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Dynamic Analysis of Foundations on Saturated Clay Using an Energy Absorbing Layer

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Abstract

In this study, a method to model the semi-infinite extension (unbounded domain) of the saturated soil is developed. In this method, the unbounded domain is replaced by an absorbing layer of finite thickness with properties that appreciably reduce the wave reflection into bounded domain. In this layer, the soil is represented by the same properties as in the soil close to the foundation (bounded domain) and a model of frequency-dependent damping is implemented.

A three-dimensional dynamic analysis of rectangular footing on a saturated soil is carried out. The foundation is subjected to four cycles of harmonic force. The coupled dynamic equations with u-p formulation based on the dynamic consolidation theory are used to simulate the soil skeleton and pore fluid responses. The solid particles of the soil are represented by linear elastic behavior. It was found that a decay in wave can be noticed when the unbounded domain of the saturated soil is represented by the energy absorbing layer. In addition, the maximum displacement of the foundation will be decreased due to using the energy absorbing layer in comparison with the elementary boundaries. The excess pore water pressure that developed during the dynamic loading will be dissipated with time in a fast rate due to using the energy absorbing layer.

Keywords: Dynamic analysis, finite elements, boundary conditions, absorbing layer.

التحليل الديناميكي للأسس على تربة طينية مشبعة بأستعمال طبقة ماصة للطاقة

الخلاصة

فـي هـذه الدراسـةِ تـم تطـوير طريقـة لتَمثيـل الإمتـدادِ نصـفِ اللانهـائي (المجـال غيـر محـدود) للتربـةِ المُشـبَعةِ. فـي هـذه الطريقـةِ يُسـتَبدلُ المجـال الغيـر محـدود بطبقـةٍ ماصـة للطاقـة ذات سُـمك محـدود وبخـواصٍ تعمـل علـى تخفـيض إنعكـاس الموجـةَ إلـى المجـالِ المحـدودِ. فـي هـذه الطبقـةِ تكـون التربـة مُمَثَّلـةُ بـنفس الخـواص كمـا فـي التربـةَ القريبـة مـن الاسـاس (المجـال المحدود) بالاضافة الى تضمينها نموذج أخماد معتمد على الترددَ.

ولبيـان تأثيرهـذه الطبقـة فقـد تـم اجـراء التحليـل الـديناميكي الثلاثـي الأبعـاد لأسـاس مسـتطيلِ علـى تربـة مُشـبَعة ومعـرض إلـى أربعـة دورات مِـنْ قـوةٍ ديناميكيـة متناسـقةِ. كمـا أن المعـادلات الديناميكيـة المُزدوَجـة و بصـيغة u-p المعتمـدة علـى نظريـةِ الانضـمامِ الـديناميكي مسـتخدمة لتمثيـل الاسـتجابة الديناميكيـة للتربـةَ وسـائل المسـام. كمـا إنّ الجزيئـاتَ الصـلبةَ للتربـةِ مُمَثَّلـة بالسـلوكِ المـرن الخطـيِّ.لقـد وُجِـدَ حـدوث إضـمحلال فـي الموجـةِ عنـد تمثيـل المجـالَ الغيـر محـدودَ للتربـةِ المُشـبَعةِ بالطبقـة الماصـة للطاقـة. بالإضـافة الـى ذلـك فـأن الإزاحـة القصـوى للأسـاس سَـتَقل عنـد اسـتخدام هـذه الطبقـة مقارنـة بالحـدودِ التقليديـة، كمـا ان ضـغط مـاءِ المسـامِ الفائضِ الذي تطوّرَ أثناء التحميل الديناميكي سوف يتبدد مع الوقتِ بنسبة سريعة.

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1. Introduction

Most structures are supported by soils and as the soil is much larger in size than the structure itself, it is considered as being unbounded, i.e., infinite in dimension. In addition, dynamic loads can be introduced through the unbounded supporting soil by the dynamic loading from machines and impacts, and act directly on the structure (Bazyar, 2007)[1].

To model a saturated soil-structure interaction problem using the finite element method, the semi-infinite soil (unbounded domain) has to be truncated to a domain of finite size. In static analysis, an artificial boundary is introduced sufficiently far away from the structure to truncate a finite region of the unbounded domain. This is not applicable to the dynamic analysis of an unbounded domain.

Various methods have been proposed for modeling the unbounded domains. In general, these approaches fall into two categories: global procedures and local procedures. The global procedures are generally rigorous. Due to their high accuracy, they can be placed immediately beyond the structure-soil interface leading to a reduction of the number of degrees of freedom in the bounded domain. Local procedures are based on the mathematical representation of wave propagation either to prevent wave reflection or to absorb waves by enforcing artificial damping.

In order to obtain results of an acceptable level of accuracy, a local procedure has to be applied at an artificial boundary sufficiently far $\bar{\mathbf{u}}$ is the displacement vector and their derivatives $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the velocity and acceleration vectors, respectively, and $\mathbf{p}$ is the pore water pressure vector. The vectors $\mathbf{f}^u$ and $\mathbf{f}^p$ include the effect of body forces and prescribed boundary conditions for the solid-fluid mixture and the fluid phase respectively.

3. Computer Procedures for Dynamic Finite Element Analysis o

DIANA-SWANDYNE III is the acronym of Dynamic Interaction and Nonlinear Analysis. This program was originally produced by the geotechnical group at the by the geotechnical group at the University of Birmingham. The program deals with three-dimensional structures, saturated soil and pore fluid interaction, and it is capable of performing analysis for static (drained and undrained), consolidation and dynamic (drained, draining and undrained). For drained analysis, the fluid phase is either neglected or its pressure is fixed at constant values. As for draining analysis, the pressure is not known for all locations and it will evolve with time. Lastly, for undrained analysis, the undrained condition can be imposed at element level or simply by using a global no-drainage condition with a drained or draining analysis.

The program uses the finite element method. The time integration is done with the generalized Newmark method (Katona and Zienkiewicz, 1985)[4]. Both tangential stiffness and secant updating methods are available for nonlinear iteration.

away from the soil-structure interface.

This study presents the development of a reliable method for the numerical simulation of dynamic saturated soil-structure interaction problems. This method is indispensable in the dynamic analysis of foundations and does not exist at present. The theoretical framework of the method is based on the frequency-dependent domain to represent the unbounded domain of the semi-infinite saturated soils.

2. Dynamic Consolidation Theory

The u-p formulation of the dynamic consolidation theory is possible for modeling of coupled problems of soil skeleton – pore fluid interaction (Zienkiewicz and Shiomi, 1984)[2]. This formulation is defined by (i) the equation of motion for the solid-fluid mixture, and (ii) the equation of mass conservation for the fluid phase, incorporating equation of motion for the fluid phase and Darcy's law. The mathematical and numerical formulation of this formulation is described in detail by Zienkiewicz et al. (1999)[3]. For the finite element method, the spatial discretization of the u-p formulation is as follows:

$$M\ddot{\bar{u}} - Q\bar{p} + K\bar{u} = f^{u} \quad \text{...... (1)}$$

$$H\bar{p} + Q^{T}\dot{\bar{u}} + S\dot{\bar{p}} = f^{p} \quad \text{..... (2)}$$

where M is the mass matrix, Q is the discrete gradient operator coupling the solid and fluid phases, K is the stiffness matrix of the solid part, H is the compressibility matrix and S is the permeability matrix.

The solution process used is the profile solver. External loadings can be given in the form of boundary displacement or pressure, boundary traction or influx and pressure loading on solid phase and all of these can be a function of time.

The program is designed forsimulating the dynamic loads induced by earthquakes and hence the applied loads are restricted at the base of the domain. To model the machine foundations, it is required to apply the dynamic load at the foundation level. Therefore, special efforts were spent by the authors to make modifications to define the loads in the specified nodes. The mesh of the foundation is inserted in the file of mesh generation and the dynamic load is ap applied as time dependent prescribed load on solid phase.

4. Radiation Damping in Soil

Radiation damping is of concern when modeling dynamic soil-structure interaction. Soil foundations can be considered unbounded with waves carrying energy away from the specific region of interest. Since finite element analyses are often used to model soil-structure interaction problems, the modeled soil foundation is discretized and finite. Therefore, this discretized region must be surrounded by boundary conditions that effectively mimic the dissipation effect of infinity, ideally absorbing all energy propagating outwards and allowing no energy back into the examined system. These boundary conditions are known as transmitting boundaries (Wang et al., 2008) [5]. Elementary boundaries (Figure 1) are conditions of zero-displacement. They do not absorb impinging waves but reflect them, keeping wave energy constrained inside the discretized region. This can be a source of error in estimating dynamic responses if the boundaries are not placed at a far enough distance for intrinsic damping to damp out the waves before hitting the boundaries.

5 Three-Dimensional Dynamic Analysis of Foundation on Saturated Soil

For some problems, it is necessary to perform three-dimensional analysis. The finite element method is found to be an effective numerical technique to perform this type of analysis (Phan et al., 1979)[6]. Such problems may include machine foundations on saturated soil. Machine foundations are usually rectangular in shape and in some cases where the ratio of the length to width is more than six; a two-dimensional idealization of the actual situation is made (Dasgupta and Rao, 1978)[7].

In this study, three-dimensional dynamic analysis of rectangular footing on a saturated soil is investigated. In order to model the dynamic behavior of saturated soil adequately, the coupled dynamic equations with u-p formulation are used to simulate the soil skeleton and pore fluid responses. For treating wave propagation problems in saturated soil, the unbounded domain is represented by energy absorbing layer as boundary condition to enable energy dissipation through radiation.

. A Model for Representing the Semi-Infinite Extension of Soil by Energy Absorbing Layer

In this section, an energy absorbing layer is developed to model the semi-infinite extension (unbounded domain) of the saturated soil. In this method, as shown in Figure (2), the unbounded domain is replaced by an absorbing layer of finite thickness with properties that appreciably reduce the wave reflection into bounded domain. The energy absorbing layer is discretized by using the standard finite element method. In this layer, the soil is represented by the same parameters as in the bounded domain and a model of frequency-dependent damping is implemented. The dimensions of the bounded domain are a function of the width of the foundation, b, so that the dimensions of the bounded domain satisfy a better location of the absorbing layer to be able to absorb the energy of the wave effectively.

The approach is applied not to the system as a whole but to the individual elements to represent the radiation damping in the soil. Now, consider that the damping matrix, C is proportional to the mass matrix, M as:

$$C = s\,M \qquad \ldots\ldots\ldots\ (3)$$

The model damping ratio is (Chopra, 1995)[8]:

$$\beta_i = \frac{S}{2\omega_i} \qquad \ldots\ldots\ldots(4)$$

where: β_i = critical damping (damping that inhibits an oscillation completely),

ω_i = modal frequency, and

s = constant of proportion.

Similarly, if the damping matrix, C is proportional to stiffness matrix, K:

$$C = s\,K \qquad \ldots\ldots\ldots\ (5)$$

and

$$\beta_i = \frac{s}{2}\,\omega_i \quad \ldots..\ (6)$$

The proportional damping starts from Equations (3) to (6). If the two types of proportional damping are combined, then:

$$C = \alpha_d\,M + \beta_d\,K \qquad \ldots\ldots\ (7)$$

$$\beta_i = \frac{s}{2\omega_i} + \frac{s\,\omega_i}{2} \qquad \ldots.\ (8)$$

If ω_o is the fundamental frequency of a system and β_s is the desired damping ratio for the range of frequencies, it can be assumed that $\beta_i = \beta_s$ at ω_o. This can be achieved by setting:

$$\alpha_d = \beta_s\,\omega_o \qquad \ldots\ldots\ldots\ (9)$$

$$\beta_d = \frac{\beta_s}{\omega_o} \quad \ldots\ldots (10)$$

The method is applied for particular elements in the energy absorbing layer, if M and K are the mass and stiffness matrices then the damping matrix, C is:

$$C = (\beta_s\ \omega_o)\ M + \frac{\quad}{\quad} \quad \ldots\ldots (11)$$

In this study, ω_o represents the frequency of the dynamic load applied on the entire system. Therefore, Equation (1) will be in the following form:

$$M\ \ddot{\bar{u}} + C\ \dot{\bar{u}} - Q\ \bar{p} + K\ \bar{u} = f^u \quad . (12)$$

Based on these formulations of the absorbing layer, the dynamic analysis of rectangular footing on saturated clay will be studied. The boundary at the bottom of the finite medium which is extending to infinity in vertical direction, and the vertical boundary of the medium which is extending to infinity in the horizontal direction are modeled by the energy absorbing layer. Then, the conventional supports are applied. Chopra (1995) mentioned that the damping ratio can be assumed to be 20%.

7. Three-Dimensional Dynamic Analysis of Foundations on Saturated Clay

The geometry of the problem consists of flexible rectangular footing of length 4.0 m, width 2.0 m and thickness 0.25 m founded on the ground surface of saturated clay. The material properties of the concrete foundation are calculated according to the ACI code (ACI-318-83)[9] as shown in Table (1). The compression strength of the concrete f'_c is 25 N/mm^2.

A foundation is founded on a saturated clay layer of a semi-infinite extension as shown in Figure (3).

A three-dimensional finite element model of 8-noded Lagrangian brick elements are used in the dynamic analysis. The brick element has eight nodes at the corners with four degrees of freedom at each node; the first, second and third degrees of freedom are for solid displacements (u) while the fourth degree of freedom is for fluid pressure (p).

The boundary conditions are applied so that, the bottom of the soil is assumed to be fixed and the constraint on displacement in X and Y directions is applied on the nodes at the boundary in Y-Z and X-Z planes, respectively. The foundation is subjected to a steady state of four cycles of a sinusoidal function of the form $F = F_o \sin(\omega t)$ with amplitude of force ± 8.0 kN and circular frequency, ω of 66.92 rad/sec as shown in Figure (4). The three-dimensional finite element model of the problem is shown in Figure (5).

A clay soil from south of Baghdad was chosen for this application. The material properties of the clay are shown in Table (2) (Al-Saady, 1989)[10]. The material parameters of the clay for the numerical model are given in Table (3).

The time step in the dynamic analysis, Δt = 0.01 sec and a total of 100 steps are performed. The generalized Newmark method proposed by Katona and Zienkiewicz (1985) has been used for time integration with algorithm parameters β_1 = 0.6 and β_2 = 0.605 for the solid phase and Θ = 0.6 for the fluid phase to obtain an unconditionally stable time-step scheme (Zienkiewicz et al., 1999). The analysis was conducted in the absence of body forces. The dynamic response of the foundation at point A (shown in Figure 5) using the elementary boundaries is shown in Figures (6) and (7).

The dynamic analysis is performed again to study the effect of the developed method of the energy absorbing layer. The result of the displacement response is presented in Figure (8). From this figure it can be noted that, the displacement response of the foundation is dissipated with time while it is oscillating due to reflecting the wave upon using the elementary boundaries (see, Figure 6). In addition, the maximum displacement of the foundation is reduced by about 10.0 %.

The excess pore water pressure at the center of the foundation (point A) using the energy absorbing layer is shown in Figure (9). It can be noticed that the excess pore water pressure is greater than the applied load. A fully coupled analysis is required to correctly model the pore water pressure response to an applied load. The pore water pressure increase under an applied load can be greater than the applied load. This phenomenon is known as the Mendel-Cryer effect (Lewis and Schrefler, 1998)[11].

In addition, the results indicate that the generated excess pore water pressure in the case of using the elementary boundaries (see, Figure 7) is larger than that when using the energy absorbing layer. This behavior can be explained by the reflecting wave at the elementary boundary that will be returned back to the system and therefore the frequency will be increased resulting in increasing the excess pore water pressure.

8. Modeling of Material Damping for the Bounded Domain

For the bounded domain, the decay of the amplitude of waves occurs from the adsorption in real soil material and so-called material damping. Zienkiewicz et al. (1999) stated that, if the solutions of the problems are in the low-strain range when purely elastic behavior is assumed, it may be necessary to add damping matrices to the solid phase. In this case, the coefficients α_d and β_d are determined by experience (Clough and Penzien, 1993) [12].

In this study, the analysis is performed using the stiffness proportional Rayleigh damping of β_d = 0.01 and the mass proportional damping α_d = 0.0. These values of the coefficients make the mean displacement zero, i.e., the behavior of the solid skeleton is linear elastic.

The dynamic response at the center of the foundation (point A) using the energy absorbing layer with material damping is shown in Figures (10) and (11). From these Figures, it can be seen that the maximum amplitude of the displacement is reduced by 18.0 % when representing the semi-infinite extension of saturated soil by the energy absorbing layer with material damping for the bounded domain compared with that using the elementary boundaries. In addition, the decay in wave has occurred and the displacement of the foundation is returning to zero, i.e. the behavior of the soil is within the elastic range, and the excess pore water pressure is reach to the steady state after the first cycle.

9. Conclusions

1. The displacement response of the foundation is dissipated with time while it is oscillating due to

reflecting the wave upon using the elementary boundaries.

2. The maximum displacement is decreased by 10.0% when modeling the semi-infinite saturated clay by the energy absorbing layer compared with that using the elementary boundaries.
3. The excess pore water pressure is dissipated with time in a fast rate by using the energy absorbing layer. In contrast, it is oscillating due to reflecting the waves at the boundary when the energy absorbing layer is ignored.
4. The maximum amplitude of the displacement is reduced by 18.0% when representing the semi-infinite extension of saturated soil by the energy absorbing layer with material damping for the bounded domain compared with that using the elementary boundaries, and the excess pore water pressure is reach to the steady state after the first cycle.

5.References .5

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Table (1): Material properties of the foundation concrete according to the ACI code (ACI-318-83).

Parameters	Value	Units
Poisson's ratio, υ	0.495	–
Modulus of elasticity, E	12.222	MPa
Bulk density, γ_{bulk}	21	kN/m^3
Void ratio, e	0.780	–
Cohesion, c	77.3	kN/m^2

Table (2): Material properties of the clay (after Al-Saady, 1989).

Parameters	Value	Symbol
Poisson's ratio, υ	0.20	–
Modulus of elasticity, E	25000	MPa
Bulk density, ρ_c	2400	kg/m^3

Table (3): Material parameters of the clay for the numerical model

Parameters	Value	Units
Mass density of soil particles, ρ_s	2957	kg/m^3
Bulk modulus of solid particles, K_s	1.0×10^{14}	MPa
Mass density of fluid, ρ_f	1000	kg/m^3
Bulk modulus of fluid, K_f	1.0×10^{3}	MPa
Coefficient of permeability, k	1.0×10^{-7}	m/sec
Porosity, n	0.438	–

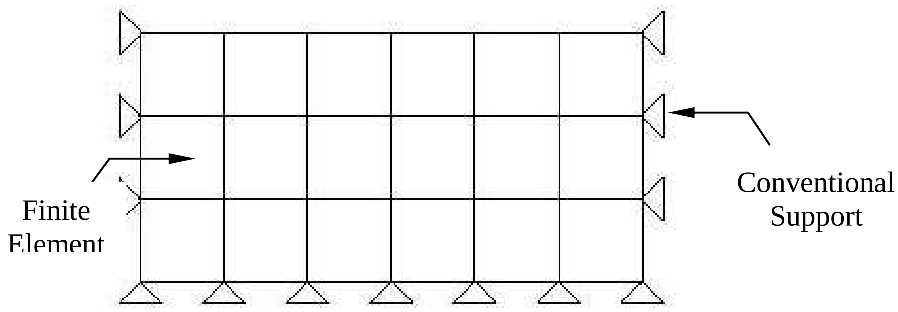

Figure (1): Elementary boundaries.

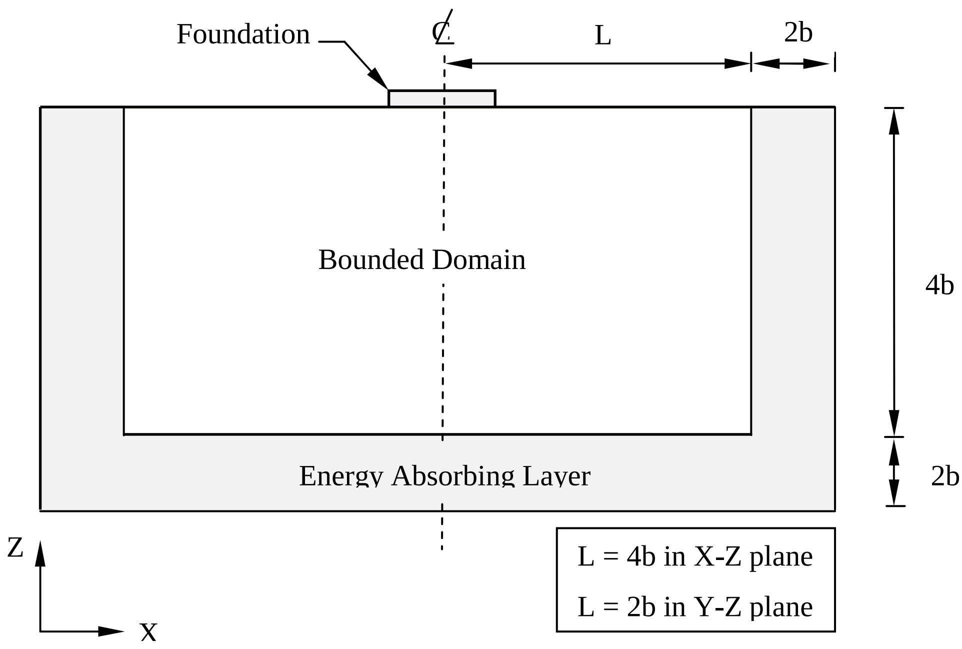

Figure (2): Basic features of the numerical model using energy absorbing layer.

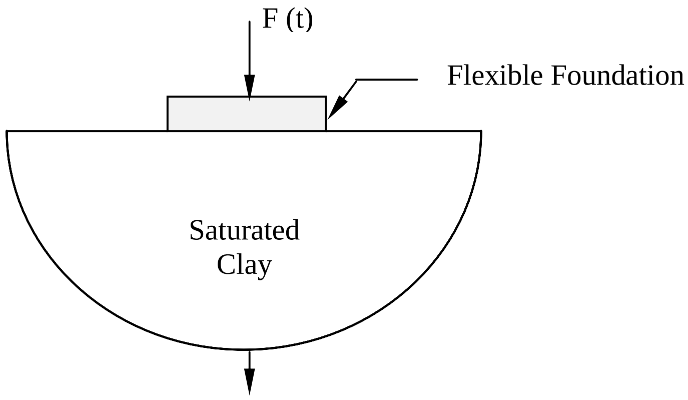

Figure (3): Elastic foundation on a semi-infinite saturated clay.

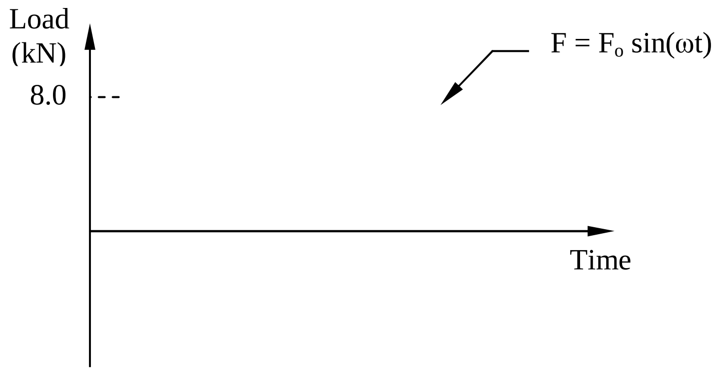

Figure (4): Steady state load of four cycles of a sinusoidal

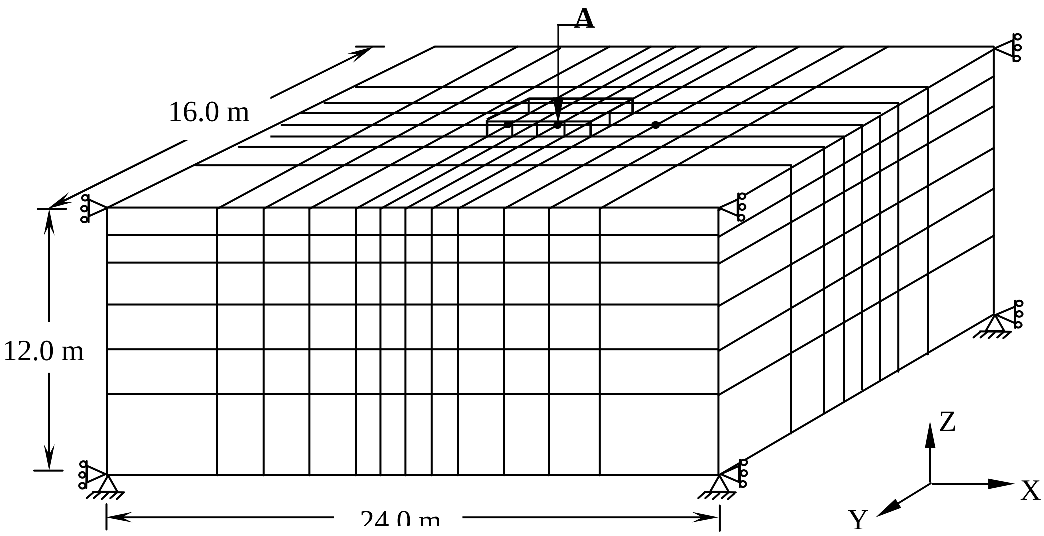

Figure (5): Three-dimensional finite element model.

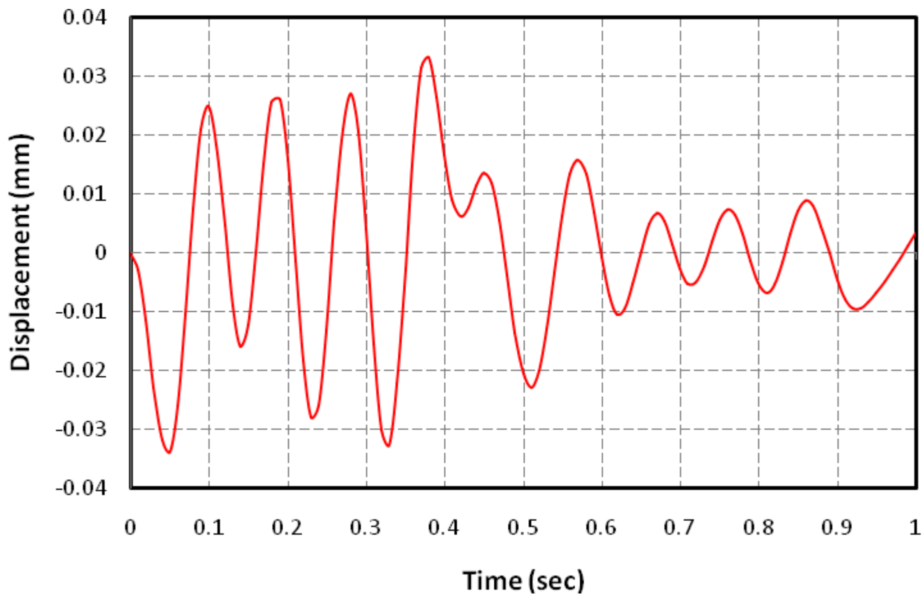

Figure (6): Displacement of the foundation at point A using elementary boundaries.

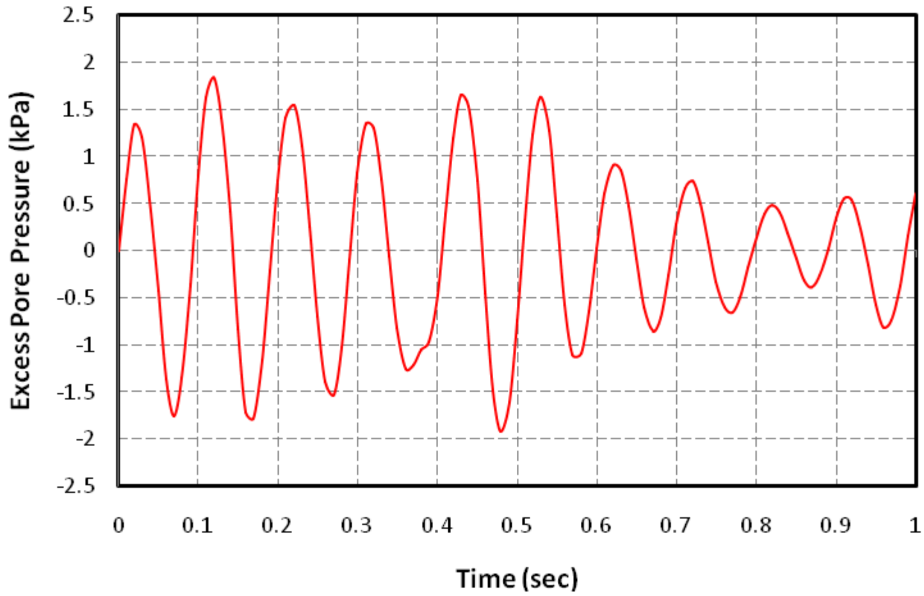

Figure (7): Excess pore water pressure at point A using elementary boundaries.

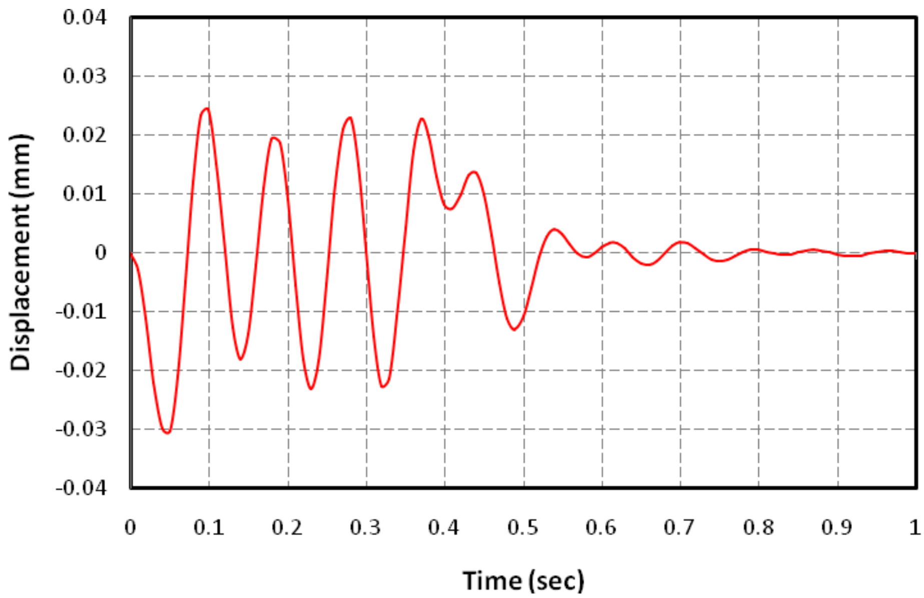

Figure (8): Displacement of the foundation at point A using energy absorbing layer.

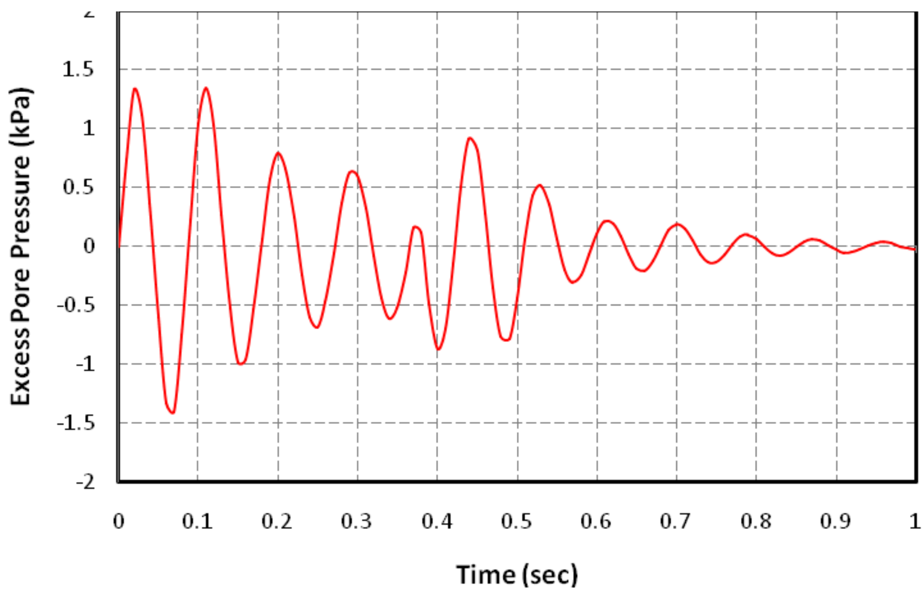

Figure (9): Excess pore water pressure at point A using energy absorbing layer.

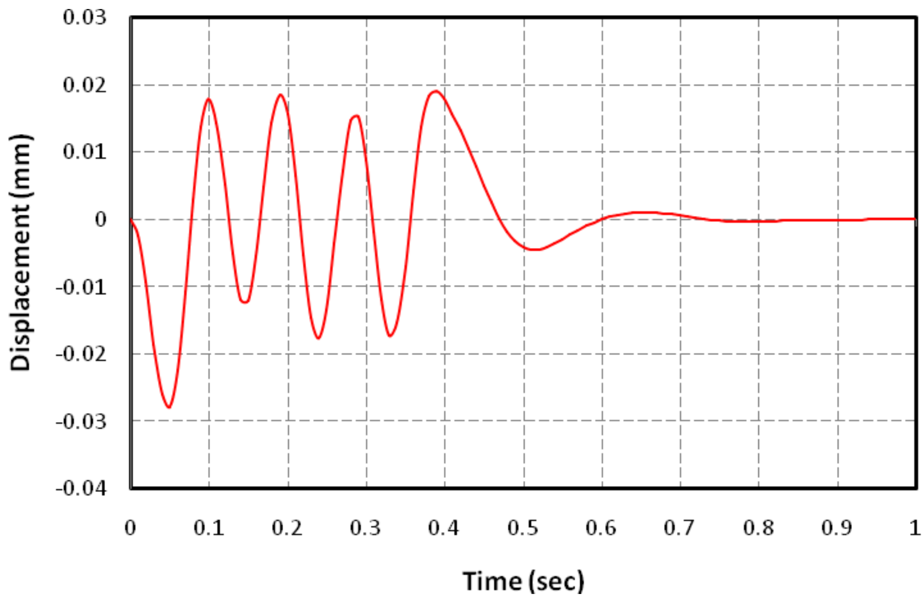

Figure (10): Displacement of the foundation at point A using energy absorbing layer with material damping.

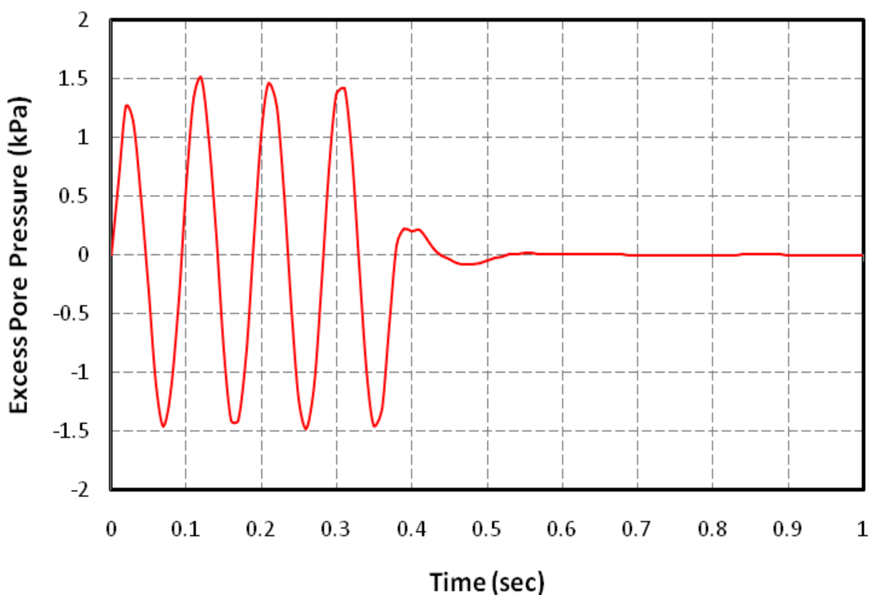

Figure (11): Excess pore water pressure at point A using energy absorbing layer with material damping.