Effect of Gaussian Aperture Width on Visibility for Different Crystal Length

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Abstract

The main goal in Quantum Cryptography is high security and this can be achieved by using single photon sources. The present work is a theoretical analysis devoted to investigate the interference pattern of biphoton amplitude generated by spontaneous parametric down conversion (SPDC) in a nonlinear crystal (BBO) pumped by femtosecond optical pulses. We have studied the visibility as a function of Gaussion aperture width for different parameters, such as the crystal length. The best visibility can be obtained at crystal length of (1.5 mm) . In our results, patterns occur and increase with increase of crystal length and decreasing aperture diameter. When using slit aperture (vertical and horizontal direction), the coincident count is more symmetry in horizontal with dimension (1×7) mm. With double crystal and parallel direction,the visibility decreases dramatically with increase in the Gaussian aperture width, while at a distance between crystals equal to (3mm) the visibility vanishes slowly at crystal thickness (L=0.5mm).

Keywords: entangled source; Quantum Cryptography; fs laser ;quantum optics ;quantum information.

Introduction

Spontaneous parametric down-conversion SPDC, namely the generation of two low-frequency photons is when a strong high-frequency pump interacts with a nonlinear crystal, is a reliable source for generating pairs of photons with entangled properties. The photons generated in SPDC exhibit a rich structure, i.e., the two-photon quantum state is described by its polarization, spatial shape, and frequency spectrum. However, the corresponding entangled states generally take advantage of only a portion of the total two-photon quantum state, [1,2].

To date most applications of parametric down-conversion in quantum systems make use of polarization entanglement as the quantum resource. Such entanglement is confined to a two-dimensional Hilbert space. On the contrary, both frequency entanglement and spatial entanglement occur in an infinite-dimensional Hilbert space, which opens a wealth of opportunities to enhance the potential of quantum techniques [3,4].

At present, the most efficient source of entangled photons is spontaneous parametric down conversion, such as KDP, LiIO3, KNbO3, LiNbO3. Recently new materials such as periodically poled lithium niobate (PPLN) have shown unprecedented effective nonlinearities. Nonlinear optical phenomena have long been used for wave mixing of light fields, such as frequency doubling or frequency summation/ difference. Parametric down conversion is the scheme in which a UV pump field is mixed with two IR fields, known as the signal and idler beam. For strong beams, all nonlinear phenomena can be understood by the classical theory of electro-magnetic waves. However, a quantum mechanical description of the interaction leads to neoclassical features at low light levels, such as the squeezing of phase quadratures of light or spontaneous photon pair production. The latter was first observed in 1970 and has been used for a range of fundamental photon experiments by Ou and Mandel. Spontaneous parametric down conversion (SPDC) allows generating different kinds of entanglement such as energy-time entanglement, momentum or mode entanglement and polarization entanglement.. One can see that the laser field is coupled by the non-linear susceptibility to two different modes $(\vec{k},P)$ and $(\acute{k},\acute{P})$. For historical reasons, the photons created in these modes are called signal and idler. There are two different cases of photon polarizations [5]:

The signal and idler photons carry the same polarization, which is called SPDC type-I phase matching.

They carry opposite polarizations, which is called type-II phase matching. Because this process takes place in a birefringent medium, the two photons are also called ordinary and extraordinary in this case. This kind of entanglement called polarization entanglement [8].

Theoretical Analysis

The nonlinear crystal which used in this work to create entangled photon pair via the SPDC process is Beta barium borate, also written β-BaB2O4 and BBO, is a common non-linear optical material. It’s useful characteristics include transparency over a large bandwidth from UV through infrared, wide phase-matching capabilities, high damage threshold, and low hygroscopic susceptibility.

The quantum state generated from SPDC, Which is concurrently entangled in frequency and transverse wave vector, leads to transverse spatial effects that can be observed in quantum interference. As the aperture diameter is increased or the aperture is brought closer to the output plane of the non-linear medium, a greater range of wave vectors is allowed to propagate through to the detectors. These transverse wave vectors introduce distinguish ability between the signal idler photons, thus reducing the visibility of the observed quantum interference. This is analogues to the temporal distinguish ability introduced by the use of a fem to second-pulsed pump, Eq. (1). It is supposed that, the two crystals are of the same material and have equal thickness L. The apertures are symmetric for both transverse directions, and the analyzers are set 45o from the optical axis. So (υpol = −1), No spectral filters are used. Under these conditions, the explicit form of the visibility function becomes: [6,7] and is valid for any linear optical
system.

$$V(\iota)=\frac{1}{1+p^2}\int_{\left(\frac{Z}{L};\frac{t}{LD}\right)}^{dz\pi_L(z)\pi_L(\frac{2t}{D}-2L-z)g^{(1)}} +$$

$$\frac{1}{1+p^2}\int_{\mathrm{Re}\left[g^{(12)}\left(\frac{Z}{L}+\frac{t}{2};\frac{T}{LD}\right)e^{-id\Delta'}\right]}^{dz\pi_L(z)\pi_L(\frac{2t}{D}-L-Z)} \quad \text{........}(1)$$

$\rho = (d_1+d)/d_1$, $\pi_L(z)$ is the unit rect function form [0, L] and,

$D = u_o^{-1} - u_e^{-1}$ is the dispersion coefficient of the nonlinear medium. It is through the g-functions that spatial temporal effects enter the equation interference pattern. Fig.(1) illustrate collinear SPDS is generated in two bulk crystals of arbitrary thickness separated by an air gap.

However, to enable swift evaluation of the integrals in this equation, we approximate a "soft" Gaussian aperture of (1/e) widths r_A and r_B. A sharp circular aperture, of this type has a diffraction pattern described by a first-order Bessel function, whereas a Gaussian aperture of the type used in the numerical simulations, has Gaussian diffraction pattern. Despite this fundamental difference, it is a fair approximation if the width r of the Gaussian is selected to roughly fit the width b of Bessel function. In our calculations, this is done by choosing $r = b/2\sqrt{2}$. This approximation offers an indispensable advantage, as it allows us to evaluate the g function analytically and thereby reduce the demand for numerical integration in making theoretical predictions. Under this approximation, an expression for the visibility function at (τ = LD) for parallel orientations of the optical axes is given by:

$$V_p(LD) = 2\frac{d_1(d_1+d)}{d_1^2+(d_1+d)^2}\frac{1}{\sqrt{1+\alpha^2}}\int_o^1 d\xi\, e^{-B(1-\xi\left[\frac{d}{s+2d_1}\right]^2}$$

$$\left[\cos C\xi^2 + D\xi + \varepsilon - \phi_\gamma{}^{(d)} + \phi_{disp}{}^{(d)}\right]$$

$$\text{.................................}(2)$$

Where

$$\left.\gamma = \frac{k_p r^2}{4d_1}\frac{d}{d_1+d}, \phi_\gamma(d) = -\arctan(\gamma)\right\} \text{...............}(3)$$

and $r^2 = (r_A^2 + r_B^2)/2;$

$$B = 2\left[\frac{k_p|M|Lr}{4(d_1+d)}\right]^2\frac{1}{1+\gamma^2}\left(1+\frac{d}{2d_2}\right)^2 \text{.............}(4)$$

$$C = \frac{kp|ML|^2}{8(d_1+d)} \frac{d}{d_1} \frac{1}{1+\gamma^2} \quad \text{.............................} (5)$$

$$D = \frac{kp|ML|^2}{2(d_1+d)} \left(1 + \frac{d}{2d_1}\right) \frac{1}{1+\gamma^2} \quad \text{............} (6)$$

$$\varepsilon = \frac{kp|ML|^2}{8(d_1+d)} \left(\frac{d}{d_1} - 4\gamma^2 \frac{d_1+d}{d} \frac{1}{1+\gamma^2}\right) \frac{1}{1+\gamma^2}$$

.... (7)

For anti parallel axes, the visibility function at τ=LD is

$$V_a(LD) = -2 \frac{d_1(d_2+d)}{d_1^2+(d_1+d)^2} \frac{1}{\sqrt{1+\gamma^2}} X \int_0^1 d\xi\, e - B(1-\xi)^2 \cos[\varepsilon(1-\xi)^2 - \phi_v(d) + \phi_{disp}(d)]$$

.... (8)

For the case of two crystals in contact (d = 0):

$B = 2(kp|M|Lr/4d_1)^2, C = o, D = -kp/ML/^2 * 2d_1, and\ \varepsilon = 0.$

From eq. (3.68), we now observe that the visibility at τ=LD for parallel optical axes is:

$$V_p(LD) = exp\left[-2\left(\frac{kp|M|Lr}{4d_1}\right)^2\right] sinc\left(\frac{kp}{2d_1}|ML|^2\right)$$

.... (9)

While for anti paralles

$$V_a(LD) = -\sqrt{2\pi} \frac{d_1}{kp|M|Lr} - erf\left(\frac{kp|M|Lr}{2\sqrt{2}d_1}\right)$$

.... (10)

Note that these visibilities depend on the aperture diameter, as well as the crystal thickness (as was seen in the preceding section), and are once markedly different from (V_a=V_p=1) as predict by the single-mode theory.

The parameter used her in the calculation of equations are:

1. Aperture diameter b = 2.5 mm, 4.0 mm, 5.0 mm.
2. L = 0.5 mm
3. λp = 2π/kp = 351.1 nm
4. M = 0.07.

Results and Discussion

The obtained from using of cascade crystal is shown in Fig. (2) which presents a plot of Vp(LD) giving in equation (2) as a function of crystal separation d for aperture diameter b=2.5mm,4mm and 5mm.the maximum of visibility is corresponding to 2.5mm,4mm and 5mm, aperture diameter respectively.

Note that for a fixed value of the distance (d), there is a reduction of visibility for increased aperture diameters. For these split L=0.5, λ_p=2π/ k_p=351.1nm and M=0.07.

A reduction of viscosity also occurs as the distance between the crystals is increased. This pattern is distinctly different from that obtained using a single-crystal configuration with the same overall nonlinear interaction length; it comprise two separate and disjoint fourth-order interference pattern.[cascaded rays. Folder theoretical].for the case of two crystal in contact (d=0),we note from equ.(9) and (10) that these visibilities depend on the aperture diameter, as well as the crystal thickness.

Equ.(9) and (10) are plotted in Figs. (3) and (4) as a function of Gaussian aperture width (r) for three different crystal thickness L = (0.5, 2.5, 4.5) mm. As predicted the plot of the visibility obtained with a parallel orientation of the crystal axes reduces faster than the visibility for an anti-parallel orientation.

Conclusion

The polarization quantum interference pattern is found to vary strongly with the spacing between the two crystals. The coincidence visibility decreases dramatically with increasing crystal length when using pulsed pump, reversing the state when used CW pump an especially with small aperture diameter (~0.25 mm) the visibility decrease slowly.

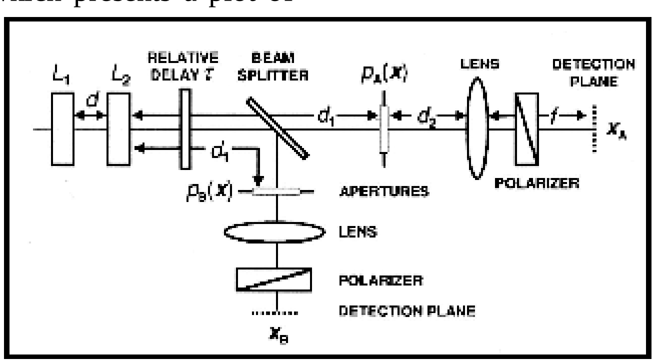

Fig. (1) Schematic representation of a polarization interferometer for which compute quantum interference pattern, [6].

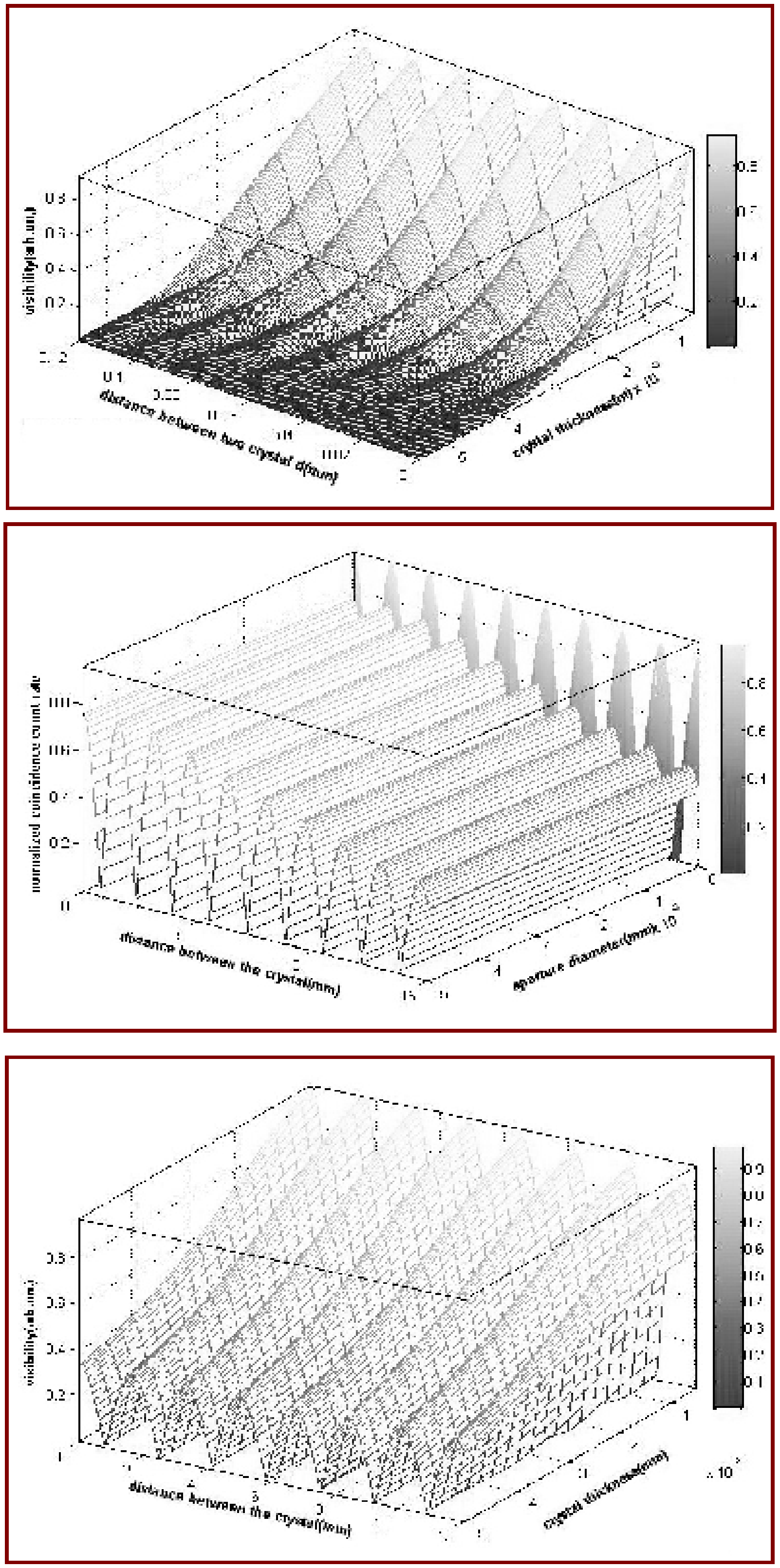

Fig.(2) visibility as a function of crystal seperation for different aperature diameter in case parallel and anti parallel (b, C) and as afunction of crystal length in parallel case (a).

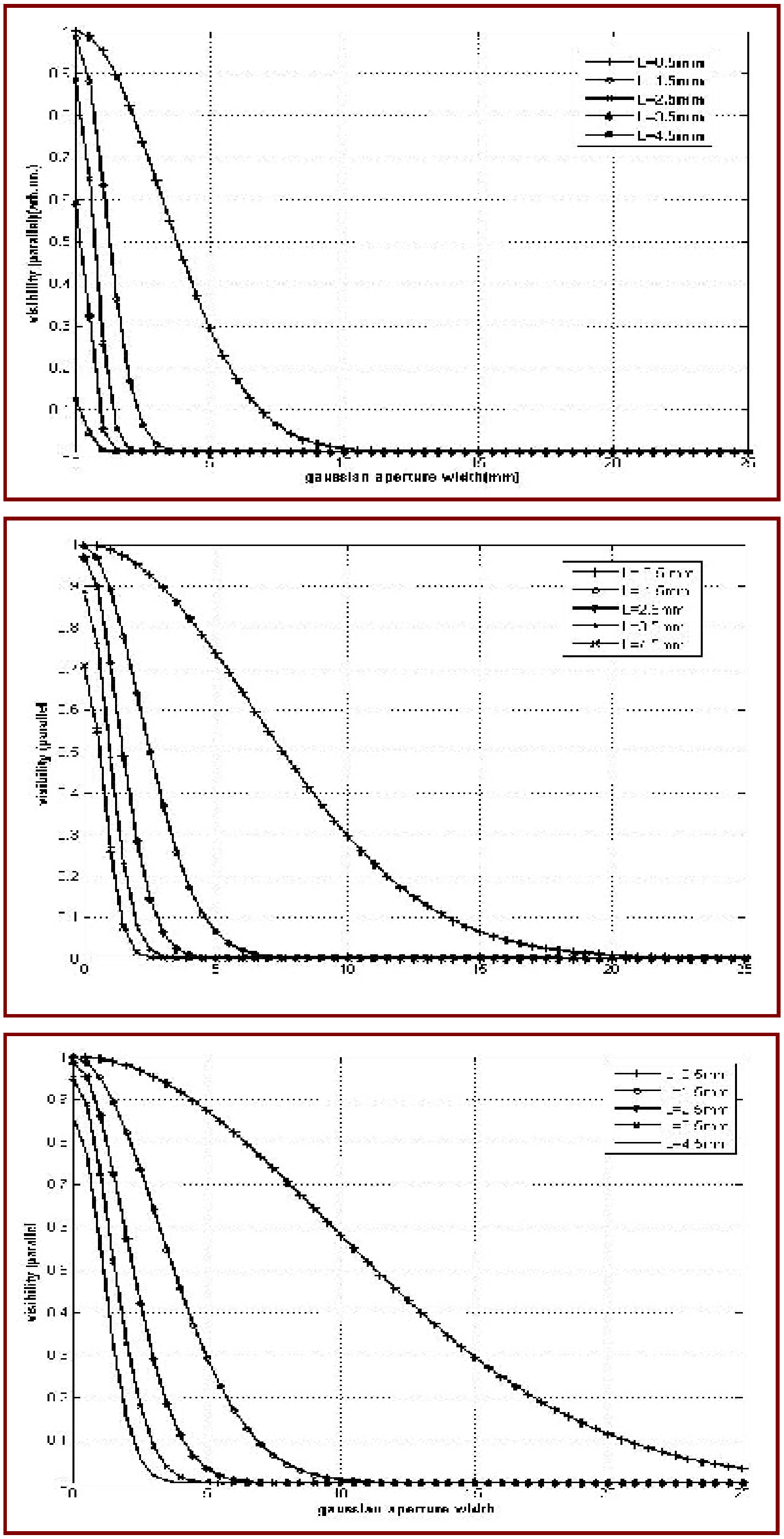

Fig.(3) visibility(parallel) as a function of Gaussian aperture width (r) for different L (a(d_1=3m), b-(d_1=2m) c-(d_1=1m))).

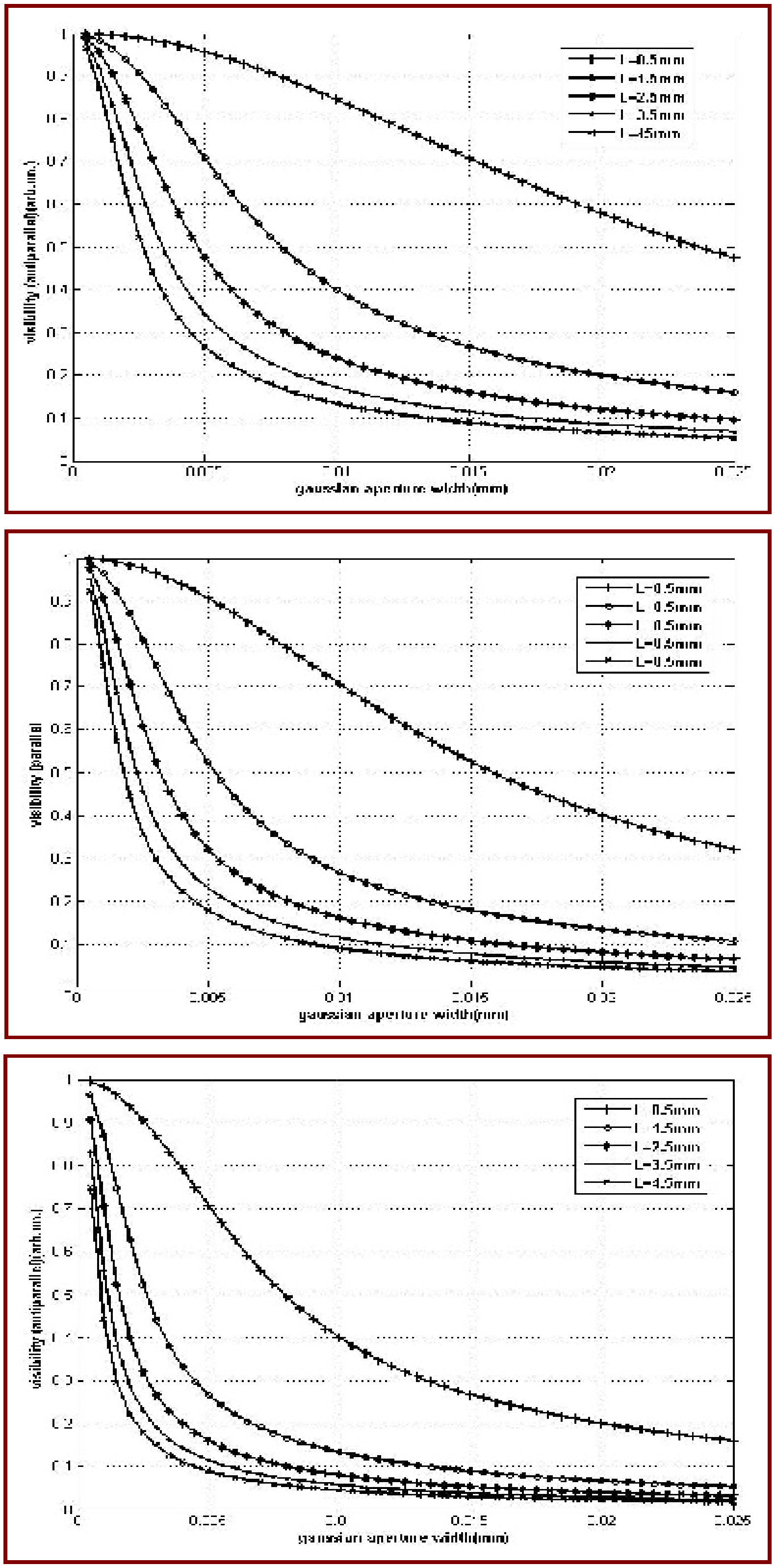

Fig.(4) visibility(antiparallel) as a function of Gaussian aperture width (r) for different L (a-(d_1=1m), b-(d_1=2m)).

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الخلاصة

تعد السرية العالية هي الهدف الرئيسي في مجال التشفير الكمي التي يمكن الحصول عليها باعتماد مصادر الفوتونات المفردة، لذا فقد تمت دراسة مصادر الفوتونات المتشابكة وبيان كيفية تأثير العناصر البصرية لمنظومات التشفير الكمي على معدل حساب التزامن وقابلية الرؤيا.

كرست التحليلات النظرية في هذا العمل لدراسة نمط التداخل لسعة الفوتونات الثنائية والمولدة بطريقة، SPDC (Spontaneous Parametric Down Conversion) في البلورات اللاخطية (BBO) والتي يتم ضخها باعتماد نبضات ضوئية قصيرة جدا. درست قابلية الرؤيا كدالة لعرض الفتحة الكاوسية ولعدة معلمات مثل طول البلورة، وشكل وحجم الفتحة، وقد أظهرت النتائج ان أفضل رؤيا يمكن الحصول عليها عندما يكون طول البلورة (1.5mm) ان النتائج التي تم التوصل اليها توضح انه في حالة استخدام بلوريتين لعملية (SPDC) فأن قابلية الرؤيا تتناقص بشكل سريع مع زيادة عرض الفتحة الكاوسية، وعندما تكون المسافة بين البؤرتين(3mm) فان قابلية الرؤيا تضمحل ببطء عند طول بلورة (L= o.5mm) .