# EFFECTS OF CROSSTALK ON WAVELENGTH DIVISION MULTIPLEXING SYSTEMS 

Dr. Hassan A. Yasser Kadim M. Hashim Shakir K. Ali


#### Abstract

The effects of ON-OFF keying of the channels on the four wave mixing (FWM) in wavelength division multiplexing (WDM) networks are studied in terms of the distribution of the actual FWM power under different probabilities of channels states. A novel technique for estimating the average FWM power in WDM network with ON-OFF keying is presented. With this technique it is possible to determine the FWM power that associated with each channel in the fiber.


الخلاصة
تأثيرات حالة القتوات من حيث الاشتغال-الإطفـاء على عمليـة مزج الموجــت الأربع في شبكات مز ج-قسمة الأطوال الموجيـة تمت دراستها في مجـال التوزيع الفعلـي لقدرة عملية مزج ج الموجــت الأربع تحت تـأثنير مختلف الاحتمـالات التّي فيهـا قنوات الاتصـال. تقنتية جديدة لاحتسـاب معدل قـدرة عملية مزج الموجات الأربع في شبكات مز ج-قنمة الأطوال الموجية مع توقع حالتي الاشتغغال والإطفـاء لجميع القتوات قـ تم اقتراحها في هـا البحث. باستخدام هذه التقنتية يكون بالامكان تحديد قلارة عملية مزج الموجـت الأربع لكل قتاة اتصال في الليف البصري.

## 1. Introduction

In wavelength division multiplexing (WDM) systems, four wave mixing (FWM) is the most influential effect [1]. When FWM occurs, three input signals generate a fourth signal, called FWM signal, which may affect the input signal operating at the same wavelength [2]. Obviously, the performance of multi-wavelength WDM system may be seriously affected by FWM crosstalk due to the interaction of various combinations of the active signal wavelengths [3].

WDM system greatly increase the total bandwidth of each optical fiber using a number of closely spaced channels at wavelengths within the typical 1540 nm to 1560
nm to take advantage of the "low loss" transmission window in optical fibers and to enable the use of erbium doped fiber amplifiers EDFA's [1,3]. Any interaction between these channels will lead to degradation of the bit error rate (BER) of the system for two reasons. Firstly, the pump channels will experience signal depletion as optical power is transferred to a different wavelengths. Secondly, if the frequency of a FWM product coincides with one of the allocated system channels, then this channel will suffer from noise. This is a particular problem for channels that are equally spaced in frequency [3,4,5]. It should be noted that each optical channel is completely independent of the other optical channels. It may run at its own rate (speed) and use its own encodings and protocols without any dependence on the other channels at all [5,6]. Channels speed for wide area network (WAN) applications are typically 2.4 Gbps in current operational WDM systems [3].

One way of minimizing the impact of FWM is to place WDM channels such that the generated signals do not fall within other WDM channels. Thus they don't interfere with other channels too much. This does help but it can not overcome the problem of noise generated in the source WDM channels by power being transferred out of them. In addition, some WDM devices are difficult to construct if wavelength spacing is uneven (reflective gratings for example) [5,6].

For a WDM system with N channels, the number of products, M , will be [7].

$$
\begin{equation*}
M=\frac{1}{2}\left(N^{3}-N^{2}\right) \tag{1}
\end{equation*}
$$

Clearly, for a typical 8-channels WDM system, unless great care is taken to allocate the channel frequencies carefully, some crosstalk will occur due to FWM. With 16 and 32channels WDM systems on the horizon, FWM is a serious consideration in a system design.

Crosstalk can be defined as a small proportion of the optical power that should have ended up in a particular channel actually ends up in an adjacent (or another) channel arrive in another they become noise in the other channel. This can have serious effects on the signal to noise ratio (SNR) and hence on the error rate of the system [3,6].

## 2. Theory

The FWM is a third order nonlinear effect in which three optical waves at frequencies $f_{i}, f_{j}$, and $f_{k}$ mix to originate a new wave at frequency

$$
\begin{equation*}
f_{i j k}=f_{i}+f_{j}-f_{k} \tag{2}
\end{equation*}
$$

Considering that the input continuous waves are not depleted by the generation of mixing products, and that the states of polarization of these waves are coincided and not changing along the propagation, the optical power of the new generated wave is given by $[8,9]$

$$
\begin{equation*}
P_{i j k}=d^{2} P_{i} P_{j} P_{k} b \eta_{i j k} \tag{3}
\end{equation*}
$$

where $\eta_{i j k}$ is the FWM efficiency and $d$ is the degeneracy factor, which takes value 1 and 2 for degenerate and non-degenerate terms, respectively, and

$$
\begin{align*}
& b=\gamma^{2} e^{-\alpha L}\left(1-e^{-\alpha L}\right)^{2} / \alpha^{2}  \tag{4}\\
& \eta_{\mathrm{ijk}}=\frac{\alpha^{2}}{\alpha^{2}+\Delta \beta_{j j k}^{2}} \tag{5}
\end{align*}
$$

where $\gamma$ is the nonlinearity coefficient, $\alpha$ is the fiber attenuation, L is the fiber length, and $P_{i}, P_{j}$, and $P_{k}$ are the input power of channels $i, j$, and $k$.

The expression of efficiency $\eta_{i j k}$ is valid for $L$ much longer than $1 / \alpha$, and the phase matching coefficient $\Delta \beta_{i j k}$, away from zero-dispersion wavelength (ZDW) is given by [5]

$$
\begin{equation*}
\Delta \beta_{i j k}=\frac{2 \pi c}{\lambda_{c}^{2}}\left(\lambda_{i}-\lambda_{k}\right)\left(\lambda_{j}-\lambda_{k}\right) D_{c} \tag{6}
\end{equation*}
$$

where $c$ is the light speed, $\lambda_{c}$ is the central wavelength, $D_{c}$ is the fiber chromatic dispersion parameter, and $\Delta \lambda_{i k}=\lambda_{i}-\lambda_{k}$ and $\Delta \lambda_{j k}=\lambda_{j}-\lambda_{k}$ are the wavelength spacing
between $i, k$ and $j, k$. Note that, away from the ZDW, Eqs.(2)-(6) accurately predict FWM power, even when the ZDW is not constant along the fiber.

This paper considers a WDM system with N equally spaced wavelengths. At a particular wavelength frequency $f_{m}$, the FWM waves generate from various combinations of interacting signals. The total FWM power at $f_{m}$ is given by

$$
\begin{equation*}
P_{\text {tot }}\left(f_{m}\right)=\sum_{f_{k}=f_{i}+f_{j}-f_{k}} \sum_{f_{i}} \sum_{f_{j}} P_{F W M}\left(f_{i}, f_{j}, f_{k}\right) \tag{7}
\end{equation*}
$$

where $i, j, k=1,2,3, \ldots \ldots \ldots . ., N, \quad i \neq m, j \neq m$ and the number $m$ takes the values: $-(N-1),-(N-2)$, $\qquad$ $-1,0,1$, $\qquad$ . $N, N+1$, $\qquad$ $.2 N-1$ which represent the indices of the new frequencies generated.

The power generated by a signal with $f_{m}$ frequency, Eq.(7), is resulted from the fact that all the channels with the frequencies $f_{i}, f_{j}$, and $f_{k}$ are in ON state. But there is possible for any channel to be in OFF state, such that Eq.(7) can be written as

$$
\begin{equation*}
\bar{P}_{\text {tot }}\left(f_{m}\right)=\sum_{f_{k}=f_{i}+f_{j}-f_{k}} \sum_{f_{i}} \sum_{f_{j}} P_{F W M}\left(f_{i}, f_{j}, f_{k}\right) \delta_{i_{\mathrm{ijk}}} \tag{8}
\end{equation*}
$$

where $\delta_{i j k}=\left\{\begin{array}{lr}1 & \text { if } \begin{array}{l}\mathrm{i}=\mathrm{j}=\mathrm{k} \\ 0\end{array} \\ \text { elsewhere }\end{array}\right.$.
In the case of channels arranged on an equally spaced grid of resolution $\Delta \lambda$, Eq.(6) takes the discrete values

$$
\begin{equation*}
\Delta \beta_{n}=n\left(\frac{2 \pi c}{\lambda_{c}^{2}}\right) D_{c} \Delta \lambda^{2} \tag{9}
\end{equation*}
$$

For the typical values $D_{c}=0.3 \mathrm{ps} / \mathrm{nm} \mathrm{km}, \lambda_{c}=1550 \mathrm{~nm}, \alpha=0.2 \mathrm{~dB} / \mathrm{km} \quad \gamma=21 / \mathrm{w} \mathrm{km}$, $L=30 \mathrm{~km}$, and each channel with power 10 mw , Eq.(9) takes the following form

$$
\begin{equation*}
\Delta \beta_{n}=0.235 n \Delta \lambda^{2} \tag{10}
\end{equation*}
$$

where $\Delta \beta_{n}$ is measured by $1 / k m$ if $\Delta \lambda$ measured by $n m$. Substituting Eq.(10) into (5) yields a quantized values of the efficiency depending on phase matching coefficient as follows

$$
\begin{equation*}
\eta_{n}=\frac{\alpha^{2}}{\alpha^{2}+\Delta \beta_{n}^{2}}=\frac{1}{1+26 n^{2} \Delta \lambda^{4}} \tag{11}
\end{equation*}
$$

Using the above parameters values and Eqs.(3) and (11) yields

$$
P_{i j k}=0.21 \begin{cases}4 \eta_{n} & \text { if } \mathrm{i}=\mathrm{j}  \tag{12}\\ \eta_{n} & \text { elsewhere }\end{cases}
$$

For example, table (1) summarizes all FWM terms falling on each frequency of an equally spaced eight-channel WDM system. Each term is represented by the indices of three channels involved in the combination. For instance 132 labels the FWM term falls on channel 2 that is generated by channels 1,2 , and 3 . Note that, since this term falls on channel 2 , its power cannot be measured by simply suppressing channel 2 since there many different terms fall on the same channel.

For each FWM term, the corresponding phase matching coefficient $\Delta \beta_{n}$ is shown in the first left column in table (1). Some FWM terms, at different frequencies, may have the same $\Delta \beta_{n}$ values, and thus the efficiency values. Not that, the vertical line between channels 4 and 5 divides table (1) into two symmetric parts. This means that the number of FWM terms and their efficiencies are the same in the two sides.

## 3. Simulation Results and Discussion

Fig.(1) represents the relation between the FWM order and the efficiency of various values for $\Delta \lambda$ of eight-channels WDM system. We find the efficiency lessen with the increase of the FWM order; that is, the effects of the remote channels upon a certain channel are much lesser that those of the neighboring channels. On the other hand, the increase of $\Delta \lambda$ leads to the increase of the efficiency sharply; therefore, it produces the increase of power levels of the generated pulses. In other words, $\Delta \lambda$ must be large enough that the transmission system can single out various channels, and $\Delta \lambda$
becomes so small that the number of channels used in WDM systems are as large as possible within the wavelengths band which is used for transmission (1540-1560 nm), determining maximum bandwidth.

Fig.(2) represents the relation between the channel index and the number of the generated components of various selected WDM systems ( $\mathrm{N}=4,6,8, \ldots \ldots, 20$ ). Increasing of the number of channels leads to increase of the number of new generated frequencies, in addition to the increase of the number of FWM components; those corresponding to frequencies. It is obvious from Fig.(2) that the central channels are those which correspond with a maximum number of the FWM components; therefore, it is the most susceptible to noise.

Increasing of the number of FWM components in a certain frequency embraces the increase in the generated power in that frequency, but not necessarily can all channels in ON state. Therefore, to calculate the generated power at each frequency depending on Fig.(2), it does not represent a perfect description of what happen in communication system. Each channel can be in ON or OFF state; that is to say for WDM with N -channels, there are $2^{N}$ different possibilities. The number of possibilities increases with increasing of N . It is the possibility that if all pulses are in ON state, it is equal to $2^{-N}$; it is then the minimum amount with the increasing the number of channels; therefore, Fig.(2) represents, in fact, a one possible from $2^{N}$ of the possibilities. For example, a communication system with 4-channels shows that the number of possibilities is 16 . So, the possibility of being all the pulses are in ON state is $\approx 6 \%$.

Accordingly, counting the power of pulses which generates in certain frequency yields to $2^{N}$ from these possibilities. The average of that power of $2^{N}$ from the possibilities represents the most acceptable value. Fig.(3) represents the relation between the channel index and the average of the power of various WDM systems. From Fig.(3), the average power at the original channels of that system oscillating round a certain value increases with the increasing of N .

## 4. Conclusions and Suggestions

The most effects of the noise on a certain channel comes as a result from the effect of the neighboring channels, and the most channels exposing to that noise are located in the center. The total power generated at some frequency does not represent an algebraic sum of the FWM components of that frequency, but in fact it represents a statistical average of $2^{N}$ from those possibilities.

To lessen the generated noise of each channel, the WDM system that consisting of N-channels may be divided into a partial sets (subsets) each of them of 4-channels which are separated by $\Delta \lambda$. Each two neighboring subsets are separated by $4 \Delta \lambda$ so as to restrict the effects of crosstalk between the channels of that partial set that consist of 4channels instead of the effects being resulting from all N -channels.

## References

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Table(1):Illustration of the generated FWM components with each transmission channel. The first left column represents the associated phase matching order of the FWM components. The numbers $i$, $j$, $k$ represents the channels number of a certain FWM process. The line between 4 and 5 channels divides the table into symmetric two parts.

| WM <br> order <br> with $\Delta \beta_{n}$ | Channel Number |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | New channels |  |  |  |  |  |  | Original channels |  |  |  |  |  |  |  | New channels |  |  |  |  |  |  |
|  | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $\Delta \beta_{1}$ |  |  |  |  |  |  | 112 | 223 | $\begin{aligned} & 132 \\ & 334 \end{aligned}$ | $\begin{aligned} & 221 \\ & 241 \\ & 441 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 332 \\ & 354 \\ & 556 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 667 \\ & 443 \\ & 465 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 576 \\ & 554 \\ & \mathbf{7 7 8} \\ & \hline \end{aligned}$ | $\begin{aligned} & 687 \\ & 665 \end{aligned}$ | 776 | 887 |  |  |  |  |  |  |
| $\Delta \beta_{2}$ |  |  |  |  |  |  | 123 | 234 | $\begin{aligned} & 143 \\ & 345 \end{aligned}$ | $\begin{aligned} & 142 \\ & \mathbf{2 5 4} \\ & \mathbf{4 5 6} \end{aligned}$ | $\begin{aligned} & 365 \\ & 567 \\ & 231 \\ & 253 \end{aligned}$ | $\begin{aligned} & 476 \\ & 364 \\ & 342 \\ & 678 \end{aligned}$ | $\begin{aligned} & 587 \\ & 453 \\ & 475 \end{aligned}$ | $\begin{aligned} & 286 \\ & 264 \end{aligned}$ | 675 | 876 |  |  |  |  |  |  |
| $\Delta \beta_{3}$ |  |  |  |  |  |  | 134 | 245 | $\begin{aligned} & 356 \\ & 154 \end{aligned}$ | $\begin{aligned} & 265 \\ & 467 \end{aligned}$ | $\begin{aligned} & 152 \\ & 376 \\ & 578 \end{aligned}$ | $\begin{aligned} & 263 \\ & 241 \\ & 487 \\ & \hline \end{aligned}$ | $\begin{aligned} & 352 \\ & 374 \end{aligned}$ | $\begin{aligned} & 485 \\ & 463 \end{aligned}$ | 574 | 865 |  |  |  |  |  |  |
| $\Delta \beta_{4}$ |  |  |  |  |  | 113 | $\begin{aligned} & 145 \\ & 224 \end{aligned}$ | $\begin{aligned} & \mathbf{2 5 6} \\ & 335 \end{aligned}$ | $\begin{aligned} & 165 \\ & 446 \\ & \mathbf{3 6 7} \end{aligned}$ | $\begin{aligned} & 153 \\ & \mathbf{2 7 6} \\ & \mathbf{4 7 8} \\ & 557 \end{aligned}$ | $\begin{aligned} & \hline 668 \\ & 387 \\ & 264 \end{aligned}$ | $\begin{aligned} & 162 \\ & 375 \\ & 331 \end{aligned}$ | $\begin{aligned} & 273 \\ & 251 \\ & \mathbf{4 8 6} \\ & \mathbf{4 4 2} \end{aligned}$ | $\begin{aligned} & 362 \\ & 384 \\ & 553 \end{aligned}$ | $\begin{aligned} & \hline 664 \\ & 473 \end{aligned}$ | $\begin{aligned} & 854 \\ & 775 \end{aligned}$ | 886 |  |  |  |  |  |
| $\Delta \beta_{5}$ |  |  |  |  |  |  | 156 | 267 | $\begin{aligned} & \hline 378 \\ & 176 \end{aligned}$ | 287 |  |  | 172 | $\begin{aligned} & 251 \\ & 283 \end{aligned}$ | 372 | 843 |  |  |  |  |  |  |
| $\Delta \beta_{6}$ |  |  |  |  |  | 124 | $\begin{aligned} & 167 \\ & 235 \end{aligned}$ | $\begin{aligned} & 278 \\ & 346 \end{aligned}$ | $\begin{aligned} & 178 \\ & 457 \end{aligned}$ | $\begin{aligned} & 164 \\ & 568 \end{aligned}$ | $\begin{aligned} & 163 \\ & 275 \end{aligned}$ | $\begin{aligned} & 476 \\ & 386 \end{aligned}$ | $\begin{aligned} & 385 \\ & 341 \end{aligned}$ | $\begin{aligned} & 182 \\ & 452 \end{aligned}$ | $\begin{aligned} & 271 \\ & 563 \end{aligned}$ | $\begin{aligned} & 832 \\ & 764 \end{aligned}$ | 875 |  |  |  |  |  |
| $\Delta \beta_{7}$ |  |  |  |  |  |  | 178 |  |  |  |  |  |  |  |  | 821 |  |  |  |  |  |  |
| $\Delta \beta_{8}$ |  |  |  |  |  | 135 | 246 | 357 | 468 | 175 | 286 | 173 | 284 | 351 | 462 | 753 | 864 |  |  |  |  |  |
| $\Delta \beta_{9}$ |  |  |  |  | 114 | 225 | 336 | 447 | 558 |  | 174 | 285 |  | 441 | 552 | 663 | 774 | 885 |  |  |  |  |
| $\Delta \beta_{10}$ |  |  |  |  |  | 146 | 257 | 368 |  | 186 |  |  | 183 |  | 361 | 742 | 853 |  |  |  |  |  |
| $\Delta \beta_{12}$ |  |  |  |  | 125 | $\begin{aligned} & 157 \\ & 236 \end{aligned}$ | $\begin{aligned} & 347 \\ & 268 \end{aligned}$ | 458 |  |  | 185 | 184 |  |  | 451 | $\begin{aligned} & 731 \\ & 652 \end{aligned}$ | 842 | 874 |  |  |  |  |
| $\Delta \beta_{14}$ |  |  |  |  |  | 168 |  |  |  |  |  |  |  |  |  |  | 831 |  |  |  |  |  |
| $\Delta \beta_{15}$ |  |  |  |  | 136 | 247 | 357 |  |  |  |  |  |  |  |  | 641 | 752 | 863 |  |  |  |  |
| $\Delta \beta_{16}$ |  |  |  | 115 | 226 | 337 | 448 |  |  |  |  |  |  |  |  | 551 | 662 | 773 | 884 |  |  |  |
| $\Delta \beta_{18}$ |  |  |  |  | 147 | 258 |  |  |  |  |  |  |  |  |  |  | 741 | 852 |  |  |  |  |
| $\Delta \beta_{20}$ |  |  |  | 126 | 237 | 348 |  |  |  |  |  |  |  |  |  |  | 651 | 762 | 873 |  |  |  |
| $\Delta \beta_{21}$ |  |  |  |  | 158 |  |  |  |  |  |  |  |  |  |  |  |  | 841 |  |  |  |  |
| $\Delta \beta_{24}$ |  |  |  | 137 | 248 |  |  |  |  |  |  |  |  |  |  |  |  | 751 | 862 |  |  |  |
| $\Delta \beta_{25}$ |  |  | 116 | 227 | 338 |  |  |  |  |  |  |  |  |  |  |  |  | 661 | 772 | 883 |  |  |
| $\Delta \beta_{28}$ |  |  |  | 148 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 851 |  |  |  |
| $\Delta \beta_{30}$ |  |  | 127 | 238 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 761 | 872 |  |  |
| $\Delta \beta_{35}$ |  |  | 138 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 861 |  |  |
| $\Delta \beta_{36}$ |  | 117 | 228 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 771 | 882 |  |
| $\Delta \beta_{42}$ |  | 128 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 871 |  |
| $\Delta \beta_{49}$ | 118 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 881 |



Fig.(1): Efficiency as a function of FWM order for various values of $\Delta \lambda$.










Fig.(2): Illustration of the number of components with each channel index for various WDM systems.










Fig.(3): The average power of each channel with channel index for various WDM systems.

