New Kinds of Open Sets

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Abstract: In this paper, we introduced new types of open sets which we called p*open set and semi p*open set. Besides, we get the following results:

(i) Every open set is p*open and semi p*open. Examples are given to show that the converse may not be true .

(ii) Preopen set and p*open set are equivalent.

(iii) Every semi p*open set is semi p open set.

§1 Introduction:

The term "preopen" was used for the first time by Mashhour A.S., Abd El-Monsef M.E., and El-Deeb S.N., in 1984 [1], then G.B.Navalagi used "preopen" term in 2000 [2]. Semi p open set was introduced in [3]. In this paper, we introduced new types of open sets which we called p*open set and semi p*open set. Besides, we get the following results:

(i) Every open set is p*open and semi p*open. Examples are given to show that the converse may not be true.

(ii) Preopen set and p*open set are equivalent.

(iii) Every semi p*open set is semi p open set.

§2 Preliminaries:

Definition 2.1[2]:(i) A subset A of a topological space X is called a preopen set if $A \subseteq int(clA)$.

(ii) The complement of a preopen set is called a preclosed set.

(iii) The family of all preopen sets of X is denoted by po(X).

(iv) The family of all preclosed sets of X is denoted by pc(X).

Remarks 2.2 [4]:

(i) The union of any family of preopen sets is a preopen set.

(ii) The intersection of two preopen sets may not be preopen set.

(iii) The intersection of any family of preclosed sets is preclosed.

(iv) The union of two preclosed sets may not be preclosed .

(v) Every open set is preopen but the converse may not be true.

(vi) Every closed set is preclosed but the converse may not be true.

Definition 2.3 [2]: The intersection of all preclosed sets containing a set A is called the preclosure of A, and is denoted by pre(cl A) .

Theorem 2.4 [4]: A subset A of a topological space X is a preclosed set if and only if A = pre(cl A).

Definition 2.5 [4]:(i) A subset A of a topological space X is called a semi p (denoted by sp) open set if there exists a preopen set U in X such that $U \subseteq A \subseteq$ pre(cl U).

(ii) The complement of a semi p open set is called a semi p closed set.

(iii) The family of all semi p open sets of X is denoted by spo(X).

(iv) The family of all semi p closed sets of X is denoted by spc(X).

Remarks 2.6 [4]:

(i) The union of any family of sp open sets is an sp open set.

(ii) The intersection of two sp open sets may not be sp open set.

(iii) The intersection of any family of sp closed sets is sp closed.

(iv) The union of two sp closed sets may not be sp closed .

(v) Every open set is sp open but the converse may not be true.

(vi) Every closed set is sp closed but the converse may not be true.

(vii) Every preopen set is sp open but the converse may not be true.

(viii) Every preclosed set is sp closed but the converse may not be true.

Definition 2.7 [4]: The intersection of all semi p closed sets containing a set A is called the semi p closure of A, and is denoted by sp(cl A) .

§3 P* & SP* open sets

Definition 3.1: :(i) A subset A of a topological space X is called a p*closed (denoted by p*closed) set if the preclosure of A is a subset of all semi p open set U which contains A. that is, whenever $A \subseteq U$ and U is semi p, then pre(clA)$\subseteq$U.

(ii) The complement of a p*closed set is called a p*open set(denoted by p*open).

(iii) The family of all p*open sets of X is denoted by p*o(X).

(iv) The family of all p*closed sets of X is denoted by p*c(X).

Theorem 3.2: Every open set is p*open.

Proof: Clear.

Remark 3.3: The converse of the above theorem may not be true as in the following example:

Let X ={1,2,3,4} , $\tau = \{\phi, X, \{1\},\{4\},\{1,4\}\}$

po(X)= $\{\phi, X, \{1\},\{4\},\{1,4\},\{1,2,4\},\{1,3,4\}\}$= p*o(X)

Theorem 3.4: Preopen set and p*open set are equivalent.

Proof:

($\Rightarrow$):First let U be a preopen set. To prove that U is p*open (i.e to prove U^c p*closed)

Let V be any sp open set such that $U^c \subseteq V$. To prove $pre(clU^c) \subseteq V$.
Since U^c is preclosed, then by 2.4 we get that $U^c = pre(clU^c)$.
So $pre(clU^c) \subseteq V$.
($\Leftarrow$): Conversely,let U be a p*open set. To prove that U is preopen (i.e $U \subseteq int(clU)$) .
Let $x \in U$, then by [5] we have {x} either open or preclosed.
If {x} is open, then $x \in \{x\} \subseteq cl(U)$.
So $x \in int(clU)$.
Hence $U \subseteq int(clU)$.
Now, if {x} is preclosed, then X-{x} is preopen.
Since $x \in U$, then $x \notin U^c$ which leads to $U^c \subseteq X\text{-}\{x\}$.
By 2.6(vii), we get X-{x} is sp open.
Because U^c is p*closed, then $pre(clU^c) \subseteq X\text{-}\{x\}$.
By [5] we have $pre(clU^c) = U^c \cup cl(int\ U^c)$
$U^c \cup cl(int\ U^c) \subseteq X\text{-}\{x\}$.
Then $cl(int\ U^c) \subseteq X\text{-}\{x\}$.
But $cl(int\ U^c) = (int(cl\ U))^c$.
Hence $(int(cl\ U))^c \subseteq X\text{-}\{x\}$
$\{x\} \subseteq int(cl\ U)$ which leads to $x \in int(cl\ U)$
Hence $U \subseteq int(cl\ U))$
Thus U is preopen.

Remark 3.5 : Because of the equivalence relation between preopen and p*open, then they have the same properties . For example, the union of any two p*closed sets may not be p*closed. For instance, the sets {1}&{2} in X ={1,2,3} , τ = {ϕ,X, {1,2}}, po(X)= {ϕ,X, {1},{2},{1,2},{1,3},{2,3}}= p*o(X), are p*closed but their union is not. And the intersection of any two p*open sets may not be p*open. Where the sets {1,3}&{2,3} in above example are p*open but their intersection is not.

So we have proved the following theorem:

Theorem 3.6: (i) The intersection of any family of p*closed sets is p*closed.
(ii) The union of any family of p*open sets is p*open.

Proof: (i)
Let{ A_α ,$\alpha \in \Lambda$} be any family of p*closed sets. To prove that $\cap_{\alpha\in\Lambda} A_\alpha$ is p*closed.
Let U be any sp open set such that $\cap_{\alpha\in\Lambda} A_\alpha \subseteq U$. To prove that $pre(cl \cap_{\alpha\in\Lambda} A_\alpha) \subseteq U$.
Since A_α is p*closed,$\forall\alpha\in\Lambda$, then by 3.4 A_α is preclosed $\forall\alpha\in\Lambda$.

By 2.2(iii) we get $\cap_{\alpha\in\Lambda} A_\alpha$ is preclosed.

By2.4 we have $\cap_{\alpha\in\Lambda} A_\alpha = \text{pre}(\text{cl} \cap_{\alpha\in\Lambda} A_\alpha)$

Hence $\text{pre}(\text{cl} \cap_{\alpha\in\Lambda} A_\alpha) \subseteq U$

Thus $\cap_{\alpha\in\Lambda} A_\alpha$ is p*closed

(ii) The proof follows immediately from (i) By the same way by taking the complement.

Definition 3.7 : A subset A of a topological space X is called a p*neighborhood of a point x in X if there exists a p*open set U in X such that $x\in U\subseteq A$.

Theorem 3.8: A subset A of a topological space X is p*open in X if and only if it is a p*neighborhood of each of its points.

Proof: (⇒):

Let A be a p* open set in X.

Then $x\in A\subseteq A \quad \forall x\in A$.

Hence A is a p*neighborhood of each of its points.

(⇐): Conversely, let A be a p* neighborhood of each of its points.

So $\forall x\in A$, there exists a p*open set U_x such that $x\in U_x \subseteq A$.

Clear $A= \cup_{x\in A} U_x$.

By 3.6(ii), we get A to be p*open.

Definition 3.9: (i) A subset A of a topological space X is called a semi p*open (denoted by sp*open) set if there exists an sp open set U in X such that $U\subseteq A\subseteq \text{sp}(\text{cl } U)$.

(ii) The complement of a semi p*open set is called a semi p*closed (denoted by sp*closed) set.

(iii) The family of all semi p*open sets of X is denoted by sp*o(X).

(iv) The family of all semi p*closed sets of X is denoted by sp*c(X).

Theorem 3.10: Every open set is sp*open.

Proof: Clear

Remark 3.11: The converse of the above theorem may not be true as in the following example:

Let $X =\{1,2,3,4\}$, $\tau = \{\phi,X, \{1\},\{4\},\{1,4\}\}$

$po(X)= \{\phi,X, \{1\},\{4\},\{1,4\},\{1,2,4\},\{1,3,4\}\}= p^*o(X)$

$spo(X)=\{\phi,X,\{1\},\{4\},\{1,4\},\{1,2,4\},\{1,3,4\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{2,3,4\}\}= sp^*o(X)$

Theorem 3.12: Every semi p*open set is semi p open set.

Proof:

let U be sp*open.

Then there exists an sp open set V such that $V\subseteq U\subseteq \text{sp}(\text{cl}V)$.

Since V is sp open, then there exists a preopen set O such that $O \subseteq V \subseteq pre(clO)$.

Then $O \subseteq U$. To prove $U \subseteq pre(clO)$

It is Clear that $sp(clV) \subseteq pre(clV)$

Since $V \subseteq pre(clO)$, then $pre(clV) \subseteq pre(clO)$.

Therefore $U \subseteq sp(clV) \subseteq pre(clV) \subseteq pre(clO)$.

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انواع جديدة من المجاميع المفتوحة

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المستخلص:

قدمنا في هذا البحث انواع جديدة من المجاميع المفتوحة والتي اطلقنا عليها اسم المجموعة المفتوحة p* والمجموعة شبه المفتوحة p* وقد حصلنا على النتائج التالية:

(i) كل مجموعة مفتوحة هي مجموعة مفتوحة p* ومجموعة شبه مفتوحة p* .واعطيت امثلة توضح بان الاتجاه المعاكس قد يكون غير صحيح.

(ii) المجموعة المفتوحة pre و المجموعة المفتوحة p* هي مجاميع متكافئة.

(iii) كل مجموعة شبه المفتوحة p* هي مجموعة شبه المفتوحة p .