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Friction Detection and Modeling for Damped Pendulum Motion using Image Processing Technique

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Abstract

Damped oscillatory motion is one of the most commonly studied movements in physics. In the present work the consideration of a system consisting of a simple pendulum, which is free to rotate, is shown including the damped oscillations, with the amplitude reducing with time till the motion ends.

The estimation of the damping oscillatory motion for four different mass are presented by using image processing through filtering the top of peaks, the angles bigger than 6 degree are atomically neglected thereafter, Finding a method to solve the harmonic differential equation that achieves mass movement. The result of experimental study shows that the observed motion has a good agreement with the predicted equations of motion. After that the effect of the weight on the hanging point (friction) is calculated. In general, the friction of the axis can be play a significant role on the damping of the pendulum depending on ball mass.

Keywords: Simple Pendulum, Friction.

الخلاصـة

الحركة التذبذبية المتضائلة هي واحدة من الحركات الأكثر شيوعًا في الفيزياء. في العمل الحالي، اخذ بالاعتبار نظام يتكون من بندول بسيط ، وهو حر الحركة و يتضمن التذبذبات المتضائلة مع السعة التي تقل مع مرور الوقت حتى تنتهي الحركة.

تم حساب الحركة التذبذبية المتضائلة لأربعة كتل مختلفة باستخدام معالجة الصور من خلال تصفية اعلى القمم، وإهمال الزاوية الأكبر من ٦ بشكلٍ اتوماتيكي وتقديم طريقة لحل المعادلة التفاضلية التوافقية التي تحقق الحركة. تظهر نتيجة الدراسة التجريبية أن الحركة الملاحظة لها اتفاق جيد مع معادلات الحركة المتوقعة. بعد ذلك يتم احتساب تأثير الوزن على نقطة التعلق (الاحتكاك) و بشكل عام ، يمكن أن يلعب احتكاك المحور دورًا مهمًا في تثبيط البندول اعتمادًا على كتلة الكرة.

1. Introduction

Friction is the tangential reaction force between two surfaces in contact, and where the complexities of friction give a models of oscillator damping focus on a single process [1]. The study of friction has played an important role in the scientific community for centuries. However, despite of its universal character, friction forces still represent a challenge in the quantitative analysis of many systems. For example, when its highly effected on the motion of an harmonic oscillators that moved in fluid(liquid or gas). Even with several different approaches, such as numerical ones[2] and heuristic arguments[3]. An analytical simple solution to the friction is still missing as demonstrated by the amount of suggestions to solve it [4]. Although friction, which dissipates energy in a system, would seem to be a stabilizing force, it has been shown that underestimating the magnitude of the friction coefficient may sometimes lead to instability in a advice system also generated limit cycles that can be located in the simulations of a simple pendulum system [5,6]. One of important application of determined friction is able to measure the gravitational acceleration very accurately, Nelson and Olsson in 1986 theoretically investigated the effect of air resistance on the bob as well as the string of a simple pendulum [7]. However, they did not discuss the relative importance of these effects. In (2012)Dunn he experimentally studied the damping effect of the string on a pendulum by varying the length of the string and the diameter of the ball and suggested that further investigation was necessary [8]. In (2015) Agarana M. C. and Iyase S. they inspected simple pendulum dynamics by putting damping into consideration.

The angular displacement of a damped pendulum with small displacement is investigates on the basis of Hermite's form of main equation of a damped simple pendulum [9]. In (2017)Pirooz Mohazzabi and Siva P. Shankar they investigated the effect of string on damping of a simple pendulum and experimentally studied the contribution to the damping of a simple pendulum from strings of various diameters [10].

The purpose of this work is to form a simulation model that is able to understand the motion of a ball which is oscillates in a simple pendulum. Where have been used massless stick instead of string. To determine the damping movement well, test requires many mass. So we will test different balls of mass(37,96,130 and300)gm , studied the effect of pivot friction on the damping and build an empirical model for the motion. Firstly, the theoretical models going to be studied using physical equations of pendulum motion. These equations of the pendulum motion are going to introduced damping pendulum motion model. After that, have been determine the specific coefficients and find the fraction of the pivot point.

2. Damped Pendulum Motion

The description of the real pendulum motion must be take into account the damping effects on motion. The major source of damping comes from the activate joint which is unable to give all the energy back during its bending motion. During the time, the motion is quite complicated and depends on the initial conditions and the phase of the applied force. Because of the damping, the temporary behavior eventually dies away [11].

2.1 Harmonic Damped Motion

Damping is any effect that tends to reduce the amplitude of the oscillations in an oscillatory system. In a vacuum with zero air resistance, such a pendulum will remain to oscillate for an indefinite period with constant amplitude. However, the amplitude of a simple pendulum oscillating in air always decreases as its mechanical energy is slowly lost due to air it is generally perceived that the main role in the intemperance of mechanical energy is played by the ball of the pendulum, and the influence of friction resistance of the pivot point [10].

So in this work, the effect of air resistance, friction of hanging point and their effect on the motion of pendulum ball have been taken in consideration.

2.2 Pendulum Motion and Friction

The simple pendulum is a traditional mechanical system. Usually it consists of a point mass attached to one end of a massless string, and the other end of the string is suspended from a fixed point. In the normal setting, a simple pendulum is excited at the point mass, and when damping is an issue, resistive forces acting on the point mass ,such as air drag, are considered [12]. Generally in mechanics, friction is represent one type of damping effect and all objects suffer frictions when they are in contact to each other. These contacts create damping in the system. Fig (1) shows a simplified view of the simple pendulum system that have been studies [13].

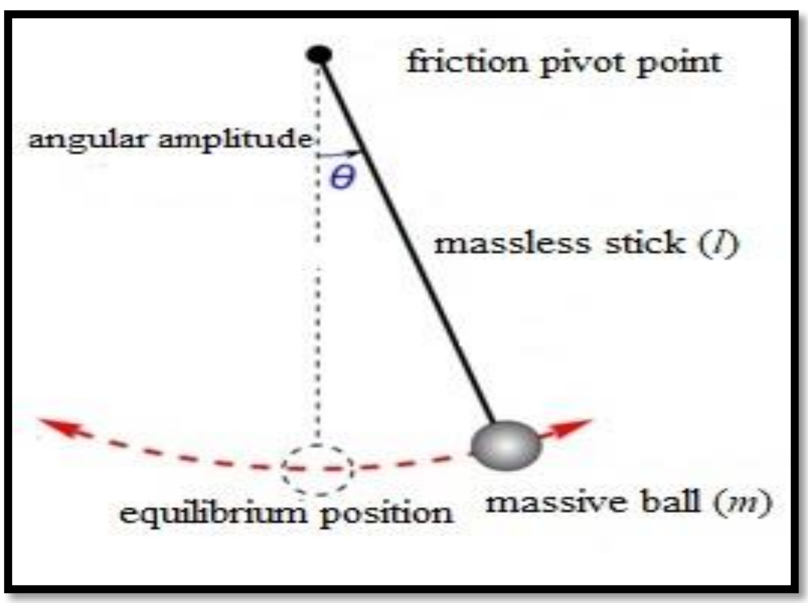

Figure 1: Simple pendulum

According to Newton's second law, the motion of a simple pendulum is [14]

$$\boldsymbol{m\ddot{\theta} = mg \sin\theta} \qquad \textbf{(1)}$$

Where $\ddot{\theta}$ represent the angular acceleration, m is mass of the ball, g is acceleration due to gravity and θ is the angular displacement.

By using the approximation $\sin\theta \approx \theta$ where $(\theta < 6^o)$, which leads to simple harmonic motion with angular frequency $w = \sqrt{\frac{g}{l}}$ where l is length of the massless rod. The equation for Simple Harmonic Oscillations (S.H.O.) with no damping and for small values of θ is approximated by [15]:

$$\boldsymbol{\theta = \theta_o \ \sin(wt)} \qquad \textbf{(2)}$$

While the equation which describes the damped S.H.O for pendulum, is[15]:

$$\boldsymbol{\theta_{(t)} = \theta_o\, e^{\lambda t} \sin(wt + \emptyset)} \qquad \textbf{(3)}$$

Where $\theta_{(t)}$ pendulum angle at time (t), θ_o is the initial angular amplitude at (t = 0), λ is the friction factor that equal to $(\frac{b}{2m})$ where b is the damping factor, *w* is the angular frequency and $\emptyset$ the phase difference. In this study, the digital image processing used to track the pendulum ball motion as a function of time. Then estimate equation of motion and determine the best empirical model

3. Determine the Damping Effect

In this work have been used a camera of (iPhone7) for recording the simple pendulum motion back and forth where the extract motion frames for only S.H.M. that started when larger angular amplitude of pendulum is ($\theta_0 < 6^0$). The distance between the camera and the pendulum is 150 cm. Where have been used four different balls of mass (37,96,130 and 300gm) the length of the rod from the center of a ball to the hanging point is 10 cm almost.

3.1 System Setting

Fig. (2) shows The arrangement of pendulum (S.H.O.) to calculate the damping factor.

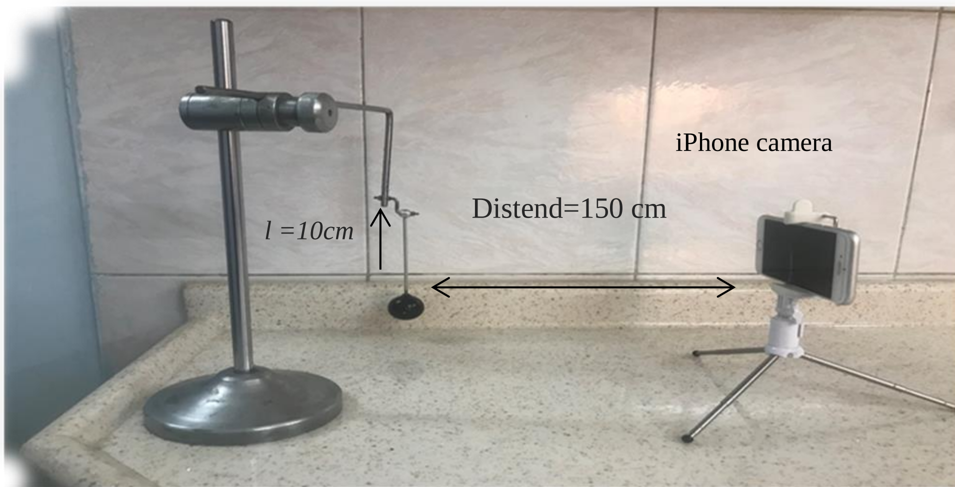

Figure 2 :The organization of the pendulum system

3.2 Recording Videos for Pendulum Motion

Four video clips data where recorded for different pendulum motion cases in the air by using different mass balls. Where the pendulum ball was removed at an angle θ larger than $6^0$and left to move continuously until angular speed reached to zero (i.e. stopped).

3.3 Pendulum Motion Detection in Videos

The motion of pendulum was analyzed by converting four video clips into still image frames as a function of time then extract only the part of pendulum motion (important region) from image frame by building an algorithm through MATLAB program see Fig.(3)

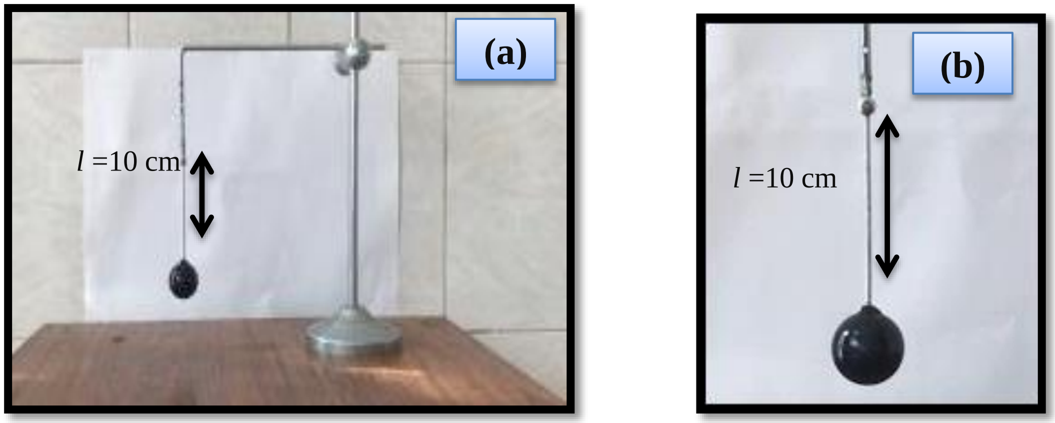

Figure 3: (a) Video frame image and (b) The extracted only the important region from video frame images.

For the extracted images, have been determined the location of the ball center in image plane (x, y) as a function of time and have been used the trigonometric functions to estimate the angle between vertical line and pendulum stick in each image frame (i.e. determine pendulum angle θ). Here the relationship between angle (θ)and frame time was plotted see Fig. (4a) by using the following equations [16] .

$$\boldsymbol{r \approx \sqrt{(x_0 - x_1)^2 + (y_{0-}y_1)^2}} \quad \textbf{(4)}$$

Where the angular displacement (θ) given by using the following equation[16]

$$\boldsymbol{\theta_{(t)} = \frac{a\sin(x_1 - x_0)}{r}} \quad \textbf{(5)}$$

From Fig. (4) have been observed, where the figure contains two parts, that the curve of motion is decreasing with increasing the time of motion. The simple harmonic motion (S.H.M.) considered at a condition $0^0 \leq \theta \leq 6^0$ for this work, it means that the signal out of this range is cut off automatically as shown in fig. (4b). It is necessary to use median filter to smooth the signal to find the peaks of the pendulum easily. The processing for performing this demonstrated in logarithm(1).

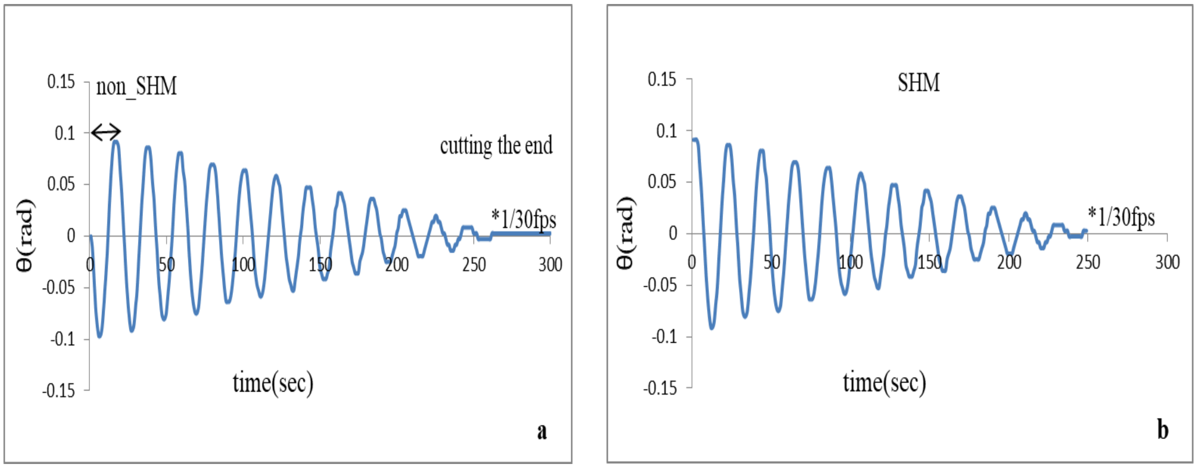

Figure 4: shows (a) The cutting the angle larger than 6^0 and cut off for the end of motion and (b) simple harmonic motion for pendulum ball after cutting the end of still image

Algorithm (1) calculate the amplitude for oscillating angle in pendulum

1. Determine the pendulum radius using MTLAB Statement (M.S).
2. Determine the invers of frame rate $t_{(1)} = {}^{1}/_{f_t}$
3. Find the no. of frames in the video (vid) using M.S. lv= vid no. of frames
4. Extract frame of ended frame from video clip (vid) using M.S. End frame =read (vid, lv)
5. Extracting the important region of pendulum motion in the image plan manually using M.S. [x, y, I, rect] = imcrop (end frame) \\ where (x, y) represent the vector location of the tow points (x_1, y_1) and (x_2, y_2) .I= extract the small part image from the frame image
6. Manually determine the suspended pendulum point (x_o, y_o)
7. Find the location and radius of pendulum ball in image plan using M.S.[cxy, radii] = imfindcircles(I1,rd,'ObjectPolarity','dark') .
8. Find the coordinate of the pendulum center ball in image plane (x_1,y_1) using x1=round(cxy(1)); and y1=round(cxy(2));
9. Compute the radius (r) of pendulum rotation motion using r=round(sqrt((x0-x1)^2+(y0-y1)^2));
10. Put idx(vlu) =vlu\\idx frame index matrix
11. Put tht(vlu)= $\sin^{-1}$((x2-x0)/r);\\ the amplitude of the pendulum motion

12. Set m= vlu +1\\the total no. of frames will be processed to estimate the pendulum motion .
13. For i= 1 to vlu
14. Sit ii= lv-i+1\\frame index will be extracted from vid ,mi=m-1\\, j=j+1\\
15. Extract frame of index (ii)using M.S, frm=read (vid, ii)
16. Extract the same important region of pendulum motion in image plane as in step (5) using M.S.

    I=imcrop (frm, [x, y], rect)
17. Find the location and radius of the pendulum ball in the frame of index (ii)using M.S.

    [cxy, radii] = imfindcircles(I2,rd,'ObjectPolarity','dark');
18. Find the coordinate of the center of pendulum ball (x2,y2)in image plane using x2=round(cxy(1)); and y2=round(cxy(2));
19. Compute oscillating angle of the frame of index ii using , tht(mi)=$\sin^{-1}$((x2-x0)/r);
20. Find peaks of the oscillating angle tht(3) as a faction of time using

    [pk1,lc1] = findpeaks(tht3)\\ top peaks, pk1= peak value and lc1= peak number
21. Find peaks of the oscillating angle tht(1) as a faction of time using

    [pk2,lc2] = findpeaks(tht3)\\ bottom peaks,.pk2= peak value and lc2= peak number
22. Find no. of peaks (np) of tht(3) using , np=length(lc1);
23. For j=1to np, if abs(lc1(j)-lc2(j))<4, for jj=1 to 3, j1=j+jj-1; if j1<(np+1)
24. if pk2(j1)>pk1(j) , pk1(j)=pk2(j1); , end if end if ,end for jj, end for j
25. convert peak theta from radius to degree using , pkd=pk1*p180; pkd', Put sp= 0
26. For 1 to np \\ to extract only the part of the harmonic motion , if pkd(i)<6 and the sp = 0; then ci =lc1(i); sp=1;end if
27. end for i , put ssp =0, for i=1:np-1//deleted the ended peaks on motion , if pk2(i)<0.005 and ssp==0, put ssp=1; put c1=lc1(i);, put c2=vlu-1; for ii=1 to 3, for cc=c1 to c2, set tht1(cc)=(tht1(cc-1)+tht1(cc)+tht1(cc+1))/3;, end for cc, end for ii, end if, Find that of the motion different of peaks, dT=diff(lc1),Find the period of motion (T), T= median (dT), ci2=lc1(np)+T
28. if ci2>vlu then, ci2=vlu, end if
29. for i=1to np-1, if abs(dT(i)-T)>3 OR, pk1(i)<0.005 then , set cm=lc1(i)+T, if cm>vlu then

30.set cm=vlu, end if, ci2=cm;, end if , for I, set j=0, for i=ci to ci2, set j=j+1, ; thtf(j)=tht1(i);thtf is the final amplitude for oscillating angle after cutting the end of simple harmonic motion for pendulum, end.

The S.H.M. part extracted automatically using algorithm (1), and determine the S.H.M. curves see Fig.5. These utilized to determine damping factor as in the follow.

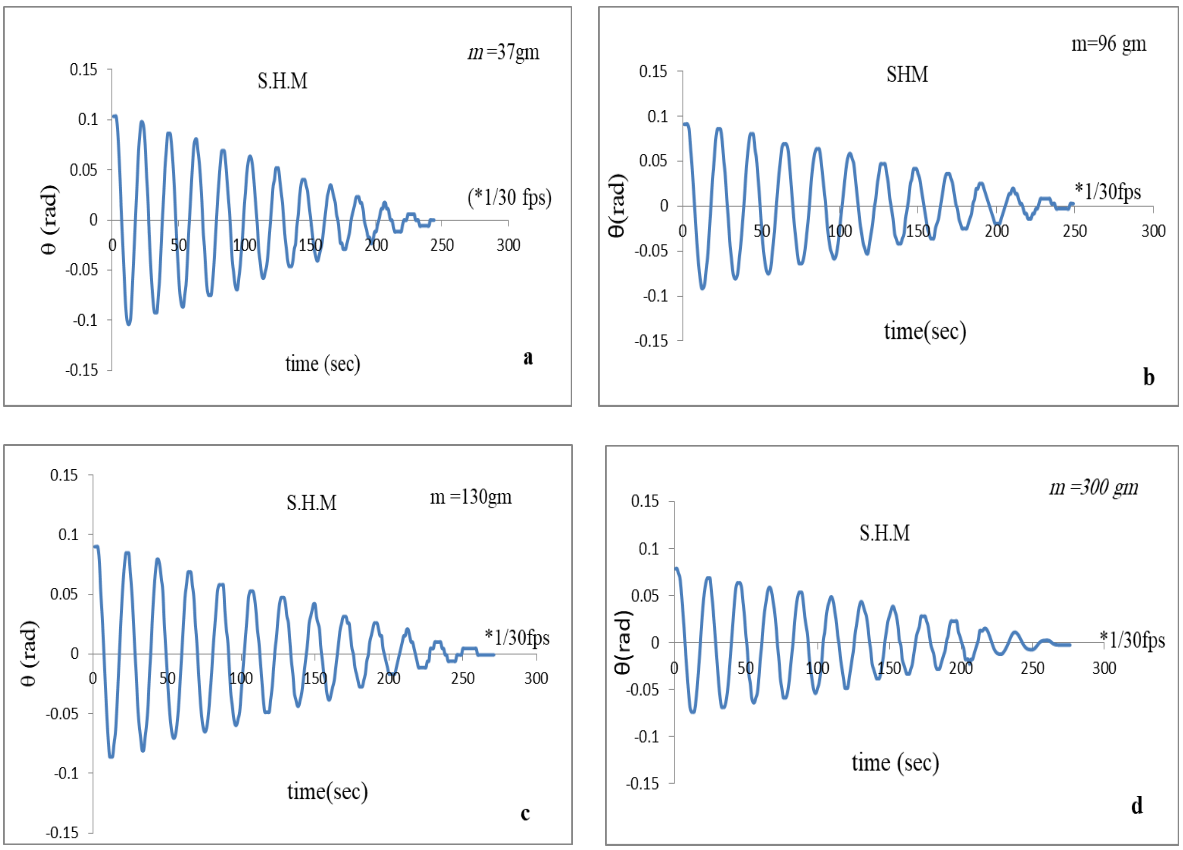

Figure 5 :Demonstrate the detected θ of damped pendulum motion for different ball masses respectively in (a, b, c and d).

Used algorithm (2) to estimate the best fitting equation, see fig (6) for each part. where have been utilized damped sinusoidal function to fit the S.H.M. curve. The damped sinusoidal function given by:

$$\boldsymbol{\theta_{(t)} = a e^{-\frac{b}{2m}t} \sin(ct + d)} \qquad \textbf{(6)}$$

Algorithm 2 (fitting pendulum ball damped motion curve) input: time index (id) , theta value of pendulum (thtf)

out put: fitting parameter (a, b, c and d)

start

1. load damped pendulum motion curve data (id, thtf)//where id represent tim index of the fram,thef represent the theta value of pendulum motion in video frame of index (id).
2. used matlab statement cftool (id,thtf),to fitting the data (id,thtf) to the damped sinusoidal function f(t)= a*exp(-b*t)*sin(c*x+d) //f(t) represent $\theta_{(t)}$ = thtf and t=id
3. save the fitting parameters (a, b, c and d)
4. end algorithm

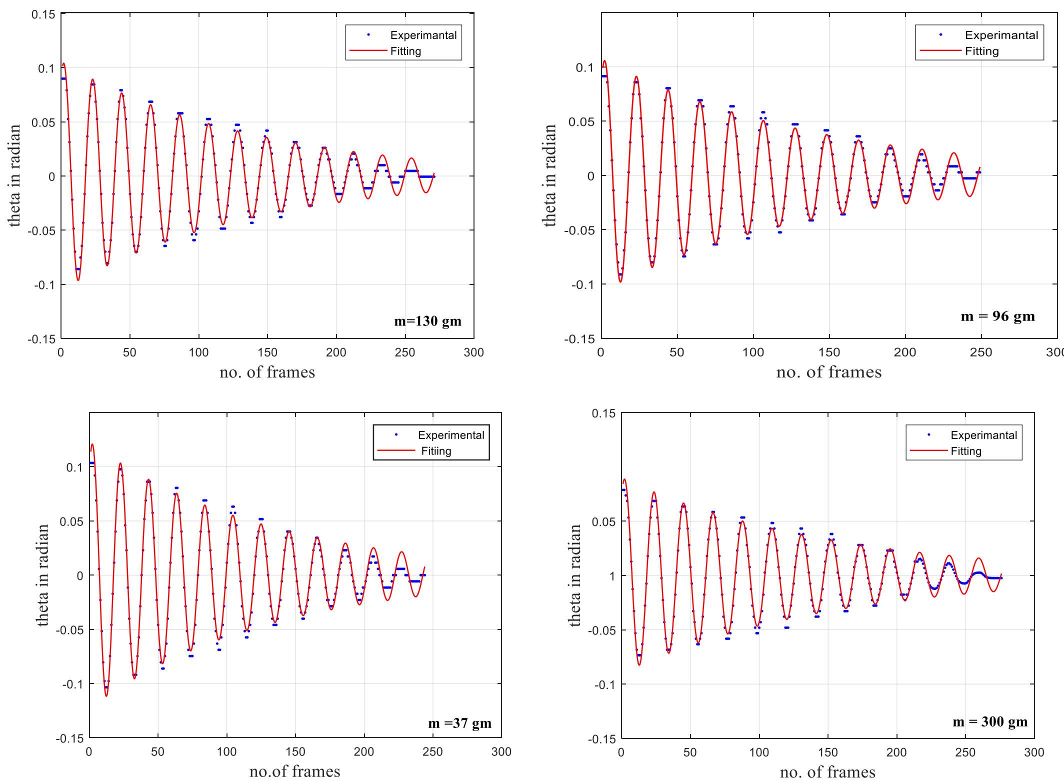

Figure 6 :Comparison of the experimental calculations of theta final with the fitting data.

where have been obtained the damping factor and other parameters, the idea of curve fitting is to find a mathematical function that fits the practical curve, by preform algorithm (2) get θ and t (frames index number) to estimate the damped pendulum motion model. The fitting parameter (a, b, c and d) are tableted in table (1) for each ball mass.

Table (1) shows the (a, b, c and d) parameters for the damping motion for the diffrent cases

Mass of balls(gm)	a	b	c	d
37	0.1229	0.007663	0.3071	0.8959
96	0.1074	0.007070	0.3011	0.8885
130	0.1030	0.006972	0.2981	0.9859
300	0.0899	0.006658	0.2930	0.6338

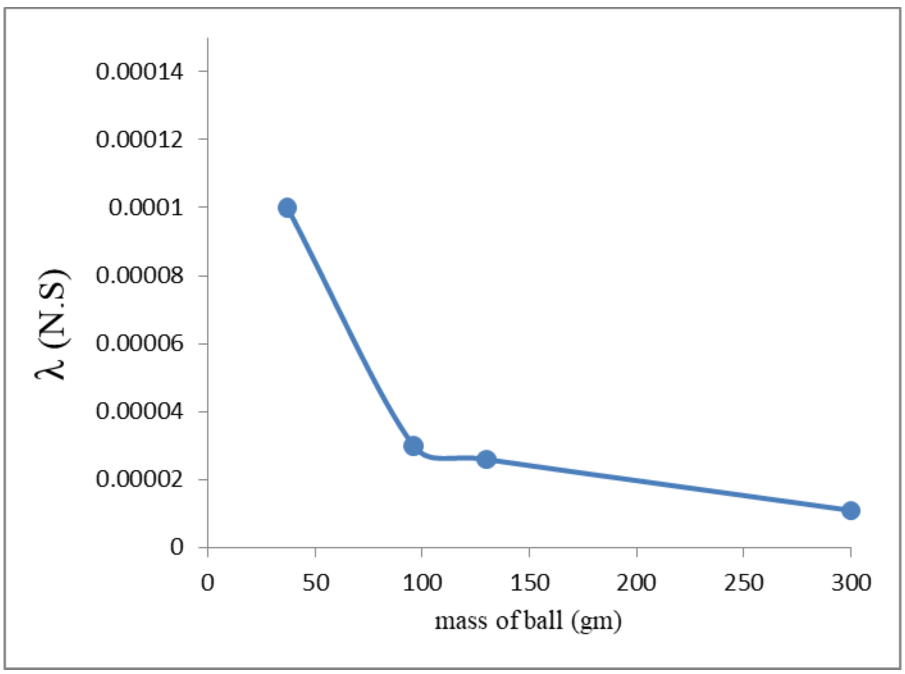

Figure 7: present the friction factor (λ) with the mass (m) of pendulum ball.

Where (b) represent the damping factor and $\lambda = (\frac{b}{2m})$ is the friction coaffsion in the hanging point it decrease as the mass of ball become increased as shown in Fig.(7). One can noted from (λ) curve the, (λ) is not constant. Its values decreases with pendulum ball mass.

The Curve Fitting MATLAB cf tool, provides functions for fitting curves and surfaces to data, supports a goodness-of-fit statistics for parametric models: The sum of squares due to error (SSE), R-square, Adjusted R-square and Root mean squared error (RMSE),these values for the 4_pendulum motion cases tableted in table (2)

Table 2: Goodness fit statistics parameter for the 4_ cases of pendulum motion.

Mass of balls(gm)	SSE	R_square	DEF	Adj_R sq	RMSE
37	0.0081	0.9836	240	0.9834	0.0058
96	0.0063	0.9845	245	0.9843	0.0051
130	0.0079	0.9803	263	0.9801	0.0055
300	0.0061	0.0982	272	0.9800	0.0047

Where can be noted the RMSE very good with the maximum value less than 0.6% for pendulum motion of mass (m = 37 gm).

4. Conclusion

From the results can be concluding that the value of damping factor is sensitive to the friction of the pivot point and effect it . Friction is inversely proportional to the mass of the pendulum, i.e. higher ball mass of pendulum give less friction and therefore less damping. Also see that effective damping motion model with error percentage less than 0.6% .

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