Adaptive Channel Equalization using Least Mean Square (LMS) algorithm with different value for step size

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Abstract

In this paper we present the whole structure for least mean square equalizer algorithm with different value for step size. After the transmitted signal arrive at the receiver, it sever from distortion because the Intersymbol Interference (ISI) that may occur in the transmission channel of the symbols of sent data and also because of the fading effects with the AWGN. Therefore, many advanced methods for detecting symbols in the receiver should be applied to retrieve the original transmitted data symbols. The coefficients of a Finite Impulse Response (FIR) filter can be adjusted by the LMS algorithm to minimize the noise and Intersymbol Interference. Simulation results show that the value of the step size effects on stability of adaptive filter, the speed of convergence and the error of steady state.

Key Words: Adaptive Equalizer, Least Mean Square (LMS) algorithm, step size, Gradient algorithm, steepest descent, Finite Impulse Response (FIR) filter.

1.1 Introduction

In any system, the message bit sequence in the transmitter can be modulated into symbols, then these symbols send out through a wireless fading channel. The received signal in the receiver is distorted by multipath fading channel. To retrieve the transmitted bits at the receiver the channel effect should be estimated. The received signal can be defined as the convolution between the frequency response of the channel and transmitted signal. More than one technique can be used to estimate the response of the channel. In any system many of the various factors must be taken into account to select technique for channel estimation. These factors include the channel time variation, performance and mathematical complexity[1]. In this paper the coefficients of a Finite Impulse Response (FIR) filter can be adjusted by the LMS algorithm. It is clear from fig. 1 that the signal u(n) is the signal which send through fading channel and the signal x(n) is considered as the input signal through the FIR filter. The adaptive filter adjust on the value of coefficient of the filter depend on the adaptive algorithm and the signal of error e(n) between the adaptive filter output y(n) and desired signal d(n). At the end the error value
e(n) reduced and the coefficients of filter w(n) will be similar to the ideal channel[2].

Figure(1): adaptive algorithm[2].

1.2 Least mean square algorithm (gradient algorithm)
An adaptive equalizer is considered as a time varying filter. The adaptive equalizer is called a transversal filter. In general adaptive filter consists of two parts an adaptive algorithm and a linear filter. The type of Linear filter is Finite Impulse Response (FIR) filter. The adaptive algorithm is the Least Mean Square (LMS) algorithm. The coefficients of a Finite Impulse Response (FIR) filter can be adjusted by the LMS algorithm to minimize the noise and Intersymbol Interference. FIR filter is the simplest filter to design. FIR filter are digital filter that have a finite impulse response[3]. The structure of an adaptive equalizer is shown in Fig.(2), where the subscript i represents a discrete time index. The value of input \( s_i \) at any time depends on the value of the noise and the radio fading channel. A single input \( s_i \) has N delay elements, N + I taps and N + I complex multipliers called weights. The filter weights also have a subscript \( i \), to indicate that it change with time. The weights of the filter are updated continuously by the adaptive algorithm[4]. The error signal \( e_i \) in the adaptive algorithm is the difference between received signal \( r_i \) and estimated of the received signal \( r'_{i} \). The adaptive algorithm uses \( y_{hi} \) to updates the equalizer weights. Therefore, the least mean squares (LMS) algorithm searches for the optimum value of filter weights. This process repeated rapidly while the equalizer attempts to converge, and more than one technique (such as gradient or steepest decent algorithms) should be used to reduce the error. At the end, the adaptive algorithm freezes the filter weights until the error signal reach to acceptable level or in case of sent a new training sequence[5][2].

A signal input for the equalizer defined as a vector \( S_i \) where
\[
S_i = [s_i \ s_{i-1} \ s_{i-2} \ldots s_{i-h}] \quad \ldots (1)
\]
The weight vector (taps) of the adaptive equalizer which represent the number of filter coefficients for FIR filter can be written as:
$y_h = [y_{0i} \ y_{1i} \ y_{2i} \ \ldots \ \ y_{hi}] \ \ \ \ \ \ \ \ \ \ (2)$

Figure(2): adaptive Equalizer using Least Mean Square algorithm[3].

The received signal at the receiver is given by (Where $w_i$ is the Additive White Gaussian Noise (AWGN)):

$\tilde{r}_i = \sum_{h=0}^{g} \hat{s}_{i-h} \ y_{i-h} + w_i \ \ \ \ldots [3]$  

$\hat{r}_i = \sum_{h=0}^{g} \hat{s}_{i-h} \ \hat{y}_{i-1,h} \ \ldots [4]$  

$e_i = r_i - \hat{r}_i \ \ \ \ \ldots [5]$  

The adaptive filter equation is given by:

$\hat{y}_{i,h} = \hat{y}_{i-1,h} + \Delta \ e_i \ \hat{s}_{i-h} \ \ \ldots [6]$  

The $\Delta$ factor is the step size and it may not be constant. It can be ranging between small values to provide an excellent estimation for slow channels, or a big value to follow the fast channel change.

1.3 SIMULATION AND RESULTS
The adaptive algorithm adjusts the coefficient iteratively in order to minimize the magnitude of error $e_i$. In this paper we use variable step size to improve the speed of convergence for adaptive filter. When we use LMS algorithm to create an adaptive filter, we must choose value for step size. The step size effects on stability of adaptive filter, the speed of convergence and the error of steady state. When the step size value is small the error of steady state will be small and the speed of convergence for the adaptive filter will decrease. When the step size value is large the speed of convergence for the adaptive filter will be improved. Therefore, when the step size value is large this might makes the adaptive unstable. In this paper we take three different value for step size and show the result for least mean square algorithm. It is obvious in Figure (3) and figure (4) when the value of step size equal to 0.0035 the difference between the estimated value and real value for the coefficient of the channel is large and there is a steady state error in the curve of the magnitude of error in the adaptive filter.

![Figure (3): Difference between actual and estimated value of the coefficient of the channel when the step size = 0.0035.](image-url)
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Figure (4): The magnitude of the error for the least mean square (LMS) estimator when the step size = 0.0035.

It is obvious in Figure (5) and figure (6) when the value of step size equal to 0.035 the difference between the estimated value and real value for the coefficient of the channel become small, the speed of convergence increase and the error of steady state increase.

Figure (5): Difference between actual and estimated value of the coefficient of the channel when step size = 0.035
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Figure (6): The magnitude of the error for the least mean square (LMS) estimator when step size = 0.035

It is obvious in Figure (7) and figure (8) when the value of step size equal to 0.055 the difference between the estimated value and real value for the coefficient of the channel is small, the speed of convergence increase and the curve for the error of steady state become unstable.
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Figure (7): Difference between actual and estimated value of the coefficient of the channel when the step size = 0.055

CONCLUSION
The Least Mean Square (LMS) algorithm requires fewer computational because it is simple and no need for matrix inversion. LMS algorithm is applied to equalize the effect of the channel. In this paper we take different value for step size in adaptive equalizer. The value for the step size must be used when the Least Mean Square (LMS) algorithm used to create an adaptive filter. From the simulation result we conclude that when the step size value is small the system will be slow but the steady state mean square error will be small. Otherwise when the step size value is large the system will be fast but the steady state mean square error will be large.

REFERENCES
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