varepsilon_c^2}$
G3 (SF)	S7	172	0.0038	$f_c = \frac{55.266\times10^3\times\varepsilon_c - 10.8512\times10^6\times\varepsilon_c^2}{1-0.205\times10^3\times\varepsilon_c + 0.00616\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-13.579\times10^3\times\varepsilon_c + 5.4078\times10^6\times\varepsilon_c^2}{1-0.6053\times10^3\times\varepsilon_c + 0.10069\times10^6\times\varepsilon_c^2}$
	S8	176	0.0040	$f_c = \frac{52.096\times10^3\times\varepsilon_c - 10.318\times10^6\times\varepsilon_c^2}{1-0.204\times10^3\times\varepsilon_c + 0.003875\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-32.868\times10^3\times\varepsilon_c + 9.834\times10^6\times\varepsilon_c^2}{1-0.68675\times10^3\times\varepsilon_c + 0.118375\times10^6\times\varepsilon_c^2}$
	S9	184	0.0043	$f_c = \frac{50.193\times10^3\times\varepsilon_c - 9.414\times10^6\times\varepsilon_c^2}{1-0.1923\times10^3\times\varepsilon_c + 0.00292\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-69.021\times10^3\times\varepsilon_c + 16.509\times10^6\times\varepsilon_c^2}{1-0.8402\times10^3\times\varepsilon_c + 0.1438\times10^6\times\varepsilon_c^2}$

Table (2): Analytical Stress – Strain Relationships for RPC Specimens in Compression (Groups G4, G5, G6 – using Polypropylene Fibers) – Taken from Ref. (1)

Group No.	RPC Mix	f_c' (MPa)	ε_o	Ascending Part $(0 \le \varepsilon_c \le \varepsilon_o)$	Descending Part $(\varepsilon_c \ge \varepsilon_o)$
G4 (Mk)	P1	118	0.0034	$f_c = \frac{37.934\times10^3\times\varepsilon_c - 10.044\times10^6\times\varepsilon_c^2}{1-0.2668\times10^3\times\varepsilon_c + 0.001384\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-14.576\times10^3\times\varepsilon_c + 4.5526\times10^6\times\varepsilon_c^2}{1-0.07118\times10^3\times\varepsilon_c + 0.1251\times10^6\times\varepsilon_c^2}$
	P2	119	0.00355	$f_c = \frac{36.437\times10^3\times\varepsilon_c - 9.3104\times10^6\times\varepsilon_c^2}{1-0.2572\times10^3\times\varepsilon_c + 0.00111\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-32.281\times10^3\times\varepsilon_c + 8.583\times10^6\times\varepsilon_c^2}{1-0.8346\times10^3\times\varepsilon_c + 0.1515\times10^6\times\varepsilon_c^2}$
	P3	120	0.0036	$f_c = \frac{36.733\times10^3\times\varepsilon_c - 9.0833\times10^6\times\varepsilon_c^2}{1-0.2494\times10^3\times\varepsilon_c + 0.001466\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-31.1\times10^3\times\varepsilon_c + 8.9167\times10^6\times\varepsilon_c^2}{1-0.8147\times10^3\times\varepsilon_c + 0.15147\times10^6\times\varepsilon_c^2}$
G5 (MS)	P4	149	0.00345	$f_c = \frac{49.321\times10^3\times\varepsilon_c - 12.0552\times10^6\times\varepsilon_c^2}{1-0.2487\times10^3\times\varepsilon_c + 0.00311\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-25.827\times10^3\times\varepsilon_c + 8.362\times10^6\times\varepsilon_c^2}{1-0.753\times10^3\times\varepsilon_c + 0.14014\times10^6\times\varepsilon_c^2}$
	P5	152	0.0036	$f_c = \frac{47.5\times10^3\times\varepsilon_c - 11.4\times10^6\times\varepsilon_c^2}{1-0.243\times10^3\times\varepsilon_c + 0.00216\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-34.749\times10^3\times\varepsilon_c + 10.5086\times10^6\times\varepsilon_c^2}{1-0.7842\times10^3\times\varepsilon_c + 0.1463\times10^6\times\varepsilon_c^2}$
	P6	153	0.0038	$f_c = \frac{45.9\times10^3\times\varepsilon_c - 10.214\times10^6\times\varepsilon_c^2}{1-0.2263\times10^3\times\varepsilon_c + 0.002493\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-38.532\times10^3\times\varepsilon_c + 11.941\times10^6\times\varepsilon_c^2}{1-0.7782\times10^3\times\varepsilon_c + 0.1473\times10^6\times\varepsilon_c^2}$
G6 (SF)	P7	168	0.0036	$f_c = \frac{53.933\times10^3\times\varepsilon_c - 12.3796\times10^6\times\varepsilon_c^2}{1-0.2342\times10^3\times\varepsilon_c + 0.003472\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-37.193\times10^3\times\varepsilon_c + 11.783\times10^6\times\varepsilon_c^2}{1-0.7769\times10^3\times\varepsilon_c + 0.1473\times10^6\times\varepsilon_c^2}$
	P8	170	0.0038	$f_c = \frac{56.6816\times10^3\times\varepsilon_c - 10.2424\times10^6\times\varepsilon_c^2}{1-0.1929\times10^3\times\varepsilon_c + 0.009\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-74.129\times10^3\times\varepsilon_c + 19.5547\times10^6\times\varepsilon_c^2}{1-0.9624\times10^3\times\varepsilon_c + 0.1843\times10^6\times\varepsilon_c^2}$
	P9	172	0.0040	$f_c = \frac{48.375\times10^3\times\varepsilon_c - 10.449\times10^6\times\varepsilon_c^2}{1-0.21875\times10^3\times\varepsilon_c + 0.00175\times10^6\times\varepsilon_c^2}$	$f_c = \frac{-80.152\times10^3\times\varepsilon_c + 20.6615\times10^6\times\varepsilon_c^2}{1-0.966\times10^3\times\varepsilon_c + 0.182625\times10^6\times\varepsilon_c^2}$

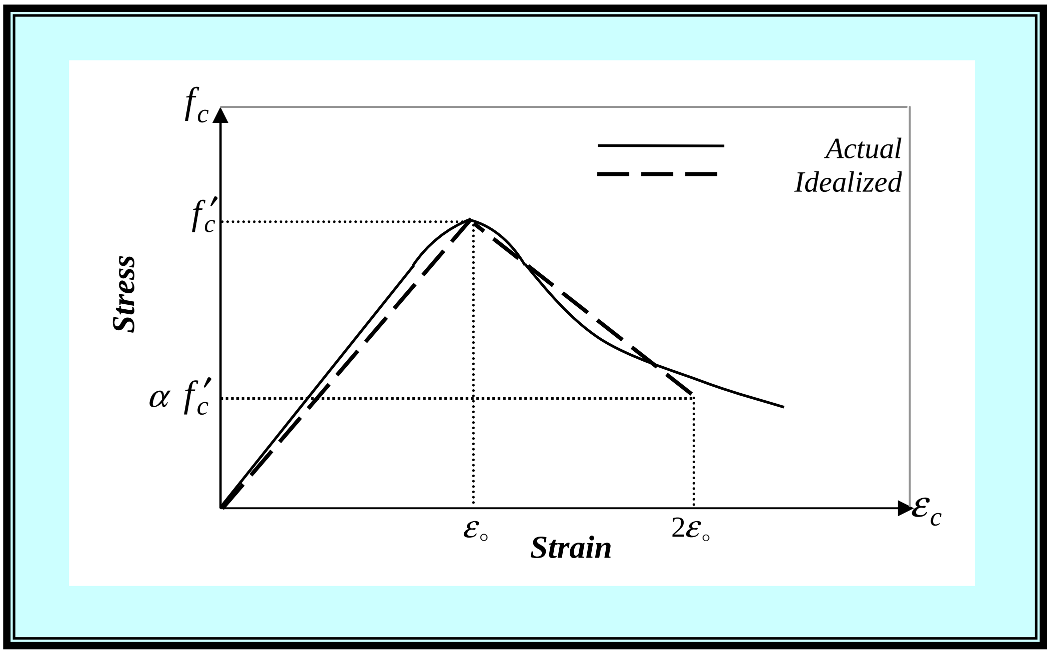

Figure (1): Stress-Strain Curve of RPC in Compression

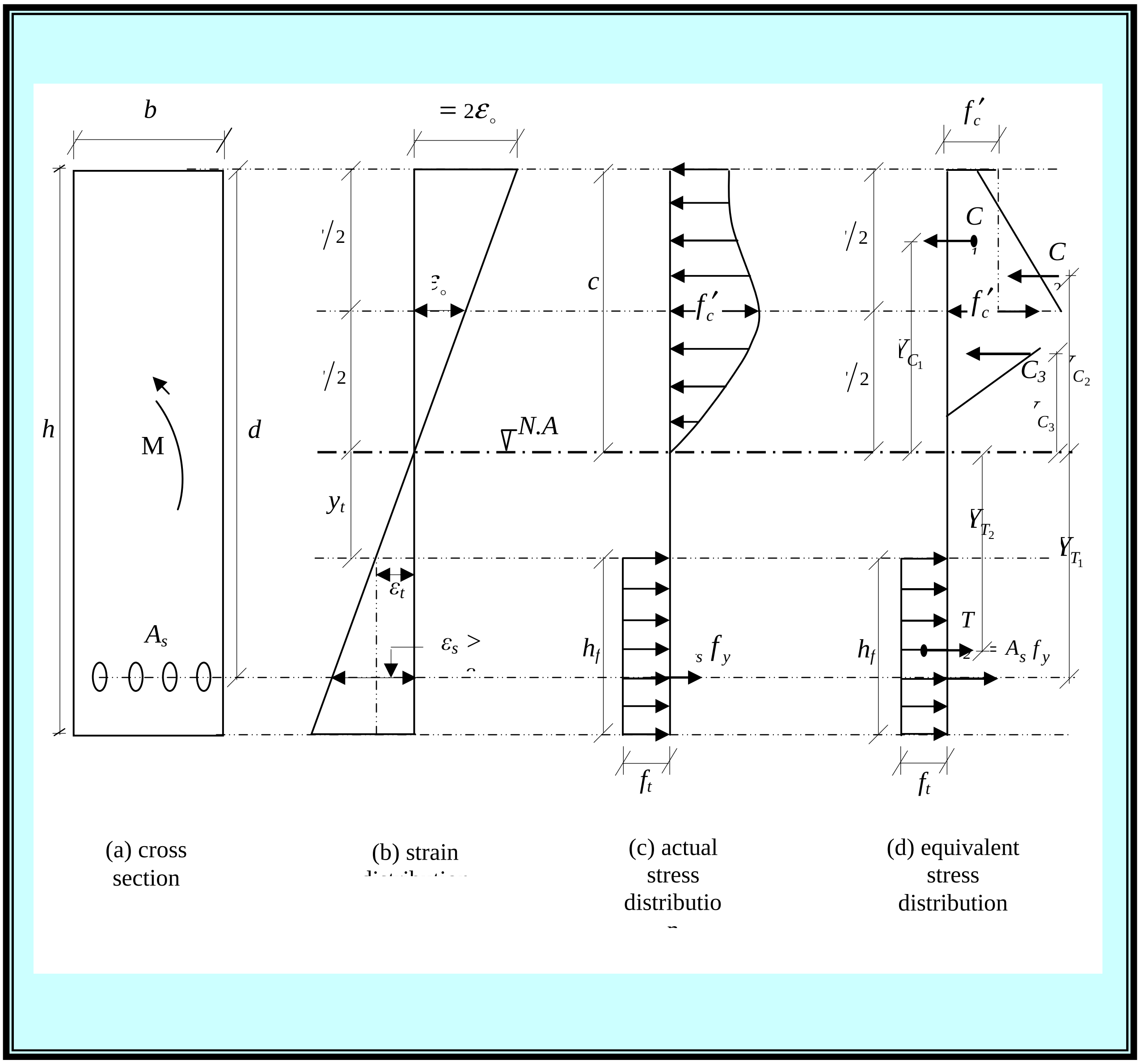

Figure (2): Proposed Strain and Stress Distributions on RPC Section at *Ultimate Stage*