Effect of Magnetohydrodynamic on Flow of Field of Oldroyd-B Fluid

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Abstract

The aim of this paper is to study the effects of magnetohydrodynamic (MHD) on flow of field of Oldroyd-B fluid between two side walls parallel to the plate .
The continuity and motion equations, for the problem under consideration are obtained. It is found that the motion equation contains fraction derivative of different order and the magnetohydrodynamic (MHD) parameter M .The effect of M upon the velocity field is analyzed ,many types of fractional models are also considered through taken different values of the fraction derivative order . This has been done through plotting the velocity field by using Mathemitca package .

Close form for the stress tensor was obtained in many cases, which have been studied before, are covered from our solution.

Keyword: Fractional derivative, Laplace transform, Fourier transform,

تأثيـر المجال المغناطيسي الهيدروديناميكي على التدفقات المتسارعة لحقل الموائع اولدرويد من نمط بي

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الخلاصة

الهدف من هذا البحث هو دراسة تاثير الحقل المغناطيسي على حقل جريان السوائل اولدرويد من نمط بي بين صفيحتين متوازيتين . معادلات الاستمرارية والحركة للمسالة المدروسة قد استخرجت , لقد استنتجنا بانه معادلة الحركة تحتوي على المشتقات الكسرية لرتب مختلفة ومعلمة الحقل المغناطيسي M .لقد حللنا تاثير M على حقل السرعة. كذلك درسنا عدة انواع من نماذج كسرية وذلك باخذ قيم مختلفة لرتبة المشتقات الكسرية , وهذا تم بواسطة رسم حقل السرعة باستخدام برنامج ماثماتيكا . حصلنا على الشكل المقرب للاجهاد بعدة حالات التي درسناها مسبقا وشملت عليها حلولنا .

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Introduction

Fluid is that state of matter, which is capable of changing shape and is capable of flowing. Fluids may be classified as real "viscous" and ideal "perfect " according to whether the fluid is capable of exerting shearing stress or not .Real fluid is called Newtonian if the relation between stress and rate of strain is linear ,otherwise is called non-Newtonian fluid.

Within the past fifty years ,many problems dealing with the flow of Newtonian and non-Newtonian fluids through porous channels have been studied by engineers and mathematicians.

The analysis of such flows finds important applications in engineering practice, particularly in chemical industries ,investigations of such fluids are desirable ,for examples oil and gas, molten plastics,liquid polymers, paints, glycerin, display non-Newtonian behavior .

The modeling of the equation governing the non-Newtonian fluids gives rise to highly nonlinear differential equations .

Many models have been proposed to describe the response of these fluids they being classified as fluids of the differential type ,rate type [1]and integral type ,the rate type models are used to describe the response of fluids that have slight memory .

Khan et al .[2] and Vieru et al.[3] have discussed some accelerated flows of a generalized Oldroyd-B fluid.The flow of a viscous fluid suddenly accelerated by a plane wall is an unsteady flow and such types of unsteady flows are studied by Erdogan [4,5].

Fetecau et al.[6] discussed the unsteady flow of a second grade fluid between two side walls perpendicular to a plate, Hayat et al [7] studied the flow of Maxwell fluid between two side wall, due to a suddenly moved plate .Fetecau et al [8] studied the unsteady flow of an Oldroyd-B fluid induced by impulsive motion of a plate between two side walls perpendicular to the plate ,Khan et al [9] studied the effect of MHD flow of a second grade fluid between two side walls perpendicular to the plate through a porous medium , Khan et al[10,11] studied the flow of a generalized second grade fluid between two side walls perpendicular to a plate with a fractional derivative model and Vieru et al [12] discussed the flow of a viscoelastic fluid with fractional Maxwell model between two side walls perpendicular to a plate Hyder Ali [13] studied the accelerated flows of viscoelastic fluid with Oldroyd-B fluid model between two side walls perpendicular to the plate .

In this paper we concerned with the problem's formulation, the continuity and the motion equations for the problem under consideration are obtained. It is found that the motion equations contains fraction derivative of different order and the MHD parameter M ,by using the Mixed Fourier sine transform and Laplace transform techniques are used to solve our problems. A closed form for the velocity and shear stress are obtained, the effects of each of fractional order derivative and the MHD parameter M upon the velocity distribution is analyzed .

(1) Statement of the problem

consider an incompressible generalized Oldroyd-B fluid occupies the space above a plane wall perpendicular to the y-axis and between two side walls and are located at z=0 and z=d initially the fluid is at rest and at time $t=0^+$ the bottom wall begins to slide with the time – dependent velocity At^a (a>0), corresponding initial and boundary condition are given by

$$u(y,z,0)=\frac{\partial u(y,z,0)}{\partial t}=0 \text{ for } y>0 \text{ and } 0\le z\le d, \qquad (1)$$

$$u(0,z,t)=At^a \quad \text{for} \quad t>0 \quad \text{and} \quad 0<z<d \qquad (2)$$

$$u(y,0,t)=u(y,d,t)=0 \text{ for } y,t>0 \qquad (3)$$

$$u(y,z,t),\partial_y u(y,z,t)\longrightarrow 0 \text{ as } y\longrightarrow\infty, z\in(0,d) \text{ and } t>0 \quad (4)$$

(2) Governing equations

Unsteady incompressible flow is governed by the continuity equation:

$$\operatorname{div}\bar{v}=0 \qquad (5)$$

and by using motion equation

$$\rho\frac{d\bar{v}}{dt}=\operatorname{div}\bar{T} \qquad (6)$$

Where ,ρ=density ,v = velocity

The Cauchy stress tensor for an incompressible fractional Oldroyd-B fluid has representation

$$T=-pI+S \quad \left(1+\lambda\frac{D^\alpha}{Dt^\alpha}\right)S=\mu\left(1+\lambda_r\frac{D^\beta}{Dt^\beta}\right)A_1. \quad (7)$$

In which P = pressure, I =identity tensor , S =extra stress tensor, μ =dynamic viscosity,

λ, λ_r = relaxation and retardation times respectively , α , β = fractional parameters such that $0 \le \alpha \le \beta \le 1$, $A_1=L+L^T$ where L is the velocity gradient

$$\frac{D^\alpha S}{Dt^\alpha} = D_t^\alpha S + (V.\nabla)S - LS - SL^T,$$

$$\frac{D^\beta A_1}{Dt^\beta} = D_t^\beta A_1 + (V.\nabla)A_1 - LA_1 - A_1 L^T \qquad (8)$$

In which D_t^α and D_t^β are fractional differentiation operators of order α and β with respect to t , respectively and may be defined as Riemann –Liouville fractional integral [14]

$$D_t^p f(t) = \frac{1}{\Gamma(1-p)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^p} d\tau, 0 \le p \le 1 \qquad (9)$$

Where $\Gamma(.)$ is the Gamma function .this model reduces to the classical Oldroyd-B fluid model for $\alpha = \beta = 1$.

We assume the velocity field under the form

$$\bar{v} = [u(y,z,t),0,0] \qquad (10)$$

which u is the velocity in x-direction.

The above velocity field automatically satisfies the constraint of incompressibility.

$$S(y, z, 0)=0 \qquad (11)$$

(the fluid being at rest up to the moment t=0) ,

We obtain $S_{yy}=S_{yz}=S_{zz}=0$

$$(1+\lambda D_t^\alpha)\tau_1 = \mu\,(1+\lambda D_t^\beta)\frac{\partial}{\partial y}u(y,z,t),$$

$$(1+\lambda D_t^\alpha)\tau_2 = \mu\,(1+\lambda D_t^\beta)\frac{\partial}{\partial y}u(y,z,t) \qquad (12)$$

Where $\tau_1 = S_{xy}$ and $\tau_2 = S_{xz}$ are tangential stress.

Using equation 10 into equation 7 and taking into account the initial condition 11,and resulting together with equation 6 gives

$$(1+\lambda D_t^\alpha)\frac{\partial u(y,t)}{\partial t} =$$

$$\nu(1+\lambda_r D_t^\beta)\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u(y,z,t) - Mu \qquad (13)$$

Where $\nu = \mu / \rho$ is the kinematic viscosity of the fluid and $M = \frac{\sigma\beta_0^2}{\rho}$ is magnetic field parameter.

(3) Calculation of the Velocity field

The velocity field can be obtained by solving the governing equation using the mixed Fourier sine transform.

So, let us denoted the mixed Fourier sine transform of the velocity component by

$$u_{sn}(\xi,t) = \sqrt{\frac{2}{\pi}} \int_0^\infty \int_0^d u(y,z,t)\sin(y\xi)\sin(\lambda_n z)dz dy, n = 1,2,.. \qquad (14)$$

And taking the Fourier mixed sine transform to equation 13 keeping in mind the boundary condition 1-3 we get,

$$(1+\lambda D_t^\alpha)\frac{\partial U_{sn}}{\partial t} + \nu(1+\lambda_r D_t^\beta)(\xi^2+\lambda_n^2)U_{sn}(\xi,t) =$$

$$\frac{A\nu t^a \xi}{\lambda n}\sqrt{\frac{2}{\pi}}(1-(-1)^n)(1+\lambda_r D_t^\beta) - M\,U_{sn}(\xi,t) \qquad (15)$$

Now applying the discrete Laplace transform method to equation 15,we get

$$U_{sn}^* = \frac{A\xi}{\lambda_n}\sqrt{\frac{2}{\pi}}(1-(-1)^n)\frac{\Gamma(a+1)}{q^{a+1}(\xi^2+\lambda_n^2)}$$

$$\frac{\nu(1+\lambda_r q^\beta)(\xi^2+\lambda_n^2)}{q(1+\lambda q^\alpha)+\nu(1+\lambda_r q^\beta)(\xi^2+\lambda_n^2)+M} \qquad (16)$$

Which can also written in the form

$$U_{sn}^* = \frac{A\xi}{\lambda n}\sqrt{\frac{2}{\pi}}\frac{(1-(-1)^n)}{(\xi^2+\lambda_n^2)}$$

$$\left(1-\frac{q+\lambda q^{\alpha+1}+M}{q(1+\lambda q^\alpha)+\nu(1+\lambda_r q^\beta)(\xi^2+\lambda_n^2)+M}\right)\frac{\Gamma(a+1)}{q^{a+1}} \qquad (17)$$

In order to get $U^*_{sn}(\xi,t)$and to avoid lengthy calculations of residues and contour integrals , we rewrite equation 17 in the series form

$$U_{sn}^* = \frac{A\xi}{\lambda n}\sqrt{\frac{2}{\pi}}\frac{(1-(-1)^n)}{(\xi^2+\lambda_n^2)}\frac{\Gamma(a+1)}{q^{a+1}} - \frac{A\xi}{\lambda n}\sqrt{\frac{2}{\pi}}\frac{(1-(-1)^n)}{(\xi^2+\lambda_n^2)}\Gamma(a+1)$$

$$\sum_{k=0}^{\infty}(-1)^k \sum_{i,j\ge 0}^{i+j=k}\frac{1}{j!}\frac{\nu^i(\xi^2+\lambda_n^2)^i M^j}{\lambda^{k+1}}$$

$$\sum_{m,L\ge 0}^{m+L=i}\frac{\lambda_r^m}{m!L!}\left(\frac{q^{-k+\beta m-a-1}k!}{(q^\alpha+1/\lambda)^{k+1}} + \lambda\frac{q^{\alpha-k+\beta m-a-1}k!}{(q^\alpha+1/\lambda)^{k+1}} + \right.$$

$$\left. M\frac{q^{-a-k+\beta m-2}k!}{(q^\alpha+1/\lambda)^{k+1}}\right) \qquad (18)$$

And use the next property of the inverse Laplace transform of equation:

$$L^{-1}\left\{\frac{s^{\lambda-\mu}k!}{(s^\lambda \mp c)^{k+1}}\right\} = t^{\lambda k+\mu-1}E_{\lambda,\mu}^{(k)}(\pm ct^\lambda), \quad (\mathrm{Re}(s) > |c|^{1/\lambda}) \qquad (19)$$

Where

$$E_{\lambda,\mu}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\lambda n+\mu)},\qquad \lambda,\mu>0 \quad (20)\ [15]$$

Represents the generalized Mittag - Leffler function [15] and,

$$E^{(k)}_{\lambda,\mu}(z)=\frac{d^k}{dz^k}E_{\lambda,\mu}(z)=\sum_{n=0}^{\infty}\frac{(n+k)!z^n}{n!\Gamma(\lambda n+\lambda k+\mu)} \quad (21)$$

Consequently, applying the inversion formulae term by term for Laplace transform equation 18 yields

$$u_{sn}(\xi,t)=\frac{A\xi}{\lambda n}\sqrt{\frac{2}{\pi}}\frac{(1-(-1)^n)}{(\xi^2+\lambda_n^2)}t^a-\frac{A\xi}{\lambda n}\sqrt{\frac{2}{\pi}}\frac{(1-(-1)^n)}{(\xi^2+\lambda_n^2)}$$
$$\Gamma(a+1)\sum_{k=0}^{\infty}(-1)^k\sum_{i,j\geq 0}^{i+j=k}\frac{1}{j!}\frac{\nu^i\left(\xi^2+\lambda_n^2\right)^i M^j}{\lambda^{k+1}}\sum_{m,L\geq 0}^{m+L=i}\frac{i!}{m!L!}\lambda_r^m$$
$$\Bigg(t^{a-\beta m+k+\alpha(1+k)}E^{(k)}_{\alpha,\alpha(1+k)+k-\beta m+a+1}(-\frac{t^\alpha}{\lambda})+$$
$$\lambda t^{a-\beta m+\alpha k+k}E^{(k)}_{\alpha,k-\beta m+a+1}(-\frac{t^\alpha}{\lambda})+$$
$$M t^{a-\beta m+\alpha k+\alpha+k+1}E^{(k)}_{\alpha,\alpha+k-\beta m+a+2}(-\frac{t^\alpha}{\lambda})\Bigg) \quad (22)$$

Inverting equation 22 by means of Mixed Fourier sine transform ,we find the velocity field is given by

$$u(y,z,t)=\frac{8At^a}{\pi d}\sum_{n=1}^{\infty}\frac{\sin\lambda_N}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_N^2}d\xi-\frac{8A\Gamma(a+1)}{\pi d}$$
$$\sum_{n=1}^{\infty}\frac{\sin\lambda_N}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}(-1)^k\sum_{i,j\geq 0}^{i+j=k}\frac{1}{j!}\frac{\nu^i\left(\xi^2+\lambda_N^2\right)^i M^j}{\lambda^{k+1}}$$
$$\sum_{m,L\geq 0}^{m+L=i}\frac{i!}{m!L!}\lambda_r^m\Bigg(t^{a-\beta m+k+\alpha(1+k)}E^{(k)}_{\alpha,\alpha(1+k)+k-\beta m+a+1}(-\frac{t^\alpha}{\lambda})$$
$$+\lambda t^{a-\beta m+\alpha k+k}E^{(k)}_{\alpha,k-\beta m+a+1}(-\frac{t^\alpha}{\lambda})+$$
$$M t^{a-\beta m+\alpha k+\alpha+k+1}E^{(k)}_{\alpha,\alpha+k-\beta m+a+2}(-\frac{t^\alpha}{\lambda})\Bigg)d\xi \quad (23)$$

Where N=2n-1 to change the origin of the coordinate system, Setting d=2h and z=z*+h and the dropping asterisk notation, then equation 23 will take the form (see appendix A1)

$$u(y,z,t)=\frac{2At^a}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)-\frac{4A\,\Gamma(a+1)}{\pi\ \ h}$$
$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi\ y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\geq 0}^{i+j=k}\sum_{m,L\geq 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}$$
$$\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^K}M^j\frac{\lambda_r^m}{\lambda}t^{\alpha k+k-\beta m+a}$$
$$\Bigg[t^\alpha E^{(k)}_{\alpha,\alpha+k-\beta m+a+1}(-\frac{t^\alpha}{\lambda})+\lambda E^{(k)}_{\alpha,k-\beta m+a+1}(-\frac{t^\alpha}{\lambda})$$
$$+M t^{\alpha+1}E^{(k)}_{\alpha,\alpha+k-\beta m+a+2}(-\frac{t^\alpha}{\lambda})\Bigg]d\xi\nu \quad (24)$$

In which $\lambda_N=(2n-1)\pi/2h$ is associated velocity field for flow of fractional Oldroyd-B fluid model between two side walls perpendicular to the plate

(i) Flow induced by constant acceleration of the plate

For this case ,Substituting a=1 into equation 24 we get the solution for flow induced by constantly accelerating plate i.e.

$$u(y,z,t)=\frac{2At}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)-\frac{4A}{\pi\ \ h}\sum_{n=1}^{\infty}(-1)^{n+1}$$
$$\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi\ y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\geq 0}^{i+j=k}\sum_{m,L\geq 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^K}M^j$$
$$\frac{\lambda_r^m}{\lambda}t^{\alpha k+k-\beta m+1}\Bigg[t^\alpha E^{(k)}_{\alpha,\alpha+k-\beta m+2}(\frac{-t^\alpha}{\lambda})+$$
$$\lambda E^{(k)}_{\alpha,k-\beta m+2}(-\frac{t^\alpha}{\lambda})+Mt^{\alpha+1}E^{(k)}_{\alpha,\alpha+k-\beta m+3}(-\frac{t^\alpha}{\lambda})\Bigg]d\xi \quad (25)$$

Now , using $\lambda_r\rightarrow 0$ into equation 24,we get

$$u(y,z,t)=\frac{2A\,t}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)$$
$$-\frac{4A}{\pi h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi\ y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\geq 0}^{i+j=k}\frac{(-1)^K}{i!\,j!}$$
$$\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^{K+1}}M^j\ \ t^{\alpha k+k+1}\Bigg[t^\alpha E^{(k)}_{\alpha,\alpha+k+2}(\frac{-t^\alpha}{\lambda})+$$
$$\lambda E^{(k)}_{\alpha,k+2}(-\frac{t^\alpha}{\lambda})+Mt^{\alpha+1}E^{(k)}_{\alpha,\alpha+k+3}(-\frac{t^\alpha}{\lambda})\Bigg]d\xi \quad (26)$$

Which is the associated velocity field for the fractional Maxwell fluid model.

For $\lambda\rightarrow 0$ into equation 17 and adopting similar procedure,

$$u(y,z,t)=\frac{2At}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)-\frac{4A}{\pi h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}(-1)^k\frac{\left(\nu(\xi^2+\lambda_N^2)+M\right)^k}{k!}\left(t^{k+1}E^{(k)}_{1-\beta,\beta k+2}(-\nu\lambda_r(\xi^2+\lambda_N^2)\ t^{1-\beta})+M t^{k+2}E^{(k)}_{1-\beta,\beta k+3}(-\nu\lambda_r(\xi^2+\lambda_N^2)\ t^{1-\beta})\right)d\xi \quad (27)$$

Which is the associated velocity field for fractional second grade fluid for constantly accelerating plate.

(ii) Flow induced by the variable acceleration of the plate

For this problem , replacing a=2 into equation 24 we find

$$u(y,z,t)=\frac{2At^2}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)-\frac{8A}{\pi h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\geq 0}^{i+j=k}\sum_{m,L\geq 0}^{m+L=i}\frac{(-1)^k}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^K}M^j\frac{\lambda_r^m}{\lambda}t^{\alpha k+k-\beta m+2}\left[t^{\alpha}E^k_{\alpha,\alpha+k-\beta m+3}\left(\frac{-t^{\alpha}}{\lambda}\right)+\lambda E^k_{\alpha,k-\beta m+3}\left(-\frac{t^{\alpha}}{\lambda}\right)+Mt^{\alpha+1}E^k_{\alpha,\alpha+k-\beta m+4}\left(-\frac{t^{\alpha}}{\lambda}\right)\right]d\xi \quad (28)$$

Which is the velocity for flow due to variable acceleration of the plate .

Using $\lambda_r\rightarrow 0$ in the above equation we get the solution for the flow induced by the variable accelerating plate for fractional Maxwell fluid model.

$$u(y,z,t)=\frac{2At^2}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)-\frac{8A}{\pi h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\geq 0}^{i+j=k}\frac{(-1)^k}{i!j!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^{K+1}}M^j t^{\alpha k+k+a}\left[t^{\alpha}E^k_{\alpha,\alpha+k+3}\left(\frac{-t^{\alpha}}{\lambda}\right)+\lambda E^k_{\alpha,k+3}\left(-\frac{t^{\alpha}}{\lambda}\right)+Mt^{\alpha+1}E^k_{\alpha,\alpha+k+4}\left(-\frac{t^{\alpha}}{\lambda}\right)\right]d\xi \quad (29)$$

Now , using $\lambda\rightarrow 0$ into equation 17 ,and taking the inverse Fourier sine and discrete Laplace transforms of use resulting one ,we get ,

$$u(y,z,t)=\frac{2At^2}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)-\frac{8A}{\pi h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}(-1)^k\frac{\left(\nu(\xi^2+\lambda_N^2)+M\right)^k}{k!}\left(t^{k+2}E^{(k)}_{1-\beta,\beta k+3}(-\nu\lambda_r(\xi^2+\lambda_N^2)\ t^{1-\beta})+M t^{k+3}E^{(k)}_{1-\beta,\beta k+4}(-\nu\lambda_r(\xi^2+\lambda_N^2)\ t^{1-\beta})\right)d\xi \quad (30)$$

Which represents the solution corresponding to the flow induced by variable accelerating plate for fractional second grade fluid.

(4) Flow over an infinite plate (limiting case $h\rightarrow\infty$)

When $h\rightarrow\infty$ in the Eq(24),the solution corresponding to the motion over an infinite constantly accelerating plate ,
and $\lambda_N\rightarrow 0$,where
$\lambda_N=\frac{(2n-1)\pi}{2h}$ we obtained

$$u(y,z,t)=At^a-\frac{2A\,\Gamma(a+1)}{\pi}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2}\sum_{k=0}^{\infty}\sum_{i,j\geq 0}^{i+j=k}\sum_{m,L\geq 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}\frac{\left[\nu(\xi^2)\right]^i}{\lambda^K}M^j\frac{\lambda_r^m}{\lambda}t^{\alpha k+k-\beta m+a}\left[t^{\alpha}E^k_{\alpha,\alpha+k-\beta m+a+1}\left(\frac{-t^{\alpha}}{\lambda}\right)+\lambda E^k_{\alpha,k-\beta m+a+1}\left(-\frac{t^{\alpha}}{\lambda}\right)+Mt^{\alpha+1}E^k_{\alpha,\alpha+k-\beta m+a+2}\left(-\frac{t^{\alpha}}{\lambda}\right)\right]d\xi \quad (31)$$

(5) Special case

1-Making $\lambda_r\rightarrow 0$ and $\alpha\rightarrow 1$in to equation 17 ,we get:

$$u(y,z,t)=\frac{2At^a}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\mathrm{Exp}(-\lambda_N y)-\frac{4A\,\Gamma(a+1)}{\pi\ \ h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_n^2}\sum_{k=0}^{\infty}(-1)^k\sum_{i,j\geq 0}^{i+j=k}\frac{1}{i!\,j!}\frac{\nu^i\left(\xi^2+\lambda_n^2\right)^i M^j}{\lambda^{k+1}}\left(t^{a+1+2k}E^{(k)}_{1,k+a+2}\left(-\frac{t}{\lambda}\right)+\lambda t^{a+2k}E^{(k)}_{1,k+a+1}\left(-\frac{t}{\lambda}\right)+M t^{a+2+2k}E^{(k)}_{1,k+a+3}\left(-\frac{t}{\lambda}\right)\right)d\xi \quad (32)$$

(6) Calculation of the shear stresses

Applying the Laplace transform to equation 8 and using the initial condition 11 ,we find that

$$\overline{\tau_1}(y,z,q)=\mu\frac{1+\lambda_r q^{\beta}}{1+\lambda q^{\alpha}}\frac{\partial\overline{u}(y,z,q)}{\partial y}\quad ,$$

$$\overline{\tau_2}(y,z,q)=\mu\frac{1+\lambda_r q^{\beta}}{1+\lambda q^{\alpha}}\frac{\partial\overline{u}(y,z,q)}{\partial z}\qquad (33)$$

The image function u(y, z, q) of u(y,z,t)can be obtained from equation 24,and consequently evaluating $\frac{\partial\overline{u}(y,z,q)}{\partial y}$ and $\frac{\partial\overline{u}(y,z,q)}{\partial z}$ from the mentioned equations and introducing into equation 33,it results that

$$\overline{\tau_1}(y,z,q)=\mu\frac{1+\lambda_r q^{\beta}}{1+\lambda q^{\alpha}}\Bigg[\frac{2A\Gamma(a+1)}{q^{a+1}h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}(-\lambda_N \mathrm{Exp}(-\lambda_N y))$$
$$-\frac{4A\,\Gamma(a+1)}{\pi\ h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi^2\cos\xi y}{\xi^2+\lambda_N^2}$$
$$\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^k}M^j\frac{\lambda_r^m}{\lambda}$$
$$\sum_{n=0}^{\infty}\frac{(n+k)!(-1)^n}{\lambda^n\, n!}\Big(\frac{1}{q^{\alpha k+\alpha n+\alpha+k-\beta m+a+1}}+$$
$$\frac{\lambda}{q^{\alpha k+\alpha n-\beta m+a+1}}+\frac{M}{q^{\alpha k+\alpha n+\alpha+k-\beta m+a+2}}\Big)d\xi\Bigg]\qquad (34)$$

$$\overline{\tau_2}(y,z,q)=\mu\frac{1+\lambda_r q^{\beta}}{1+\lambda q^{\alpha}}\Bigg[\frac{2A\,\Gamma(a+1)}{q^{a+1}h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{-\lambda_N\sin\lambda_N z}{\lambda_N}$$
$$\mathrm{Exp}(-\lambda_N y)-\frac{4A\,\Gamma(a+1)}{\pi\ h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{-\lambda_N\sin\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi\sin\xi y}{\xi^2+\lambda_N^2}$$
$$\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^k}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^K}M^j\frac{\lambda_r^m}{\lambda}$$
$$\sum_{n=0}^{\infty}\frac{(n+k)!(-1)^n}{\lambda^n\, n!}\Big(\frac{1}{q^{\alpha k+\alpha n+\alpha+k-\beta m+a+1}}+$$
$$\frac{\lambda}{q^{\alpha k+\alpha n-\beta m+a+1}}+\frac{M}{q^{\alpha k+\alpha n+\alpha+k-\beta m+a+2}}\Big)d\xi\Bigg]\qquad (35)$$

Lengthy but straight forward computations ,allow us to determine $\tau_1(y,z,t)$, $\tau_2(y,z,t)$ from equation 34 and 35. (appendixA2), the property (appendix A3) (that used for the convolution product of two functions) and the simple decomposition

$$\frac{1+\lambda_r q^{\beta}}{1+\lambda q^{\alpha}}=1+\frac{\lambda_r}{\lambda}\frac{q^{\beta}}{q^{\alpha}+1/\lambda}-\frac{q^{\alpha}}{q^{\alpha}+1/\lambda}$$

Final form of $\tau_1(y,z,t)$, $\tau_2(y,z,t)$,(see also appendix A2)

$$\tau_1^{*}(y,z,q)=\frac{2A\mu\, t^{a}}{h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}(-\lambda_N \mathrm{Exp}(-\lambda_N y))$$
$$-\frac{4A\,\Gamma(a+1)}{\pi\ h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi^2\cos\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}$$
$$\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^k}M^j\frac{\lambda_r^m}{\lambda}t^{\alpha k+k-\beta m+a}$$
$$\Big[t^{\alpha}E^{k}_{\alpha,\alpha+k-\beta m+a+1}\big(\frac{-t^{\alpha}}{\lambda}\big)+\lambda E^{k}_{\alpha,k-\beta m+a+1}\big(-\frac{t^{\alpha}}{\lambda}\big)+$$
$$Mt^{\alpha+1}E^{k}_{\alpha,\alpha+k-\beta m+a+2}\big(-\frac{t^{\alpha}}{\lambda}\big)\Big]d\xi$$
$$+\Big[\frac{\lambda_r}{\lambda}\frac{2A\mu\,\Gamma(a+1)}{h}R_{\beta\text{-}a\text{-}1,\alpha}\big(-\frac{1}{\lambda},0,t\big)\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}$$
$$(-\lambda_N \mathrm{Exp}(-\lambda_N y))\Big]-\Big[\frac{\lambda_r}{\lambda}\frac{4A\,\Gamma(a+1)}{\pi h}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}$$
$$\int_0^{\infty}\frac{\xi^2\cos\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^k}M^j\frac{\lambda_r^m}{\lambda}$$
$$\int_0^{t}s^{\alpha k+k-\beta m+a}\Big[s^{\alpha}E^{k}_{\alpha,\alpha+k-\beta m+a+1}\big(\frac{-s^{\alpha}}{\lambda}\big)+$$
$$\lambda E^{k}_{\alpha,k-\beta m+a+1}\big(-\frac{s^{\alpha}}{\lambda}\big)+Mt^{\alpha+1}E^{k}_{\alpha,\alpha+k-\beta m+a+2}\big(-\frac{s^{\alpha}}{\lambda}\big)\Big]$$
$$\times R_{\alpha,\beta}(\text{-}1/\lambda,0,t-s)ds\,d\xi-\Big[\frac{2A\mu\,\Gamma(a+1)}{h}R_{\alpha\text{-}a\text{-}1,\alpha}\big(-\frac{1}{\lambda},0,t\big)$$
$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}(-\lambda_N \mathrm{Exp}(-\lambda_N y))\Big]+\Big[\frac{4A\,\Gamma(a+1)}{\pi\ h}$$
$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos\lambda_N z}{\lambda_N}\int_0^{\infty}\frac{\xi^2\cos\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}$$
$$\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^k}M^j\frac{\lambda_r^m}{\lambda}\int_0^{t}s^{\alpha k+k-\beta m+a}$$
$$\Big[s^{\alpha}E^{k}_{\alpha,\alpha+k-\beta m+a+1}\big(\frac{-s^{\alpha}}{\lambda}\big)+\lambda E^{k}_{\alpha,k-\beta m+a+1}\big(-\frac{s^{\alpha}}{\lambda}\big)+$$
$$Mt^{\alpha+1}E^{k}_{\alpha,\alpha+k-\beta m+a+2}\big(-\frac{s^{\alpha}}{\lambda}\big)\Big]*R_{\alpha,\alpha}(\text{-}1/\lambda,0,t-s)ds\,d\xi\qquad (36)$$

$$\tau_2^*(y,z,q)=\frac{2A\mu t^{a}}{h}\sum_{n=1}^{\infty}(-1)^{n+1}(-\lambda_N \mathrm{Sin}\lambda_N z)\mathrm{Exp}(-\lambda_N y))$$

$$-\frac{4A\,\Gamma(a+1)}{\pi h}\sum_{n=1}^{\infty}(-1)^{n+1}(-\lambda_N \mathrm{Sin}\,\lambda_N z)\int_0^{\infty}\frac{\xi\ \mathrm{Sin}\xi y}{\xi^2+\lambda_N^2}$$

$$\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^k}M^j\frac{\lambda_r^m}{\lambda}$$

$$t^{\alpha k+k-\beta m+a}\left[t^{\alpha}E^k_{\alpha,\alpha+k-\beta m+a+1}\left(\frac{-t^{\alpha}}{\lambda}\right)+\right.$$

$$\left.\lambda E^k_{\alpha,k-\beta m+a+1}\left(-\frac{t^{\alpha}}{\lambda}\right)+Mt^{\alpha+1}E^k_{\alpha,\alpha+k-\beta m+a+2}\left(-\frac{t^{\alpha}}{\lambda}\right)\right]d\xi$$

$$\left[\frac{\lambda r}{\lambda}\frac{2A\mu\,\Gamma(a+1)}{h}R_{\beta-a-1,\alpha}\left(-\frac{1}{\lambda},0,t\right)\right.$$

$$Mt^{\alpha+1}E^k_{\alpha,\alpha+k-\beta m+a+2}\left(-\frac{t^{\alpha}}{\lambda}\right)d\xi+\left[\frac{\lambda r}{\lambda}\frac{2A\mu\,\Gamma(a+1)}{h}\right.$$

$$\times R_{\beta-a-1,\alpha}\left(-\frac{1}{\lambda},0,t\right)\sum_{n=1}^{\infty}(-1)^{n+1}(-\lambda_N \mathrm{Sin}\lambda_N z)\mathrm{Exp}(-\lambda_N y))]$$

$$-\left[\frac{\lambda r}{\lambda}\frac{4A\,\Gamma(a+1)}{\pi\ \ h}\sum_{n=1}^{\infty}(-1)^{n+1}(-\lambda_N \mathrm{Sin}\lambda_N z)\int_0^{\infty}\frac{\xi\ \ \mathrm{Sin}\xi y}{\xi^2+\lambda_N^2}\right.$$

$$\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^k}{j!m!L!}\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^k}M^j\frac{\lambda_r^m}{\lambda}$$

$$\int_0^t s^{\alpha k+k-\beta m+a}\left[s^{\alpha}E^k_{\alpha,\alpha+k-\beta m+a+1}\left(\frac{-s^{\alpha}}{\lambda}\right)+\right.$$

$$\left.\lambda E^k_{\alpha,k-\beta m+a+1}\left(-\frac{s^{\alpha}}{\lambda}\right)+Mt^{\alpha+1}E^k_{\alpha,\alpha+k-\beta m+a+2}\left(-\frac{s^{\alpha}}{\lambda}\right)\right]\times$$

$$R_{\alpha,\beta}(-1/\lambda,0,t-s)dsd\xi-\left[\frac{2A\mu\,\Gamma(a+1)}{h}R_{\alpha-a-1,\alpha}\left(-\frac{1}{\lambda},0,t\right)\right.$$

$$\left.\sum_{n=1}^{\infty}(-1)^{n+1}(-\lambda_N \mathrm{Sin}\,\lambda_N z)\mathrm{Exp}(-\lambda_N y))\right]+\left[\frac{4A\,\Gamma(a+1)}{\pi\ \ h}\sum_{n=1}^{\infty}(-1)^{n+1}\right.$$

$$(-\lambda_N \mathrm{Sin}\,\lambda_N z)\int_0^{\infty}\frac{\xi \mathrm{Sin}\xi y}{\xi^2+\lambda_N^2}\sum_{k=0}^{\infty}\sum_{i,j\ge 0}^{i+j=k}\sum_{m,L\ge 0}^{m+L=i}\frac{(-1)^K}{j!m!L!}$$

$$\frac{(\nu(\xi^2+\lambda_N^2))^i}{\lambda^k}M^j\frac{\lambda_r^m}{\lambda}\int_0^t s^{\alpha k+k-\beta m+a}\left[s^{\alpha}E^k_{\alpha,\alpha+k-\beta m+a+1}\left(\frac{-s^{\alpha}}{\lambda}\right)+\right.$$

$$\left.\lambda E^k_{\alpha,k-\beta m+a+1}\left(-\frac{s^{\alpha}}{\lambda}\right)+Mt^{\alpha+1}E^k_{\alpha,\alpha+k-\beta m+a+2}\left(-\frac{s^{\alpha}}{\lambda}\right)\right]\times$$

$$R_{\alpha,\alpha}(-1/\lambda,0,t-s)dsd\xi$$

Where

$$R_{a,b}(c,d,t)=\sum_{n=0}^{\infty}\frac{c^n(t-d)^{(n+1)a-b-1}}{\Gamma[(n+1)a-b]}$$

(6) Results and discussion

The effect of each parameter upon the velocity field will be considered .For simplicity ,in the middle of the channel ,let z=0 will be setting equal to zero .Also ,to see the effects of any parameter ,this parameter will be taken in some range ,while the other parameters will kept fixed .regards the generalized Oldroyd-B fluid,

The following results are observed :

1-As β increasing there is decreasing in the velocity field ,see figure 1: figure 2 and figure 3

2-As λ_1 increasing there is decreasing in the velocity field ,see figure 4 : figure 5 and figure 6. 3-As α increasing there is decrease in the velocity field ,see figure 7: figure 8 and figure 9

4-As t increasing there is decreasing in the velocity field ,see figure 10 and figure 11.

5-As λ increasing there is decreasing in the velocity field, see figure 12 and figure 13.

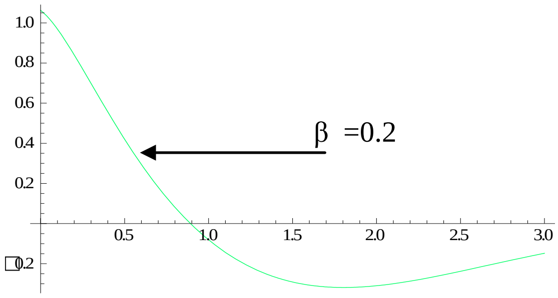


Figure 1- z=0, A=1 , ν=1, h=1 ,α=0.3 a=0.2, t=2, λ1=4, M=1, λ=12

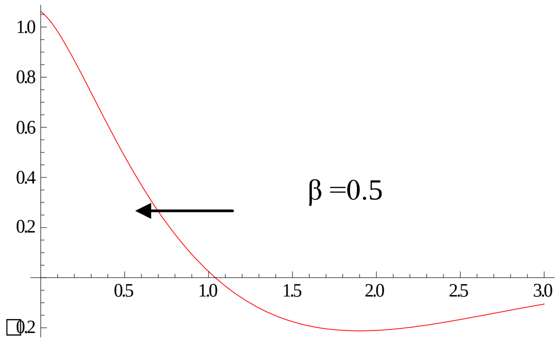


Figure 2- z=0, A=1 , ν=1, h=1 ,α=0.3 a=0.2, t=2, λ1=4, M=1, λ=12

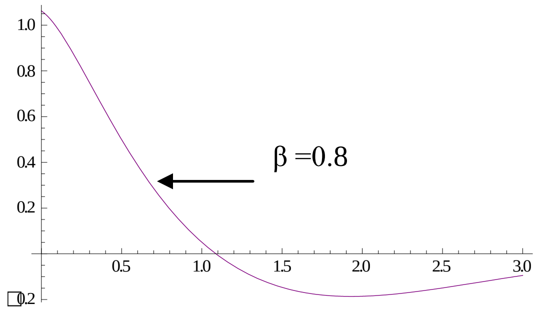
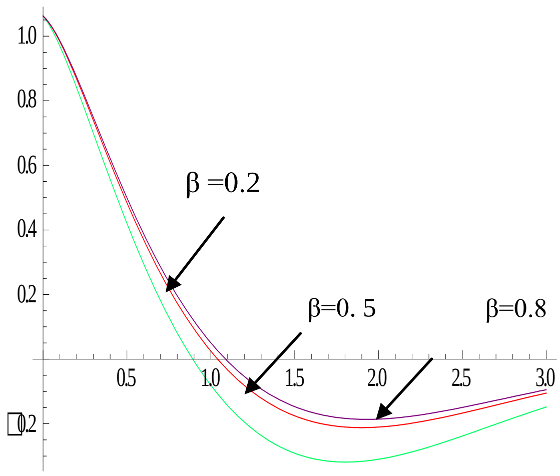


Figure 3- z=0, A=1 , ν=1, h=1α=0.3, a=0.2, t=2, λ1=4, M=1, λ=12



Together Figure (1:2:3)

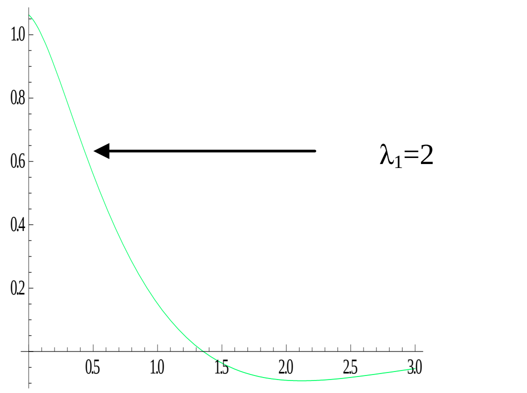


Figure 4- z=0, A=1, ν=1, h=1, α=0.3, β=0.8, a=0.2, t=2, M=1, λ=12

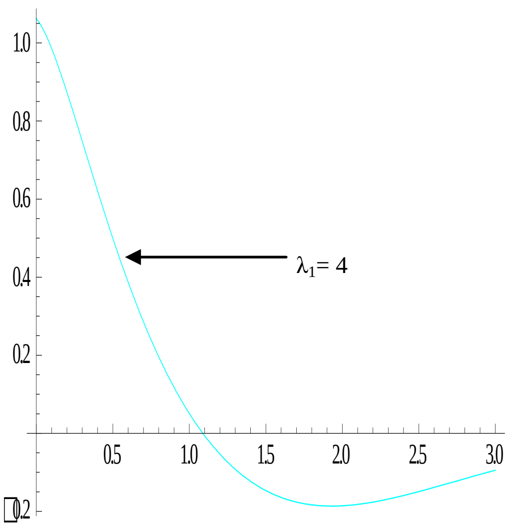


Figure 5- z=0, A=1, ν=1, h=1, α =0.3 , β=0.8, a=0.2, t=2, M=1, λ=12.

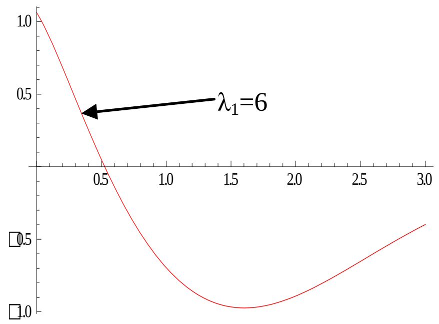
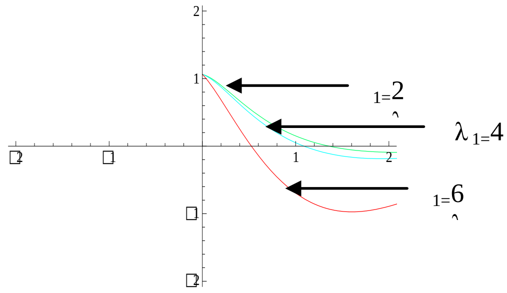


Figure 6- z=0, A=1, ν=1, h=1, α=0.3 , β=0.8, a=0.2, t=2, M=1, λ=12.



Together figures (4,5,6)

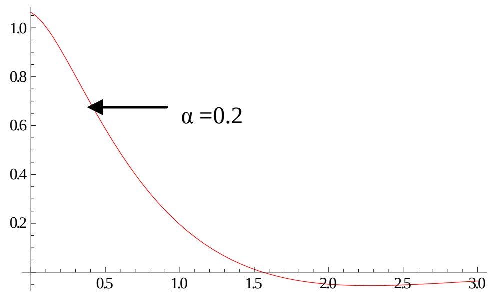


Figure 7- z=0, A=1, ν=1, h=1, β=0. 8a=0.2, t=2, λ_1=4, M=1, λ=12

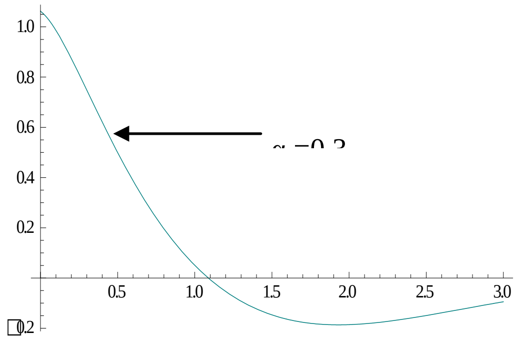


Figure 8- z=0, A=1, ν=1, h=1, β=0.8 a=0.2, t=2, λ_1=4, M=1, λ=12

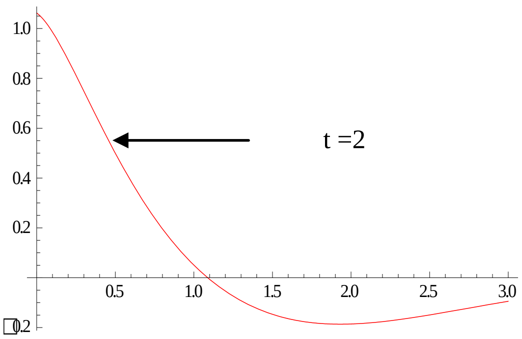


Figure 9 - z=0, A=1, ν=1, h=1, β=0.8 a=0.2, t=2, λ_1=4, M=1, λ=12

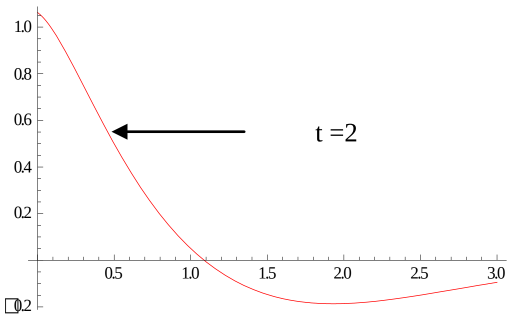
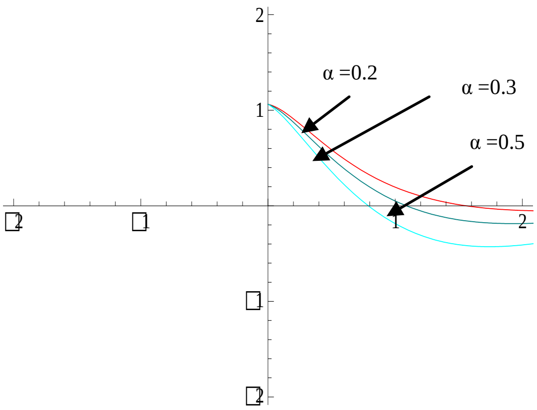
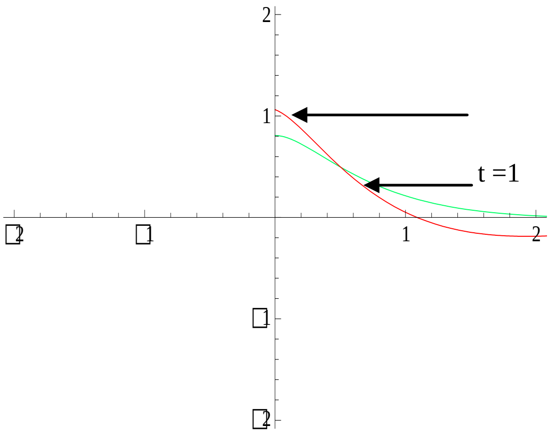


Figure 11-z=0, A=1, ν=1, h=1, α=0.3, β=0.8, a=0.2, λ_1=4, M=1, λ=12, t=1.



Together figures (7, 8 , 9)



Together figures(10,11)

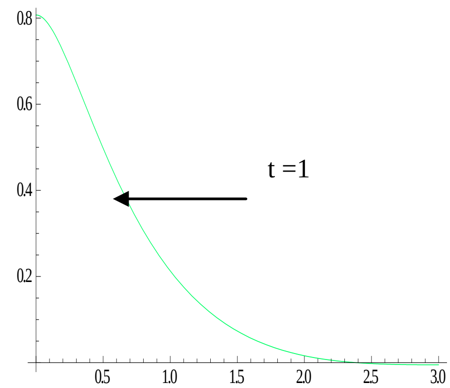


Figure 10- z=0, A=1, ν=1, h=1, α=0.3, β=0.8, a=0.2, λ_1=4, M=1, λ=12, t=1

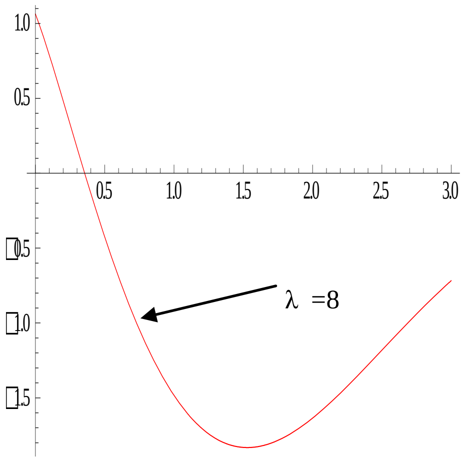


Figure 12- z=0,A=1,ν=1, h=1,α=0.3 ,β=0.8, a=0.2, t=2, M=1, λ1=4

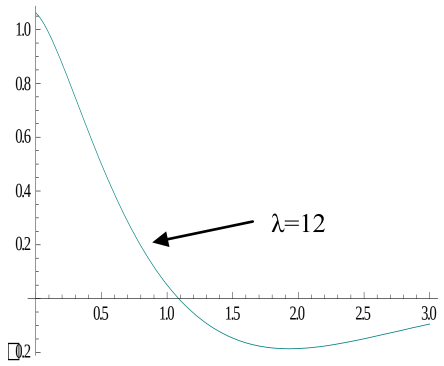
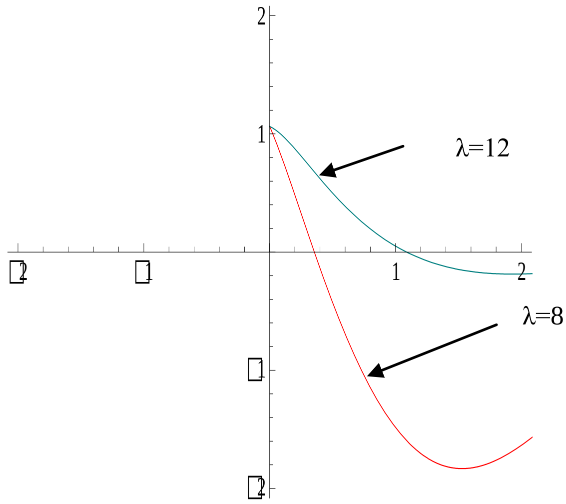


Figure 13- z=0, A=1, ν=1, h=1, α=0.3 , β=0.8, a=0.2, t=2, M=1, λ1=4



Together figures (12,13)

Appendix A:

Some relations used in our text

$$\int_0^\infty \frac{\xi \sin(\xi y)}{\xi^2 + a^2} = \frac{\pi}{2} e^{-ay}, \mathrm{Re}(a) \ge 0 \qquad (A1)$$

$$L^{-1}\left\{\frac{q^b}{(q^a - c)}\right\} = R_{a,b}(c,0,t) : \mathrm{Re}(a-b) > 0, \mathrm{Re}(q) > 0, \qquad (A2)$$

$$(u_1 * u_2)(t) = \int_0^t u_1(t-s)u_2(s)ds = \int_0^t u_2(t-s)u_1(s)ds$$

$$= L^{-1}(\bar{u}_1(q)\bar{u}_2(q)) \qquad (A3)$$

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