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The Role of Instability Threshold in Lorenz-Haken Laser System

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Abstract

We have investigated the role of instability-threshold (the second-threshold) on the dynamical behavior of the Lorenz-Haken laser system .Here, we report theoretical results of instabilities leading to chaos at low instability threshold , obtained through variation the laser system control parameters over a wide range of laser operating conditions.

Keywords: Laser instability, period-doubling , chaos, Lorenz-Haken model.

Introduction

Instabilities play an important role in a large number of fields for Example in hydrodynamics, ecology, economy, chemistry, biology, and semiconductor physics [1-5]. Also, in quantum optical systems, such as lasers, a great variety of instabilities can be found [6,7]. Compared to hydrodynamics, the nonlinear equations are, at least for simple models, rather simple. Therefore, the investigation of the instabilities in lasers is easier to carry out than in most other fields.

The type of instabilities in the dynamical systems depends on the control parameters and the nonlinear dynamics model used. In laser systems, the instabilities change mainly with increasing pumping strength. The variations of other laser operating parameters are also playing important roles on the dynamics of the laser system. Of these parameters are the asymmetry of the medium gain profile and the inhomogeneous broadening which is introduced in the original Lorenz-Haken equations (as additional parameters) to produce more generalized equations. Homogeneously broadened single-mode lasers are well-known to exhibit self-pulsing instabilities and chaotic dynamics under the combined conditions of large ratio of gain over losses and low cavity quality [8,9]. In fact, a two-level homogeneously broadened single-mode laser in resonance (with a symmetric Lorentzian gain profile) is realistically described by the classical Lorenz-Haken model [9-11] which has served as a prototype model for investigating instabilities and chaos in continuous dynamical systems. However, the conditions for observing instabilities in such systems require the bad-cavity condition in conjunction with a gain considerably above first (lasing) threshold, thus making the experimental realization of this unfeasible for most lasers. The above mentioned requirements for the occurrence of Lorenz-Haken instability phenomenon have been realized only in a very few laser systems of high gains and narrow linewidths, satisfying the bad-cavity condition. The ideal candidates of these lasers are the optically pumped mid and far infrared laser systems (MIR and FIR lasers) [12-14]. The single –mode semiconductor laser is also a good example that achieves the condition for the optical instabilities and can easily exhibit different types of dynamic instability (such as pulsations and chaotic behaviors) at low excitation power [5].

The bad-cavity condition was originally derived by Korobkin and Uspenskii [15] and it implies that the electric field relaxation rate (k) is more strongly than the sum of the polarization relaxation rate ($\gamma_\perp$) and the population inversion relaxation rate ($\gamma_{\parallel}$), i.e., $k > \gamma_\perp + \gamma_{\parallel}$ (or in the present paper, $\sigma > b + 1$). In contrast to the basic model of homogeneously broadened lasers, the single-mode instabilities are realized with relative ease in an inhomogeneously broadened system although the onset of instabilities still requires a bad cavity but with rates considerably reduced to those needed for the corresponding homogeneous case. As a result of this situation, it is found that, the threshold for the observation of the pulsing instabilities and chaos (or the second-threshold) in the inhomogeneously

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broadened laser is much lower than for homogeneously broadened laser [16].

In fact, the different types of laser systems are generally classified into three categories (or classes), A, B, and C [17]. This classification is based on the relations between cavity and active medium relaxation rates (k , $\gamma_\perp$, and $\gamma_{\shortparallel}$). For class-A lasers, $\gamma_\perp \cong \gamma_{\shortparallel} >> k$, therefore the polarization and the population inversion can be adiabatically eliminated. For class-B lasers, $\gamma_\perp >> k \geq \gamma_{\shortparallel}$, so that the polarization of the active medium adiabatically follows the cavity field and may be eliminated from consideration. Most solid-state lasers, semiconductor lasers, and certain molecular lasers belong to this class. For class-C lasers, all variables (the electric field, the polarization, and the population inversion) have comparable relaxation rates ($\gamma_\perp \cong \gamma_{\shortparallel} \cong k$) .

The general property of class-B lasers distinguishing them from class-A lasers is that they readily exhibit relaxation oscillations leading themselves to modulations. This is why under moderate strength of modulation the dynamical response of such lasers becomes strongly nonlinear, and the lasers display a rich variety of nonlinear phenomena. In order to consider the more general case, the standard Lorenz-Haken model, based on a two-level laser system with a symmetric Lorentzian lasers gain profile, was modified by introducing two additional parameters describing the effects of the asymmetry of the gain profile (represented by the asymmetry parameter α) and the inhomogeneous broadening (represented by the detuning parameter θ) in the Lorenz-Haken equations [18]. The resulting set of equations is named the generalized Lorenz-Haken model.In this paper we study the effects of the additional parameters on the dynamical behaviors of the laser system and on the laser instability-threshold (the second laser threshold).

Theoretical Considerations

The numerical studies of dynamical behaviors of the semiconductor laser system have been obtained using the following generalized Lorenz-Haken rate equations for the amplitudes of electric field (X), atomic polarization (Y), and population inversion (Z) [18]:

$$\dot{X} = \sigma (Y - X) \qquad \ldots (1)$$

$$\dot{Y} = - (1 + i\,\theta) [Y - (1 - i\,\alpha) X Z] \qquad \ldots (2)$$

$$\dot{Z} = b (r - Z) - \mathrm{Re} (X^{*} Y) \qquad \ldots (3)$$

Where $\sigma = k / \gamma_\perp$, $b = \gamma_{\shortparallel} / \gamma_\perp$ and r is the pump parameter. α and θ are new control parameters, where α is the linewidth enhancement factor which controls the coupling between the amplitude and the phase variations (it is related to the medium refractive index) and θ controls the inhomogeneous broadening of the resonance. Re and (*) in Eq. (3) are denoting the real part and the complex conjugate, respectively.

Results and Discussion

We have numerically solved the Lorenz-Haken equations (1)-(3) for selected values of the dynamical system control parameters using the fourth-order Runge-Kutta method.

Let us start with the case of two-level resonant laser system (when $\alpha = \theta = 0$), where the generalized Lorenz-Haken equations reduce to the well-known standard (or classical) Lorenz-Haken equations [9]. Here we study the effects of varying the values of the laser operating control parameters, σ , b , and r on the dynamical behaviors of the laser system.

Figs. 1 to 3 (a to c) illustrate the effects of σ on the behaviors of the laser system for different values of the pump parameter (r). The value of σ is changed over the chosen small range σ = 3.0 – 4.0 , when b = 1.0. In each figure, the left side represents the laser output intensity ($| X |^2$) as a function of time (the electric-field intensity evolution), while the right side represents the phase-space portrait (the laser output intensity ($| X |^2$) against the population inversion) corresponding to the time intensity series in the left side. In Fig.1 , when σ = 3.0 , the relaxation oscillations toward a steady-state behavior seen in Figs.1(a) and (b) develop to a chaotic behavior in (c) when the pump parameter (r) increases gradually over the range r = 16 – 21. We find that the behavior of the laser system becomes completely chaotic when the pump parameter increases to approximately 21 (where this value represents the second laser

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threshold ($r_{th.2}$) or the laser instability threshold). As the value of σ increases to 3.5 (Fig. 2), the second threshold (at which instability starts to appear) significantly drops down, where the pump parameter reduces to 18. The damped oscillations behavior in Figs.2 (a) and (b) convert to a chaotic behavior in Fig.2 (c). When the value of σ is further increased ($\sigma = 4.0$) the instability threshold reduces more further, here $r_{th.2} = r \cong 16.5$, as shown in Fig. 3. The phase – space portraits corresponding to the time series plots in Figs.1 to 3 (a to c) are shown in the right side of these figures, where the appearance of a fixed point attractor [19], as shown in Fig. (a), is always associated with damped oscillations, while the chaotic oscillations behavior leads to the appearance of the strange attractor trajectory [19], as shown in Fig. (c). It is clearly seen that the dynamical behaviors in the three figures (Figs.1 to 3) are qualitatively similar, namely , the laser system is always driven directly into the chaotic state from the damped oscillations state and remains there when the pump parameter still increases.

Now, if we keep the parameter σ fixed at a chosen value ($\sigma = 3.0$) and vary the parameter b over the selected range b = 0.20 – 0.75 , we find (roughly) a similar behavior to that we have seen in the preceding figures (i.e., Figs.1 to 3), and this is clearly seen in Figs.4 to 6 (a to c). From these figures, we can easily find that the laser instability threshold is lower than for the case of varying the value of σ , here $r_{th.2}$ is dropped to 12.5 (Fig.4 (c)).

Let us now consider the case of including the parameters α and θ in the Lorenz-Haken equations, i.e., using the generalized Lorenz-Haken model. Let us take the case when $\alpha = \theta \neq 0$ (the detuned case) and set $\sigma = 3$ and b =1 (are fixed). Figs.7 to 10 illustrate the effect of variation $\alpha = \theta$ on the dynamical behaviors of the laser system. In Fig.7, when $\alpha = \theta = 0.5$, the behavior changes from damped oscillations to chaos when the pump parameter increases from r = 14.0 (Fig.7 (a)) to $r = r_{th.2} \cong 16.3$ (Fig. 7 (c)), similar in its features to that seen in the previous figures (the resonant case, $\alpha = \theta = 0$). Here, the laser instability starts to appear at $r_{th.2} = r \cong 16.3$. When we continue to increase the pump parameter (r), over the chosen range r = 27.0-35.0., the laser system starts to display a completely different sequence of behaviors and this is clearly evident in Figs.7 (d to f). In Fig.7 (d) we notice periodic oscillations behaviors of period six when r = 27.0 , changes to period – three oscillations when r = 3.0 , shown in Fig.7 (e). When r increases further to 35.0 , the laser system driven again to the chaotic regime and a fully chaotic behavior appears, as shown in Fig.7 (f). The phase-space portraits (attractor trajectories) corresponding to the intensity evolution plots in Fig.7 are shown in the right side of this figure. As $\alpha = \theta$ increases to 0.75 , the dynamical system starts to display a different sequence of behavior transitions as shown in Fig.8. Here, the damped oscillations behavior is directly changed to periodic oscillations of period two (Fig.8 (c)), when $r_{th.2} = r \cong 13.25$, and evolves towards period four and period twelve oscillations (Figs.8 (d) and (e)), and eventually approaches to chaotic behavior when = 25.0 , as shown in Fig.8 (f). We notice that our system follows the well-known universal route to chaos, the so-called period-doubling route (or Feigenbaum scenario [20]).

When the value of $\alpha = \theta$ increases to 1.0 , the dynamical behavior begins to follow the sequence transitions seen in Fig.9 , as r varies over the range r = 8.0 - 65.0 . In this case the damped oscillations (Figs.9 (a) and (b)), change to stable period-one oscillations, as shown in Fig.9 (c), then they change to period-two and period-four oscillations, as shown in Figs.9 (d) and (e). Increasing the value of r to 35.0 leads to a reverse sequence of transitions (period – doubling bifurcation), where the period – two oscillations (when r = 35.0) back to period – one oscillations (when r = 65.0), as shown in Figs.9 (f) and (g) , respectively. The corresponding attractor trajectories to the laser intensity evolution are shown in the right side of Fig.9. These are : fixed point attractors ((a) and (b)), single – limit cycle ((c)), two limit cycles ((d)) , four limit cycles ((e)), and again two limit cycles and single limit cycle ((f) and (g)) , respectively.

As $\alpha = \theta$ increases to 2.0 , the sequence of behavior transitions changes as illustrated in Fig.10 . The damped oscillations in Figs. (a) and (b) convert to stable period –one oscillations, when r varies to 5.1.

It is interesting to note here that the behavior of the laser system remains unchanged, namely that the laser system persists to exhibit stable periodic oscillations of period – one over the whole range of increasing pump parameter (r = 5.1 – 30.0) and the corresponding attractor trajectory (orbit) always shows single – limit cycle (single orbit). Such a case of behavior plays an important role in the operation of laser system because over

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this range of the selected values for the control parameters (the laser operating conditions) we are able (easily and nicely) to control the laser system simply by varying the laser operating control parameters.

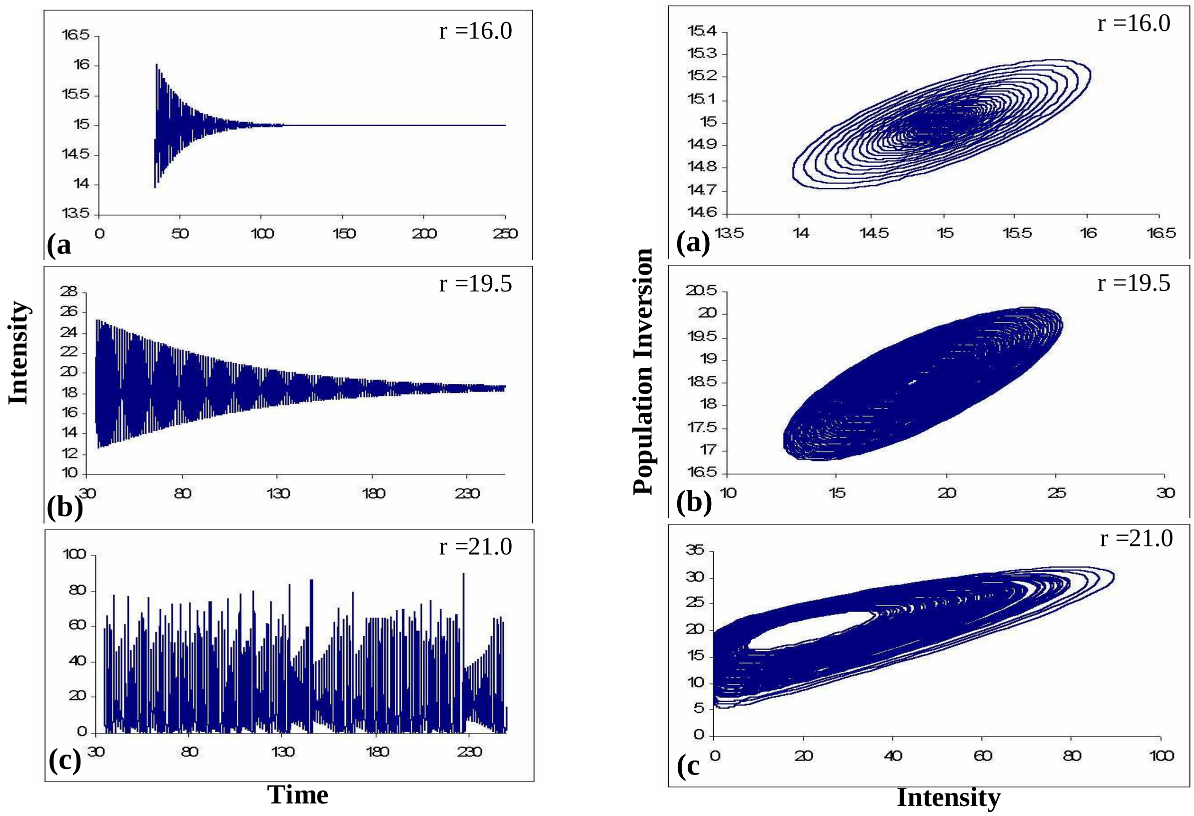

Fig.1. Left side : Intensity time series (normalized laser field intensity versus normalized time) at α = θ = 0 , b =1.0 , and σ = 3.0 for different values of r . Right side : The corresponding phase – space portraits (laser field intensity against population inversion).

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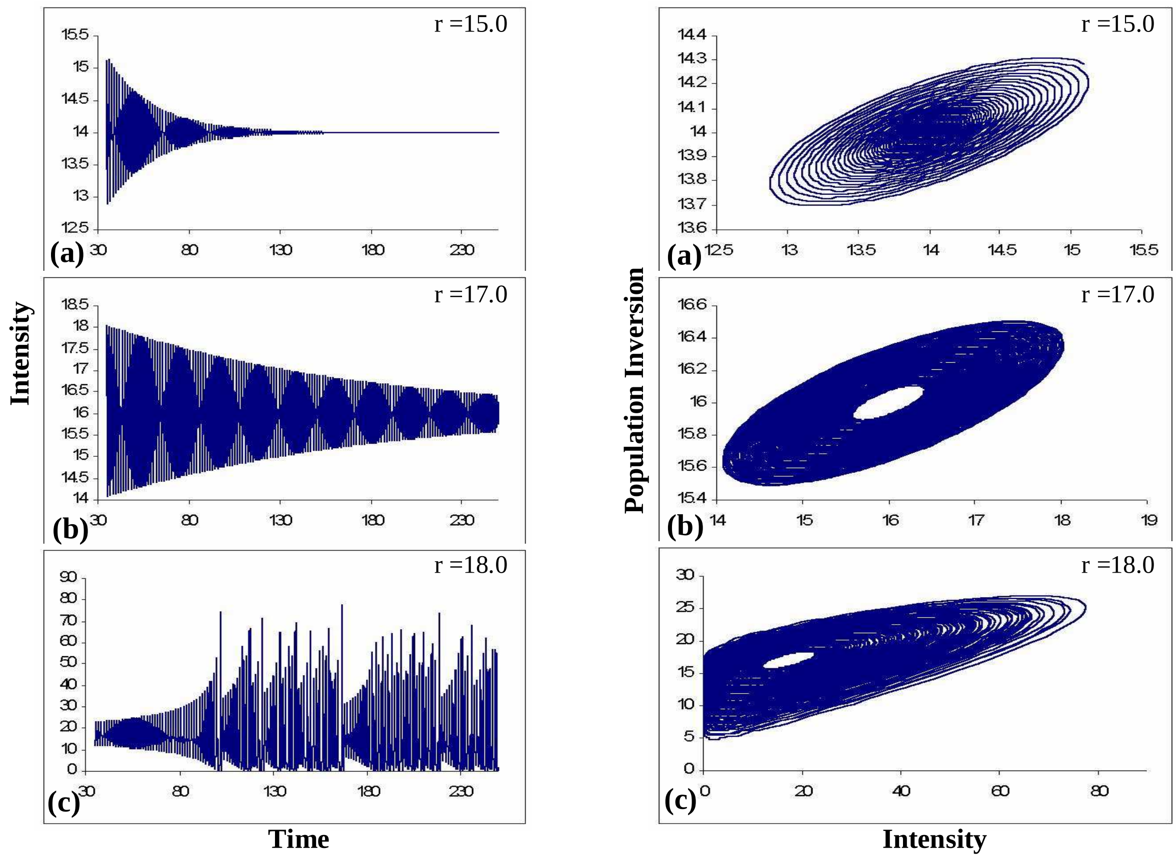

Fig.2. Same as Fig.1, but with σ = 3.5 .

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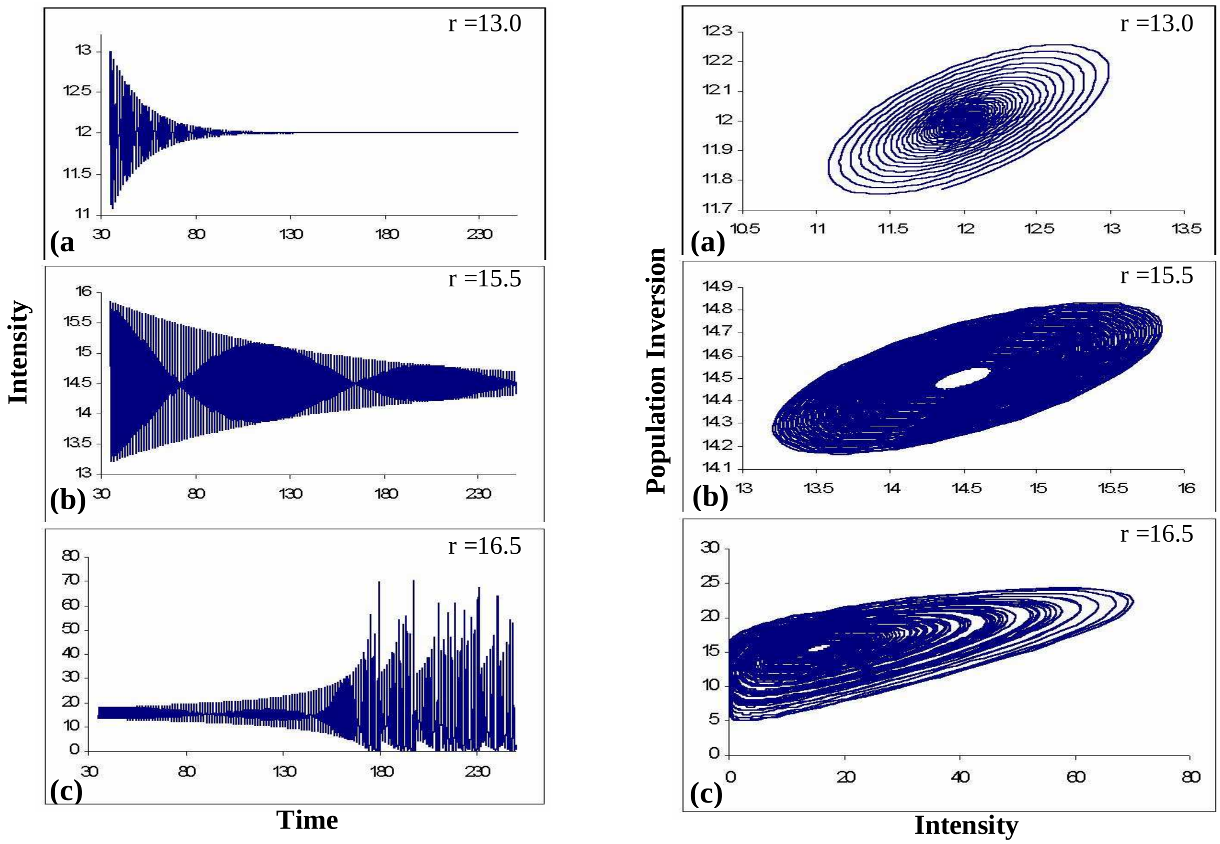

Fig.3. Same as Fig.1, but with σ = 4.0 .

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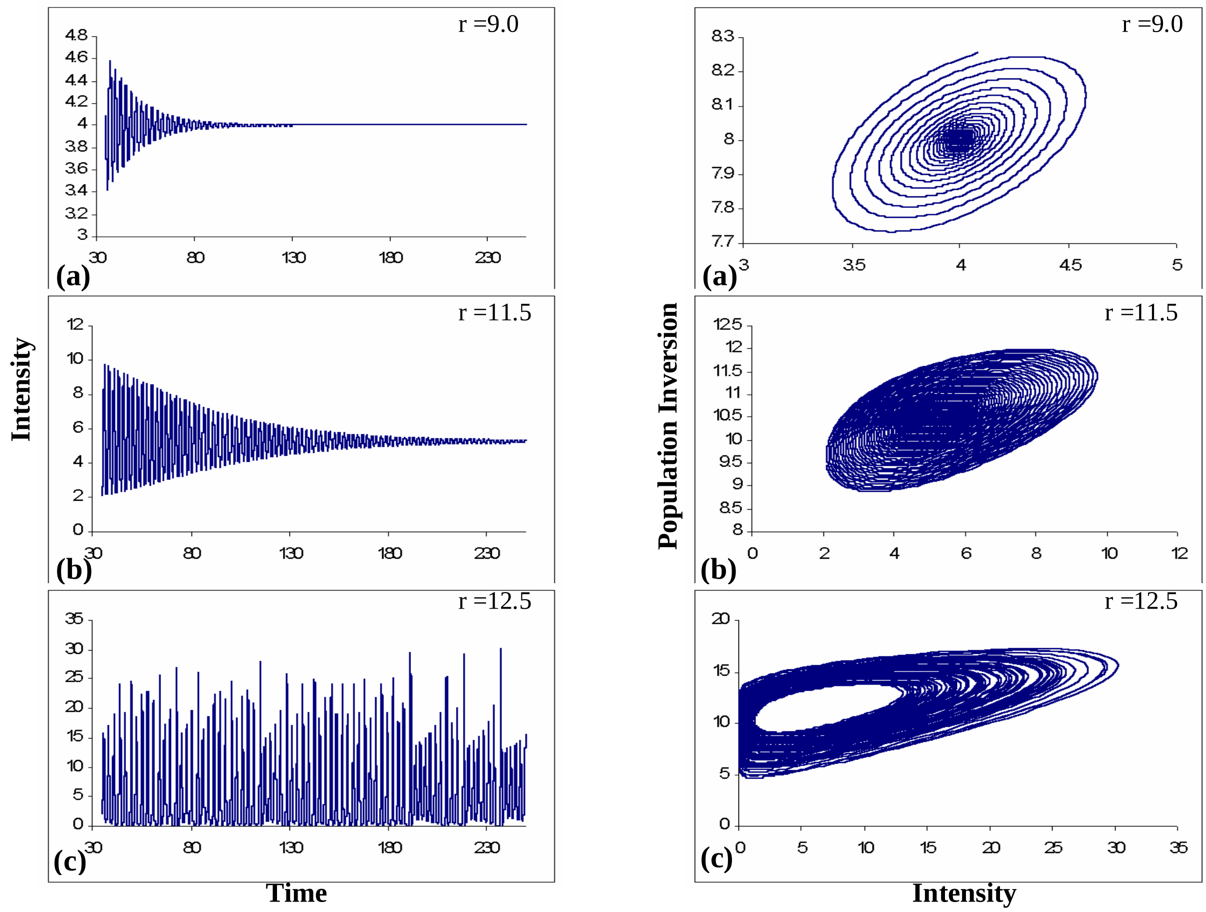

Fig.4. Same as Fig.1, but with b = 0.20 .

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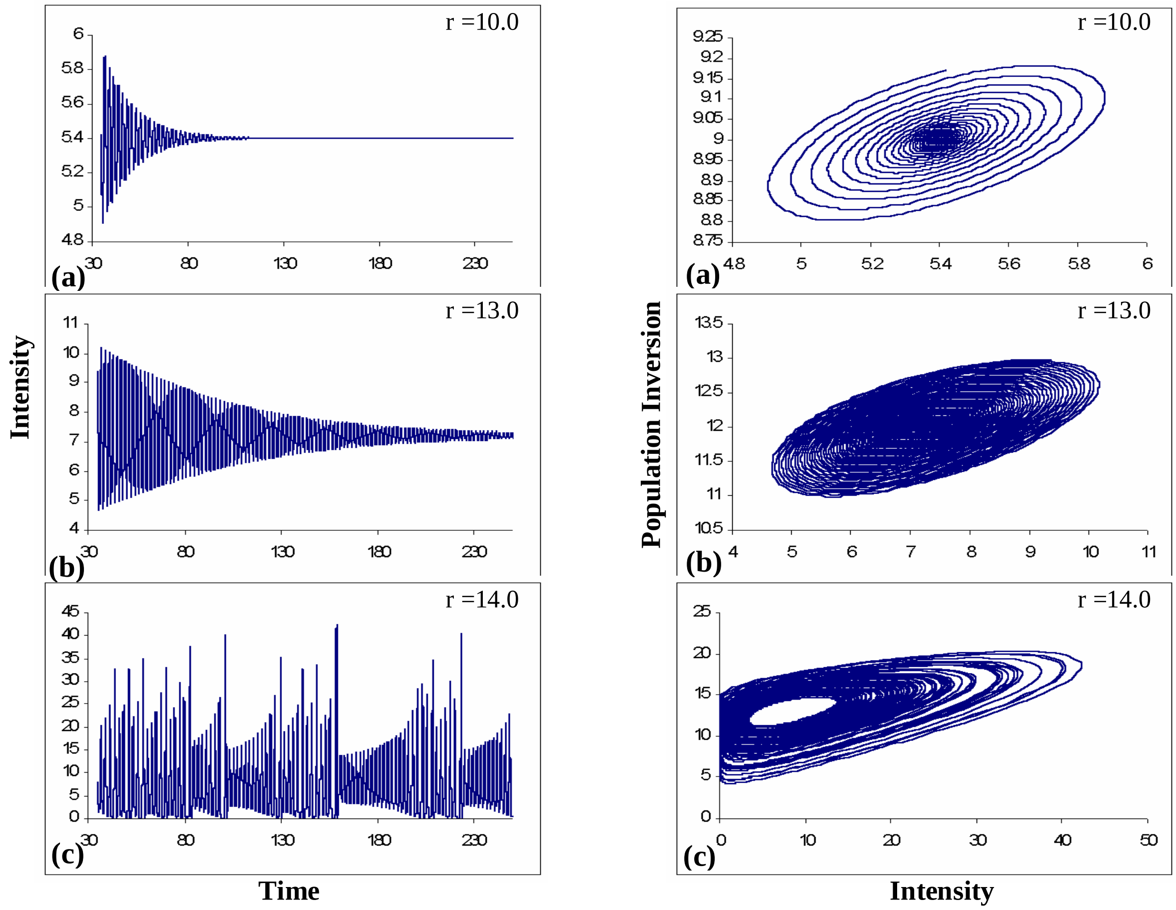

Fig.5. Same as Fig.1, but with b = 0.50 .

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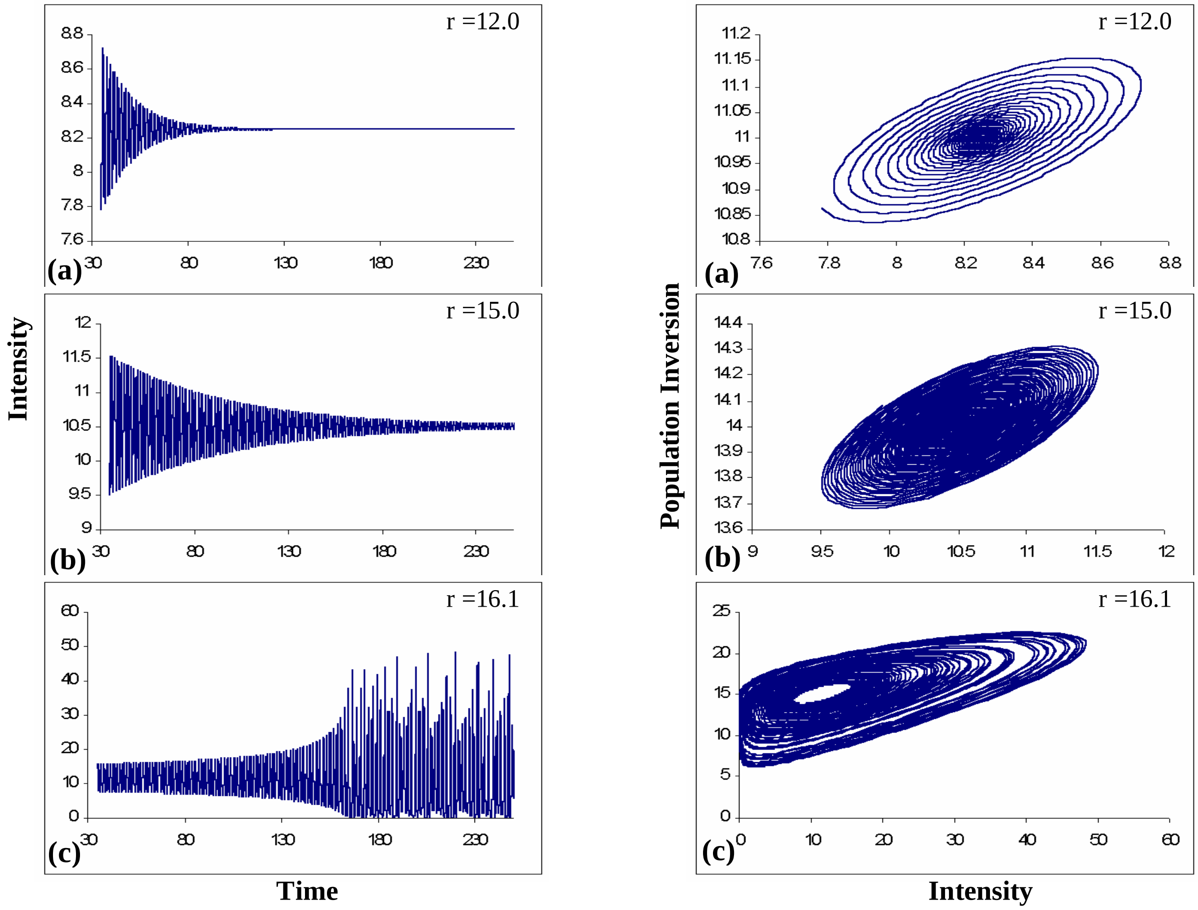

Fig.6. Same as Fig.1, but with b = 0.75 .

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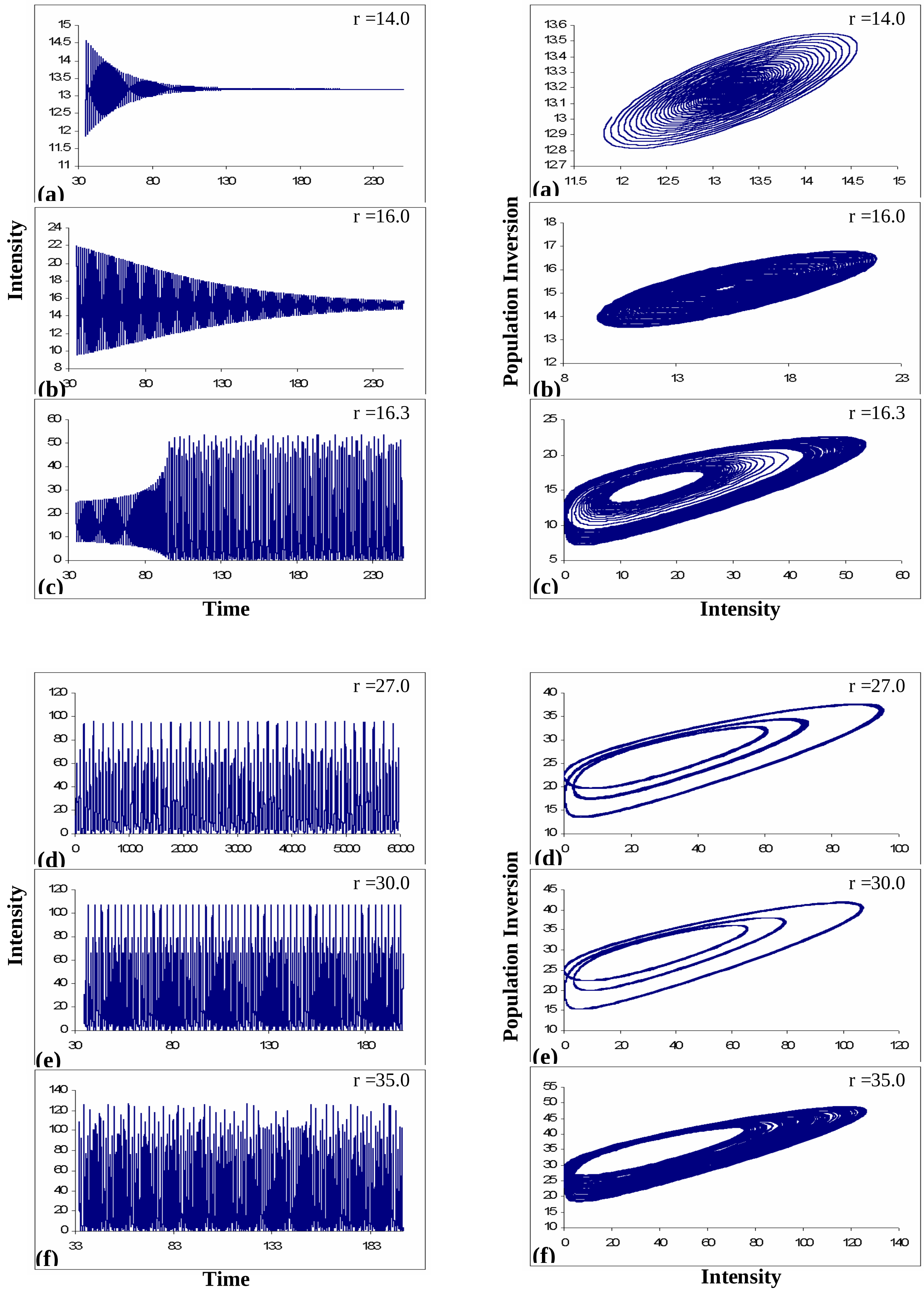

Fig.7. Left side : Intensity time series (normalized laser field intensity versus normalized time) at α = θ = 0.5 , b =1.0 , and σ = 3.0 for different values of r . Right side : The corresponding phase – space portraits (laser field intensity against population inversion).

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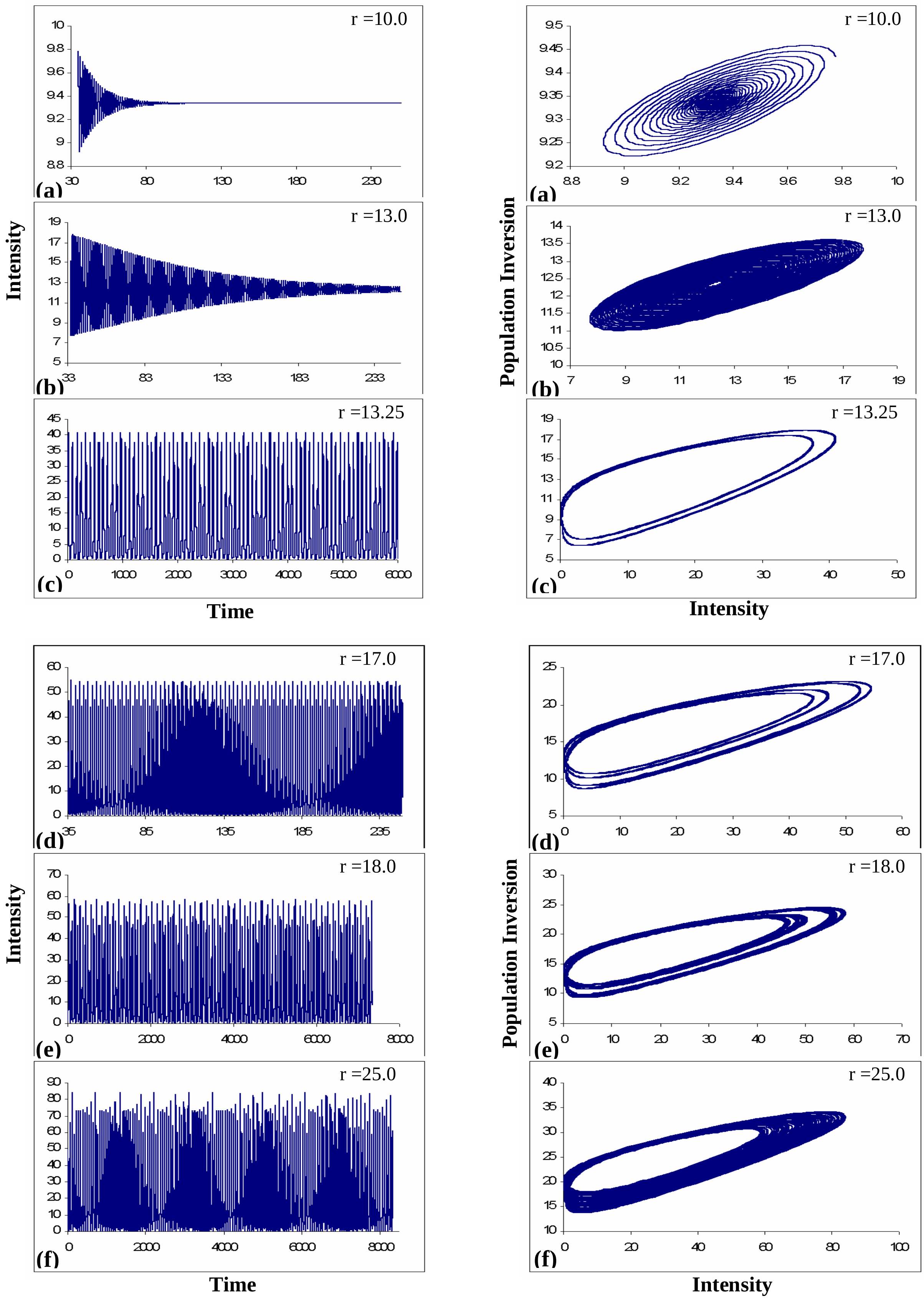

Fig.8. Same as Fig.7, but with $\alpha = \theta = 0.75$.

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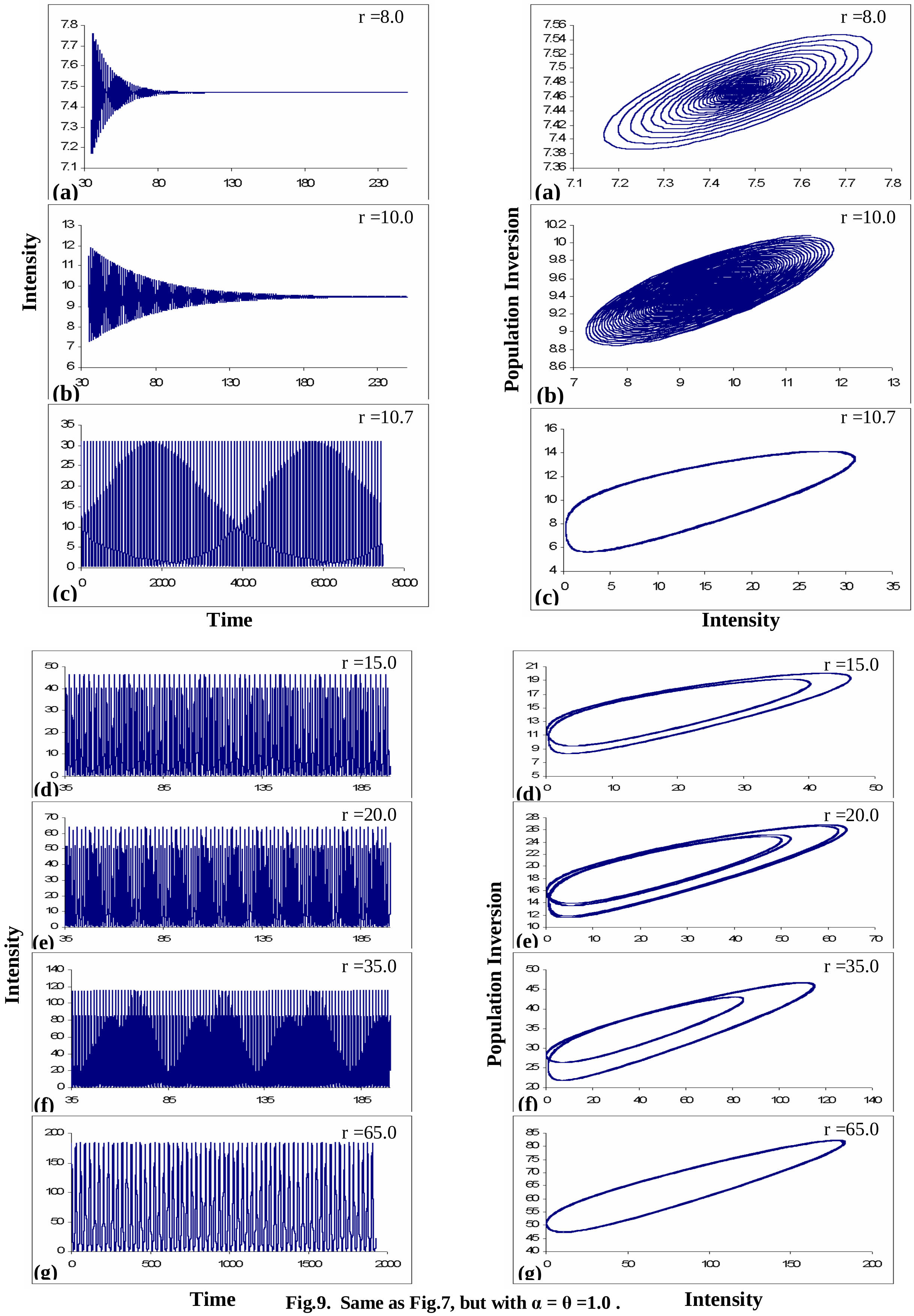

Fig.9. Same as Fig.7, but with $\alpha = \theta = 1.0$.

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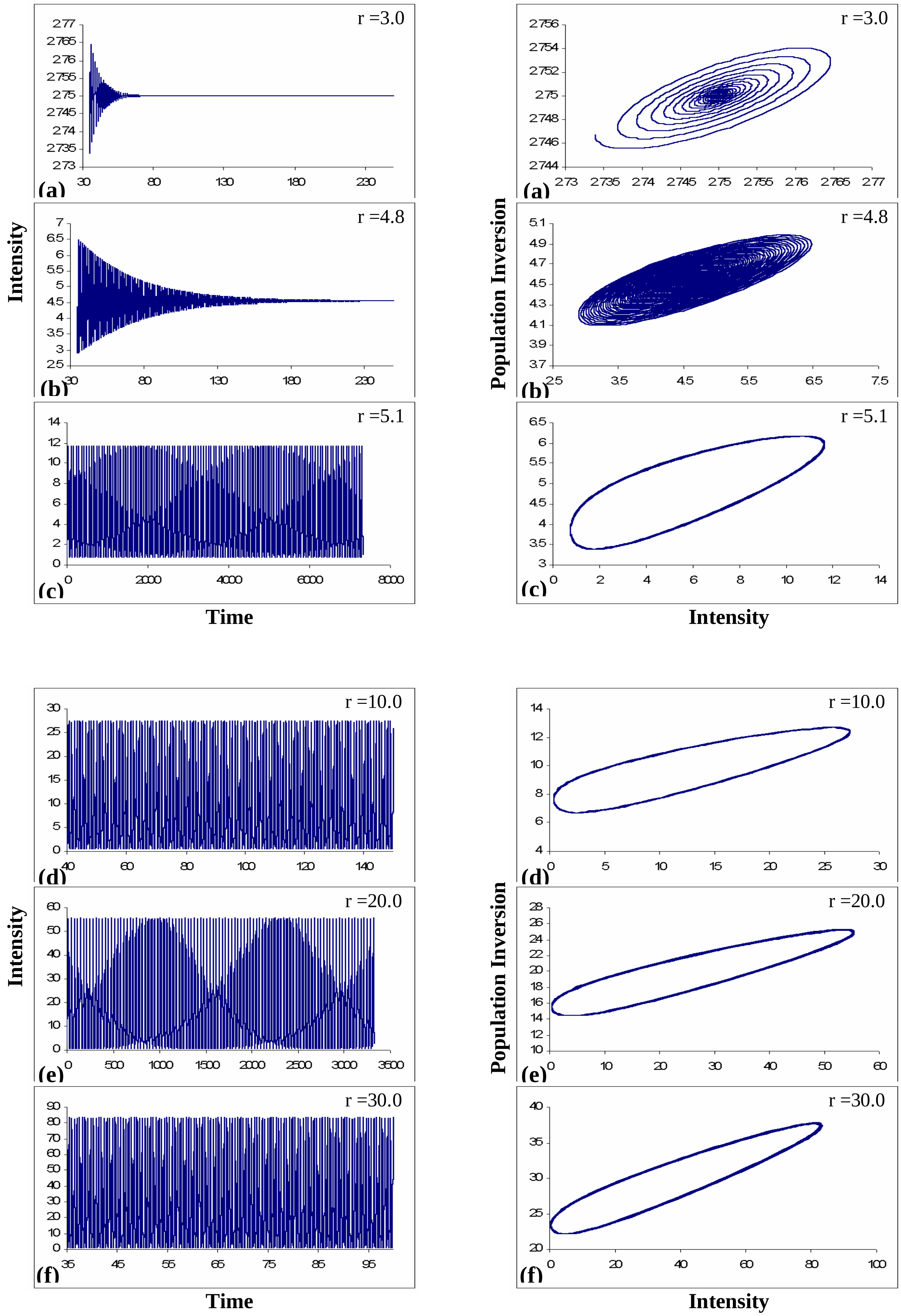

Fig.10. Same as Fig.7, but with $\alpha = \theta = 2.0$.

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Conclusions

We have investigated the effects of the main laser control parameters on the dynamical behavior of the laser system. We have found that including the asymmetry of the medium gain parameter (α **)** and the detuning parameter (θ) in the classical (standard) Lorenz – Haken equations leads to a completely different dynamical behavior to that obtained by using the classical Lorenz – Haken equations. In the case of resonant laser system (when $\alpha = \theta = 0$), we find that the laser system is always driven into the chaotic regime even with a high excitation level
(i.e., strong pump power), while the variation of the pump parameter leads to periodic oscillations (in addition to the chaotic oscillations) when the parameters α and θ are taken into account. Perhaps the most interesting and important case of the several cases of our study is the case shown in Fig.10 , where the dynamical laser system exhibits only stable periodic oscillations of period – one as the pump power increases. In this case the laser system becomes highly stable making the output of the laser easily (and nicely) controllable only by choosing and adjusting the suitable values of the laser operating control parameters. This is practically important in the laser applications and in the study of the properties of the nonlinear media where high stability is required.

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دورة عتبة اللااستقرارية في منظومة ليزر لورنس – هاكن

عماد الدين حسين السعيدي و فرات احمد السيمري
قسم الفيزياء ، كلية التربية ، جامعة البصرة

الخلاصة

قمنا بدراسة دورة عتبة – اللااستقرارية (العتبة – الثانية) على السلوك (الديناميكي) لمنظومة ليزر لورنس – هاكن . نقدم هنا نتائج نظرية للااستقرارية المؤدية الى الفوضى عند عتبة استقرارية منخفضة ، تم الحصول عليها من خلال تغير عوامل تحكم منظومة الليزر على مدى واسع لظروف عمل الليزر .

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