

INCIDENT SIGNAL PARAMETERS ESTIMATION ON ANTENNA UNIFORM LINEAR ARRAY ⁺

تخمين ثوابت الإشارة الساقطة على مصفوفة الهوائيات الخطية المنتظمة

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Abstract:

This paper presents a tool for the modeling, analysis, and simulation for extraction of the Directions of Arrival (DOA) of several signals impinging on uniform linear arrays. The first part of this paper describes the Multiple Signal Classification (MUSIC) technique of direction of arrival estimation and simulation method. The second part presents some illustrative simulation cases of MUSIC-DOA estimation by using Graphical User Interface (GUI). Simulation results are analyzed by using Matlab program in order to conclude on the algorithm's accuracy and reliability. To demonstrate the versatility and accuracy of the developed tool, it is used to study the effect of changing a number of parameters related to the signal environment as well as the antenna array.

Keywords: Direction of Arrival (DOA); Multiple Signal Classification (MUSIC); Graphical User Interface (GUI).

المستخلص:

يُقدّم هذا البحث أداة للعرض، التحليل، والمحاكاة لاستخراج اتجاهات الوصول لإشارات متعددة ساقطة على مصفوفة الهوائيات المنتظمة. يصف الجزء الأول من هذا البحث تقنية التصنيف المتعدد للإشارات من تحديد اتجاه الوصول وطريقة المحاكاة. يعرض الجزء الثاني بعض حالات المحاكاة الموضحة من تقنية التصنيف المتعدد للإشارات من تحديد اتجاه الوصول باستخدام واجهة المستخدم (GUI). تم تحليل نتائج المحاكاة باستخدام برنامج Matlab لكي تستنتج على الدقة وأعطائية الخوارزمية. لعرض تعددية الاستعمال ودقة الوسيلة المتطورة، يُستعمل لدراسة تأثير تغيير عدد من البارامترات المتعلقة ببيئة الإشارة بالإضافة إلى مصفوفة الهوائيات.

Introduction:

The purpose of direction-of-arrival (DOA) estimation is to use the data received on the downlink at the base-station sensor array to estimate the directions of the signals from the desired mobile users as well as the directions of interference signals. The results of DOA estimation are then used by adjusting the weights of the adaptive beam-former so that the radiated power is maximized towards the desired users, and radiation nulls are placed in the directions of interference signals. Hence, a successful design of an adaptive array depends highly on the choice of the DOA estimation algorithm which should be highly accurate and robust [1].

Parameters estimation of several signals received by a communications system is a critical aspect of both the mobile channel characterization process and the source localization

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problem. In order to locate the signal sources, there are two fundamental parameters that should be extracted: the amplitude and the Direction-of-Arrival (DOA) of the impinging wave front [2].

The Multiple Signal Classification (MUSIC) technique was first proposed by Schmidt in (1981). The term MUSIC is used to describe experimental and theoretical techniques involved in determining the parameters of multiple wave fronts arriving at an antenna array from measurements made on the signals received at the array elements [3].

Within this class of algorithms, the MUSIC technique has received the most attention and has been widely studied [4-6].

MUSIC algorithm is applicable to arrays with arbitrary geometry and the price paid for this generality is that the array response must be measured and stored for all possible combinations of source parameters. In this paper, a novel subspace method with higher resolution is presented with GUI matlab program, the MUSIC technique can resolve very weak sources in vicinity of strong ones and its resolution is not sensitive to power level differences of adjacent sources.

The Data Model of DOA Estimation:

The MUSIC technique starts by constructing a real-life signal model. Consider a number of plane waves from k narrow-band sources impinging from different angles θ_i , $i = 1, 2, \dots, k$, impinging into a Uniform Linear Array (ULA) of N equi-spaced sensors, as shown in Fig (1).

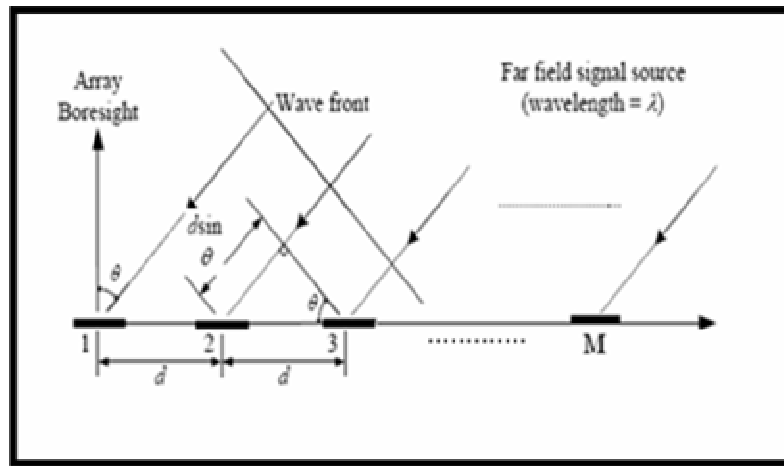


Fig. (1) Uniform linear array.

Let an array of M sensors receive k narrowband plane waves from far-field emitters with the same known center frequency f_0 . With the narrow band assumption the i^{th} signal complex envelop representation can be shown as: [4]

$$s_i(t) = u_i(t) e^{j(2\pi f_0 t + \varphi_i(t))} \quad i = 1, 2, \dots, k \quad (1)$$

where $u_i(t)$ and $\varphi_i(t)$ are slowly varying functions of time that define the amplitude and phase of i^{th} signal, respectively. Slowly varying means $u_i(t) = u_i(t - \tau)$ and $\varphi_i(t) \approx \varphi_i(t - \tau)$ for all possible propagation delays τ between array sensors. As a result of this the effect of a time delay on received waveforms is simply a phase shift, i.e.

$$s_i(t - \tau) \approx s_i(t) e^{-j2\pi f_0 \tau} \quad (2)$$

then $y_m(t)$, the complex signal output of the m^{th} sensor at time t , can be written as: [4]

$$\begin{aligned} y_m(t) &= \sum_{i=1}^k a_m(\theta_i) s_i(t - \tau_m(\theta_i)) + n_m(t) \\ &= \sum_{i=1}^k a_m(\theta_i) s_i(t) e^{-j2\pi f_o \tau_m(\theta_i)} + n_m(t) \end{aligned} \quad (3)$$

$m = 1, 2, \dots, M$

where $\tau_m(\theta_i)$ is the propagation delay between a reference point and the m^{th} sensor for the i^{th} wave-front impinging on the array from direction θ_i , $a_m(\theta_i)$ is the corresponding sensor element complex response (gain and phase) at frequency f_o , k is the number of sources present, and $n_m(t)$ is additive noise at m^{th} sensor element. Using vector notation for the outputs of M sensors, the data model becomes

$$y(t) = \sum_{i=1}^k a(\theta_i) s_i(t) + n(t) \quad (4)$$

where

$$a(\theta_i) = [1, e^{j((\frac{2\pi d}{\lambda}) \sin \theta_i)}, \dots, e^{j((\frac{2\pi d}{\lambda(M-1)}) \sin \theta_i)}]^T \quad (5)$$

$$y(t) = [y_1(t), \dots, y_M(t)]^T \quad (6)$$

$$n(t) = [n_1(t), \dots, n_M(t)]^T \quad (7)$$

The $M \times 1$ vector $a(\theta_i)$ is known as array response or array steering vector for direction θ_i , with defining matrix $A = [a(\theta_1), \dots, a(\theta_k)]$ and $k \times 1$ vector $s(t) = [s_1(t), \dots, s_k(t)]^T$, the equation (4) can be written as

$$y(t) = A s(t) + n(t) \quad (8)$$

where $n(t)$ is additive noise, the DOA estimation problem is finding θ_i , $i = 1, 2, \dots, k$ from a finite number (N) of data samples or snapshots of $y(t_j)$ taken at times t_j , $j = 1, 2, \dots, N$.

For solving the MUSIC technique the following assumptions are made:

a- The number of sources is known and is less than the number of sensors, i.e., $K < M$.

b- The sources are uncorrelated zero mean stationary processes with the $k \times k$ diagonal covariance matrix [4]

$$R_s = E[s(t)s(t)^H] = \text{diag}\{\sigma_1^2, \sigma_2^2, \dots, \sigma_k^2\} \quad (9)$$

where σ_i^2 denotes the power (variance) of i^{th} source, E is mathematical expectation operator and H denotes conjugate transpose.

c- The additive white noise at each sensor is a stationary zero mean complex white Gaussian noise process. The noise processes of different sensors are uncorrelated and

$$R_n = E[n(t)n(t)^H] = \sigma_n^2 I_M \quad (10)$$

where σ_n^2 is the noise power at each sensor and I_M is an identity matrix of order M .

d- The noise and signal waveforms are uncorrelated.

e- The array response vector $a(\theta)$ is known for all θ and the array is configured in such a way that the matrix A in equation (8) has full column rank, i.e., $\text{rank}(A) = k$.

From the above assumptions, the $M \times M$ covariance matrix of received data can be written as: [5]

$$R = E[y(t)y(t)^H] = A R_s A^H + \sigma_n^2 I_M \quad (11)$$

where $E[\bullet]$ denotes expectation, H is the hermitian operator and $R_s = E[s(t)s(t)^H]$ is the signal covariance matrix. Assuming that the each signal is a sample of a random process with zero mean, the signal covariance matrix can be written [7]

$$R_s = E[s(t)s(t)^H] \quad (12)$$

$$R_s = E \begin{bmatrix} s_1(t)s_1(t)^H & s_1(t)s_2(t)^H & \dots & s_1(t)s_p(t)^H \\ s_2(t)s_1(t)^H & s_2(t)s_2(t)^H & \dots & s_2(t)s_p(t)^H \\ \vdots & & \ddots & \vdots \\ s_p(t)s_1(t)^H & \dots & & s_p(t)s_p(t)^H \end{bmatrix} \quad (13)$$

$$R_s = E \begin{bmatrix} \sigma_1^2 & \sigma_1 \sigma_2 \rho_{12} & \dots & \sigma_1 \sigma_p \rho_{1p} \\ \sigma_2 \sigma_1 \rho_{21} & \sigma_2^2 & \dots & \sigma_2 \sigma_p \rho_{2p} \\ \vdots & & \ddots & \vdots \\ \sigma_p \sigma_1 \rho_{p1} & \dots & & \sigma_p^2 \end{bmatrix} \quad (14)$$

where $\sigma_p^2 = E[|s_p(t)|^2]$ is the variance of the p^{th} signal and $\rho_{ik} = E[s_i(t)s_k(t)^H]/(\sigma_i \sigma_k)$ is the correlation coefficient between the i^{th} and the k^{th} signal. If the signals are fully correlated, $|\rho_{ik}| = 1 \forall i, k$ and R_s has rank one. On the other hand, if $|\rho_{ik}| < 1$ for $i \neq k$, R_s has full rank.

Because R is a conjugate symmetric matrix its Eigen-decomposition can be expressed in terms of unitary matrices:

$$R = U \Lambda U^H \quad (15)$$

where U is the Eigen-vector and Λ is Eigen-values as:[8]

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & & \lambda_M \end{bmatrix} \quad (16)$$

and $\lambda_1 \geq \lambda_2 \geq \dots, \lambda_M > 0$. The columns of U contain the orthonormal Eigen-vectors corresponding to the Eigen-values in Λ .

Substituting equation (11) into (15) gives:

$$A R_s A^H + \sigma_n^2 I_M = U \Lambda U^H \quad (17)$$

For subspace methods, it is necessary that the emitter be non-coherent (R_s must be full rank). If the k emitters are non-coherent, then [8]

$$Re\{A(\theta)R_s A(\theta)^H\} = Re\{A(\theta)\} \quad (18)$$

Let the signal subspace (\mathcal{S}) defined as $Re\{A(\theta)\}$ and the noise subspace (\mathcal{S}^\perp) as the orthogonal complement of the signal subspace. The dimension of \mathcal{S} is k , and the dimension of \mathcal{S}^\perp is $M - k$. Thus if a vector lies in \mathcal{S}^\perp , then it is an Eigen-vector of R corresponding to an Eigen-value equal to σ_n^2 . Since $A(\theta)R_s A(\theta)^H$ is positive semi-definite, it can also be shown that any Eigen-vector corresponding to an Eigen-value greater than σ_n^2 must belong to \mathcal{S} ; therefore, it can partition the Eigen value/vector pairs in equation (11) into: [8]

$$R = U \Lambda U^H \quad (19)$$

$$= [U_s \quad U_n] \begin{bmatrix} \Lambda_s & 0 \\ 0 & \Lambda_n \end{bmatrix} [U_s \quad U_n]^H \quad (20)$$

$$= U_s \Lambda_s U_s^H + U_n \Lambda_n U_n^H \quad (21)$$

The k columns of U_s are the Eigenvectors corresponding to the k largest Eigen values ($\lambda_1 \geq \lambda_2 \geq \dots, \lambda_k > \sigma_n^2$) and the $M-k$ smallest Eigen values ($\sigma_n^2 \geq \lambda_{k+1} \geq \lambda_{k+2} \geq \dots, \lambda_M$). From the size of their corresponding Eigen value, $Re\{U_s\} = \mathcal{S}$ and $Re\{U_n\} = \mathcal{S}^\perp$. Equation (21) shows that basis vector for signal and noise subspace can be determined from the Eigen-decomposition of the spatial covariance matrix. However, in practice the spatial covariance matrix is unknown, and it must be estimated from noisy measurements. An estimate, \hat{R} , of the spatial covariance matrix R can be obtained by first sampling the complex envelope at rate $1/T$ and by then performing the time average: [8]

$$\hat{R} = \frac{1}{N} \sum_{t=1}^N y(tT) \times y(tT)^H = A(\theta) \hat{R}_s A(\theta)^H + \hat{R}_n I_M \quad (22)$$

where N is the number of samples. The emitters are assumed to be at fixed DOAs throughout this N sample interval. \hat{R}_s and \hat{R}_n are estimates of emitter and noise covariance. For a finite number of samples, \hat{R}_n will not be diagonal and vectors orthogonal to $A(\theta)^H$ will not necessarily be Eigenvectors of \hat{R} . Thus we can only obtain estimates of the basis vectors for \mathcal{S} and \mathcal{S}^\perp by performing an Eigen-decomposition on equation (22): [8]

$$\hat{R} = \hat{U} \times \hat{\Lambda} \times \hat{U}^H \quad (23)$$

$$= \hat{U}_s \hat{\Lambda}_s \hat{U}_s^H + \hat{U}_n \hat{\Lambda}_n \hat{U}_n^H \quad (24)$$

The k columns of \hat{U}_s are the Eigen-vectors corresponding to the k largest Eigen- values, and $M - k$ columns of \hat{U}_n are the Eigenvectors corresponding to the $M - k$ smallest Eigen-values.

In practice situations, only noisy measurements of the array output are available, and the basis vector of signal subspace must be estimated from equation (24). The signal subspace estimate, \hat{S} , is not the same as S , and there may be no intersections between A and \hat{S} . However, some elements of the A are expected to be closer to \hat{S} than other elements. The k elements of A which are close to \hat{S} can be used as estimates of the intersection between A and S . One measure of the closeness between $a(\theta) \in A$ and \hat{S} can be obtained by projection $a(\theta)$ onto the estimated noise subspace \hat{S}^\perp . The logic is that if $a(\theta) \in A$ is close to \hat{S} , then $a(\theta)$ should be nearly orthogonal to \hat{S}^\perp . Thus the norm squared of the projection onto \hat{S}^\perp is close to zero. This measure of closeness leads to the MUSIC spectrum: [8]

$$P_{MUSIC}(\theta) = \frac{1}{A(\theta)^H \times \hat{U}_n \times \hat{U}_n^H \times A(\theta)} \quad (25)$$

The denominator in equation (25) is simply the norm squared of the projection of $A(\theta)$ onto \hat{S}^\perp . If $A(\theta = \theta_k) \in \hat{S}$, then the denominator is near zero and $P_{MUSIC}(\theta = \theta_k)$ is peaked. The k peaks of P_{MUSIC} identify the k array response vectors which are most orthogonal \hat{S}^\perp . MUSIC estimates can be obtained by mapping these k array response vectors to the appropriate DOAs. A simplified flow chart of the MUSIC algorithm applied for signal parameter estimation is shown in Fig. (2).

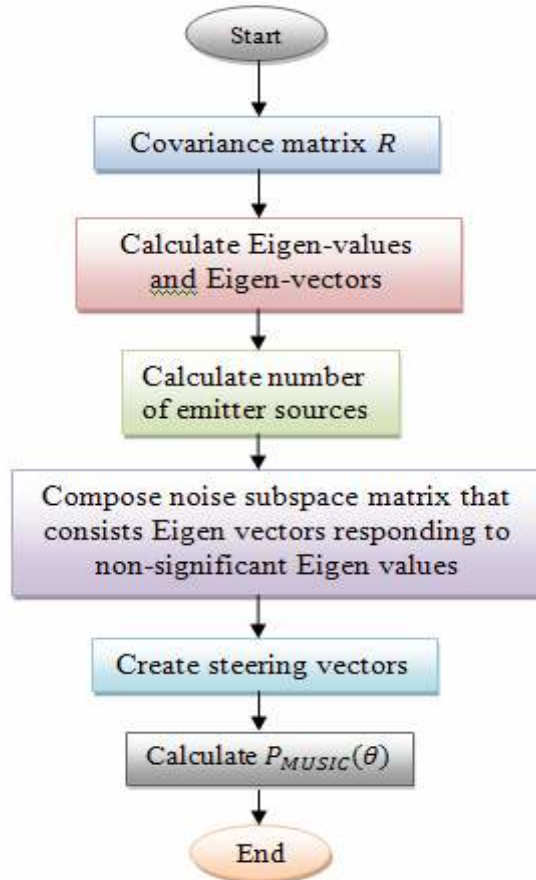


Fig. (2) Flow chart of DOA-MUSIC algorithm.

Graphical User Interface (GUI):

The MUSIC technique has been implemented using Matlab program. A GUI has also been built to ease the simulation. A layout of the GUI is depicted in Fig. (3). The user can input the signal parameters including the number of snapshots N , the number of emitter sources K , and their angles of arrival θ_i .

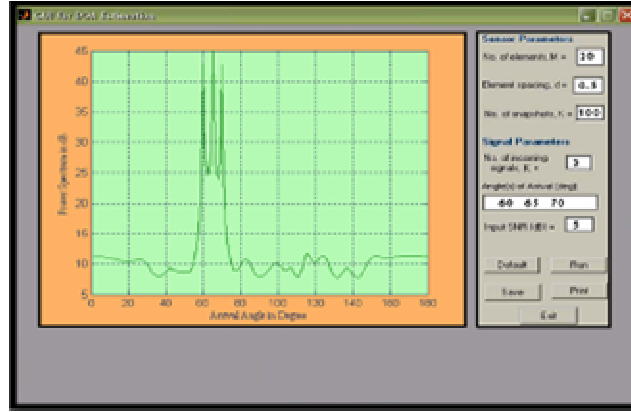


Fig.(3) Layout of graphical user interface.

Simulation Results:

In this paragraph, the performance of the Direction of Arrival estimation is examined. There are several cases simulated using software developed to function under Matlab 2009a environment. The receiver antennas are supposed to be uniform linear arrays. Different cases examine the effect a distance (d) between sensors, number of sensor and number of snapshots on performance of the DOA.

a. Impact of Element Spacing (d)

The element spacing d also has a significant impact on the shape of the radiation pattern. It is evident that the more elements an array has or alternatively the larger the array gets, the better the characteristics of the radiation pattern as far as its shape and degrees of freedom. The radiation patterns of a two-element array with element spacing of $\lambda/2$ are shown in Figures (4, 5), it is seen for $d/\lambda = 0.5$ there is one main lobe and nulls at $\theta = \pm 90$ off broadside. These nulls occur because at that DOA the signal wave-front must travel exactly $\lambda/2$ between two sensors and; therefore, exact canceling of the resulting phasor sum.

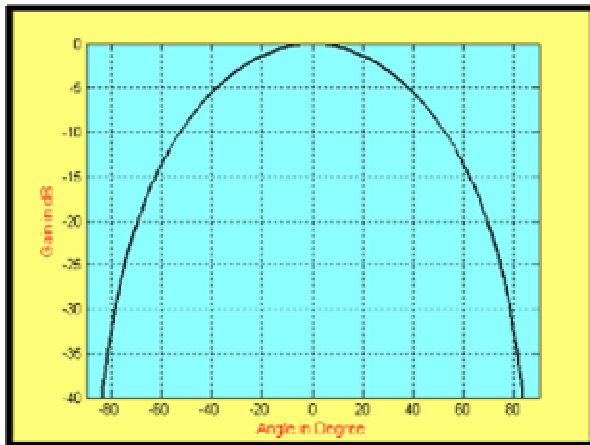


Fig. (4) Normalized direction pattern, $M = 2$, $d = \lambda/2$.

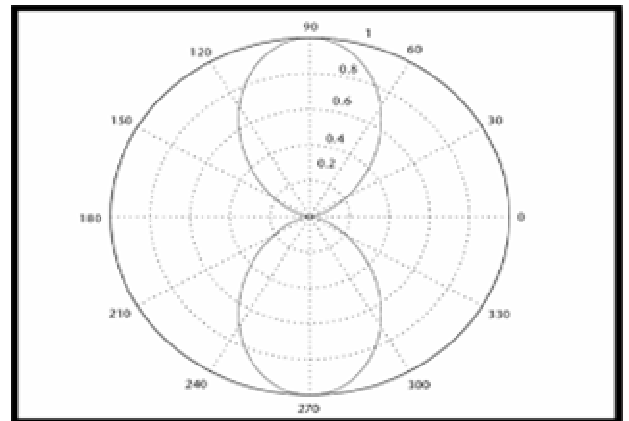


Fig. (5) Polar pattern, broadside array, $M = 2$, $d = \lambda/2$

b. Impact of Number of Sensors (M):

A simulated narrow-band emitter is used with $M=12$, $N=100$ (constant) and suppose the emitter signals with 5dB additive noise are impinging on sensors array from azimuth of 60° , 62° , 64° degrees. The MUSIC searches for the peaks is illustrated in Fig. (6). The MUSIC technique will improve its performance at increasing M to 15 and 20 respectively as shown in Figs. (7, 8). The following effects on the radiation pattern can be observed at increasing M :

- The width of the main lobe decreases; in other words, it becomes narrower. This is crucial for the applications of smart antennas when a single narrow beam is required to track a mobile or cluster of mobiles.
- The number of side-lobes increases. In addition, the level of the first and subsequent side-lobes decreases compared with the main lobe. Side-lobes represent power radiated or received in potentially unwanted directions.
- The number of nulls in the pattern increases. In interference cancellation applications, the directions of these nulls as well as the null depths have to be optimized.

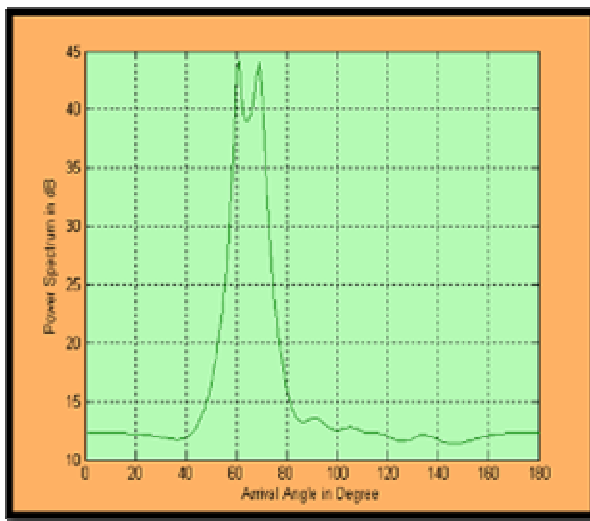


Fig.(6) DOA-MUSIC for three closely spaced emitter sources with $M=12$, $N=100$

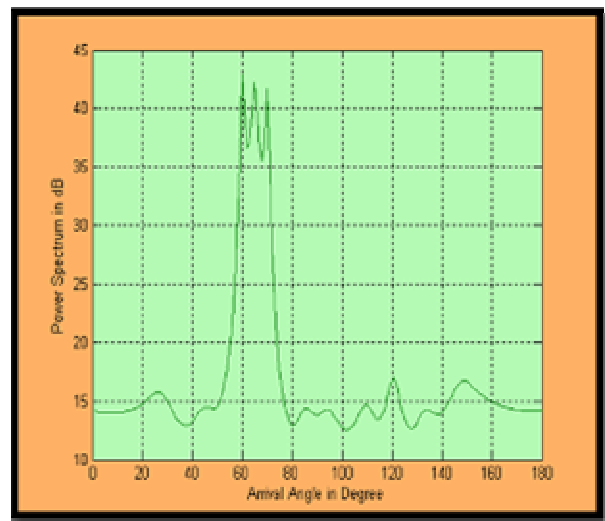


Fig.(7) DOA-MUSIC for three closely spaced emitter sources with $M=15$, $N=100$

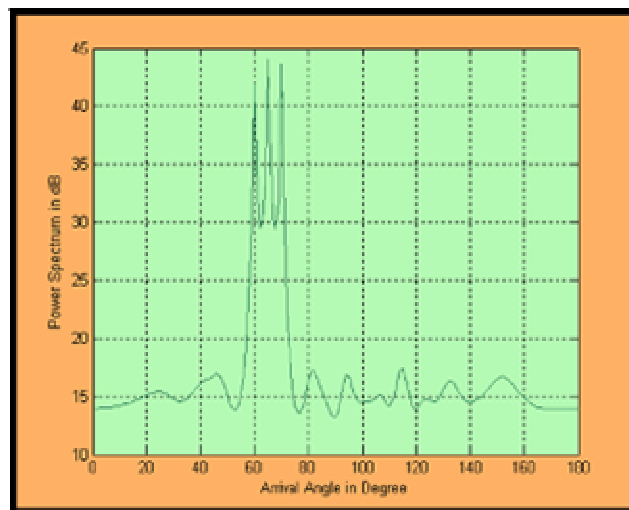


Fig.(8) DOA-MUSIC for three closely spaced emitter sources with $M=20$, $N=100$

For three very closely spaced emitter sources, with the previous assumptions are impinging on a sensor array from azimuth of 60° , 62° and 64° degrees. The MUSIC searches

for the peak is illustrated in Fig. (9), from the graph, it can easily be noticed that there is only one peak in MUSIC spectrum. The failure in the MUSIC technique is due to the few sensors M used in this simulation. The MUSIC technique will also improve its performance at increasing M to 15 and 20 respectively as shown in Figs. (10, 11).

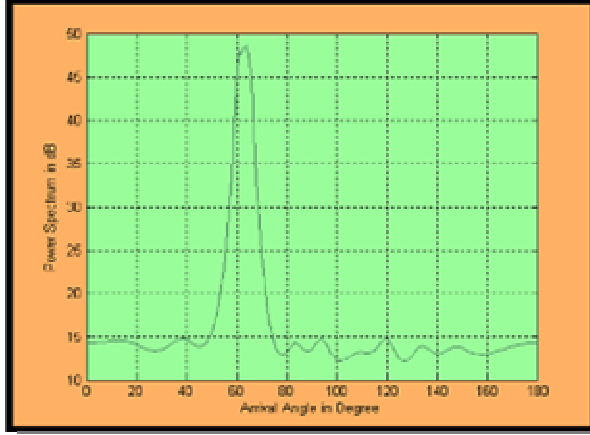


Fig.(9) DOA-MUSIC for three very closely spaced emitter sources 60°, 62°, and 64° with $M=12$, $N=100$.

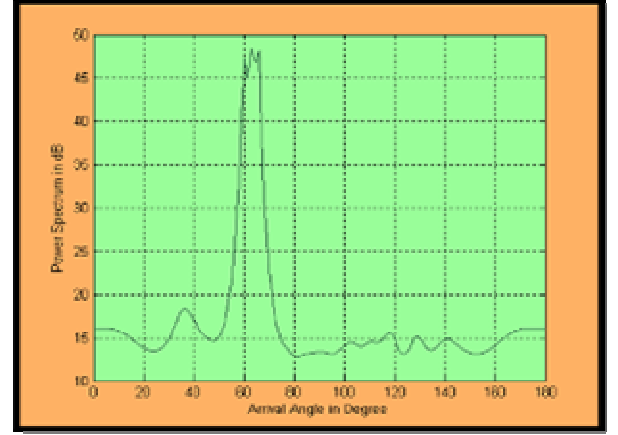


Fig.(10) DOA-MUSIC for three very closely spaced emitter sources 60°, 62°, and 64° with $M=15$, $N=100$

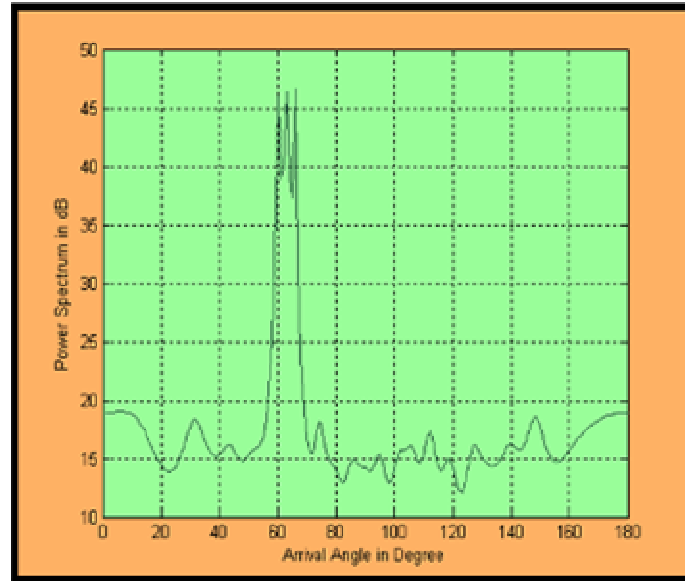


Fig.(11) DOA-MUSIC for three very closely spaced emitter sources 60°, 62°, and 64° with $M=20$, $N=100$

c. Impact of Number of Snapshots (N):

As the number of snapshots (N) is increased to 300 and 600 instead of 100, the MUSIC technique will also improve its performance to limit three peaks as shown in Figures (12, 13), with survival $M=12$ (constant).

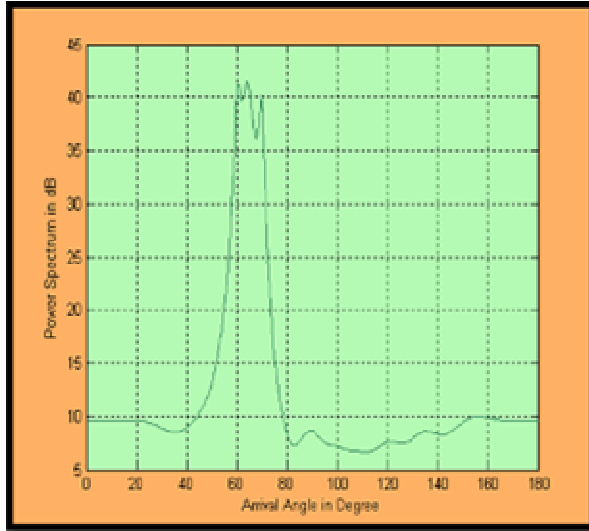


Fig.(12) DOA-MUSIC for three closely spaced emitter sources with $M=12$, $N=300$.

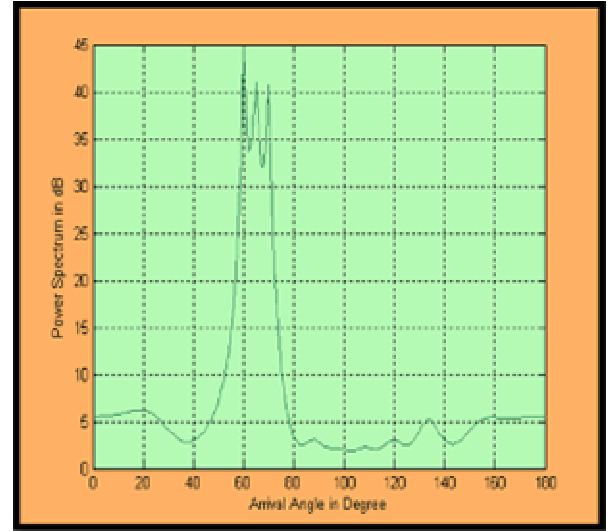


Fig.(13) DOA-MUSIC for three closely spaced emitter sources with $M=12$, $N=600$.

For three very closely spaced emitter sources 60° , 62° and 64° , with increasing N to 300 and 600 instead of 100 with same previous assumptions of M are applied. The MUSIC gives the following spectrum as shown in Figures (14, 15) respectively.

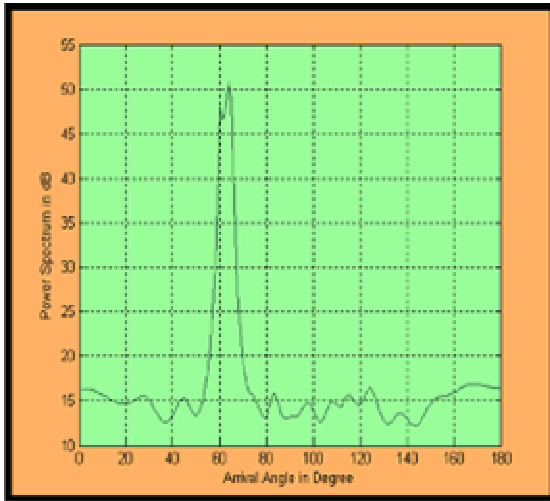


Fig.(14) DOA-MUSIC for three very closely spaced emitter sources 60° , 62° , and 64° with $M=12$, $N=300$

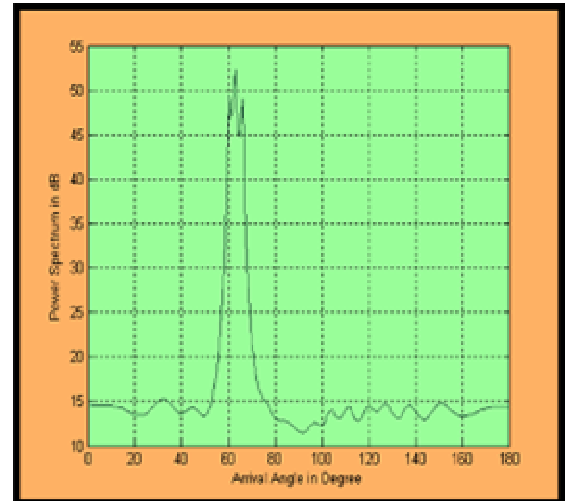


Fig.(15) DOA-MUSIC for three very closely spaced emitter sources 60° , 62° , and 64° with $M=12$, $N=600$

Conclusions:

A number of numerical experiments were conducted to investigate the effect of various parameters on the performance of the MUSIC algorithm and its ability to resolve incoming signals accurately and efficiently. Many conclusions can be derived in this paper; the most important results can be summarized as follows:

1. The better the characteristics of the radiation pattern as far as its shape and degrees of freedom at $d = \lambda/2$ because of the mutual coupling effects for closely spaced elements. To overcome this problem, the spacing between the elements of the sensor array must be increased resulting in a better resolution of the estimated peaks, as shown in Figures (4, 5) for which $d = 0.5\lambda$.
2. The efficiency of the MUSIC technique increases with the increasing number of sensors (M). It is evident that using more sensors (M) improves there solution of the algorithm in detecting the incoming signals as shown in Figures (8, 11). This is achieved, however, at the expense of computational efficiency and hardware complexity of the sensor array.
3. The efficiency of the MUSIC technique increases with the increasing number of snapshots (N). the performance improves significantly as the emitter sources move away from each other
4. Spectrum searching is required.

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