

## LEARN ABILITY OF SKILLED MOTI4ON FOR ROBOT SYSTEMS WITH INTERNAL MODEL

*Khulood Moosa Omran*  
*Arab Gulf Studies center / Basrah University*

---

### ABSTRACT

In this paper a simple learning control scheme using internal model for trajectory tracking of robot manipulators is presented. The proposed learning control structure consists of an internal model based controller plus a feed forward learning control for linear time varying system. The two degree of freedom control structure consists of an inner disturbance controller for the system and used an outer tracking controller. Instead of a quadratic artificial potential of joint angles, this paper introduces a new type of a quasi – natural potential in robot dynamics, which induces a type of sinusoidal position feedback with saturation in servo – loops. The proposed algorithm is used for improving the performance at the next trail on the basis of the previous operation data. Examples are given to show the effectiveness of the proposed algorithm. An analysis of the convergence proof together with simulation results are presented. The robustness of the algorithm against error in initial settings is studied through computer simulation. Simulation results show good performance for the proposed algorithm.

---

### 1. INTRODUCTION

Learning control has received attention as an alternative approach for controlling uncertain dynamic systems in a simple manner [S.Arimoto et al 1984 , Z, Qu et al 1993, C. Cheah and D.Wang 1995]. The concept of learning control differs from that of conventional classical and modern control techniques, where a control law is

implemented and fixed during operation of the system [Dutton and Conroy 1996, K. M. Omran ,1996, K.E. Avrachen kov and R.W] . In fact, actual operation data of the input and the output will never be used in direct modification of the control law itself [Longman 2003, A. Tayebi 2004, R. Kelly 2005, T.Oomen et al 2009]. In contrast, the learning control concept stands for the

repeatability of operation of a given objective system and the possibility of improvement of the control input on the basis of previous actual operation data. In this work, the internal model concept is used and it is enhanced with a learning capability. Such a scheme has the robustness property against variation in system parameters or disturbance. The internal model controller has been used in process control and it has different schemes. Fig. 1 shows a block diagram for internal model controller [S. Arimoto and T. Naniwa, 1995]. It follows from this figure that if  $G_m = G$ , then the signal  $u_e$  does not depend on the control signal  $u_r$  and it will be identical to the disturbance  $d$ , then by using a perfect compensation of disturbance is then obtained if  $G_d$  is chosen as the inverse of  $G$ . A two – degree of freedom control structure using internal model is shown in fig 2. In this structure the feedback part is used for disturbance rejection, while the feed forward controller is enhanced by adding a learning capability. A part of the feed forward signal is updated iteratively by repeating trial. It is common to use a filter in the loop as shown in fig.3. The internal model structure ensures that the trajectory in the first trial starts near the desired one and so the convergence is made faster [S.P Chan, 1995, W.E. Dixon and J.chen,2003]. As shown in fig.1,  $G$  and  $G_m$  represent the plant and internal model, respectively, further  $G_d$  and  $G_r$  are the controllers which are designed for disturbances rejection and trajectory tracking

respectively. The two degree of freedom controller is enhanced by learning capability for good trajectory tracking if  $G_d = G_m^{-1}$  then fig.2 can be obtained from fig.1 .

Fig.2 shows the equivalent block diagram for the internal model structure with learning.

Instead of a quadratic artificial potential of joint angles, this paper introduces a type of artificial potential based on functions  $\sin(\theta)$  and  $\cos(\theta)$  defined by [P. Lucibello et al 2003,]:

$$\text{Sin}(\theta) = \begin{cases} \text{Sin } \theta, & \|\theta\| < \pi/2 \\ 1, & \theta \geq \pi/2 \\ -1, & \theta \leq -\pi/2 \end{cases}$$

$$\text{Cos}(\theta) = \begin{cases} \text{Cos } \theta, & \|\theta\| < \pi/2 \\ -\theta + \pi/2, & \theta \geq \pi/2 \\ \theta + \pi/2, & \theta \leq -\pi/2 \end{cases}$$

It is quite easy to see that

$$d \cos(\theta) / d \theta = - \sin(\theta) \text{ and}$$

$1 - \cos(\theta) \geq 0$ . Hence, the feedback  $\sin(\Delta q_i)$  for residual position signal  $\Delta q_i = q_{di} - q_i$  with desired position  $q_{di}$  and actual one  $q_i$  at joint  $i$  induces an artificial potential  $1 - \cos(\Delta q_i)$ , which is non – negative and attains the minimum at  $\Delta q_i = 0$ . It is straight forward generalize this to a potential of multi-variables  $\Delta q = (\Delta q_1, \dots, \Delta q_n)^T$ , which behaves like a natural potential induced by the gravity.

## **2.LEARNING CONTROL**

The ideal principles for the concept of learning control are summarized by the following conditions [De Luca and G.Ulivi, 1992, S. Arimoto , 1990]:

b1. Every trial ends in a fixed time duration  $T > 0$ .

b2. A desired output  $y_d(t)$  is given a priori over that time duration  $t \in [0, T]$ .

b3. Repetition of the initial setting is satisfied, that is, the initial state  $q_k(0)$  of the objective system can be set the same at the beginning of each trial:

$$\text{for } k=1,2,\dots, q_k(0) = q_0$$

b4. Invariance of the system dynamics is ensured throughout repeated exercises.

b5. Every output  $y_k(t)$  can be measured and hence the residual error signal :

$$\Delta y_k(t) = y_k(t) - y_d(t) \dots(1)$$

can be utilized in construction of the next input  $w_{k+1}(t)$ .

b6. For a given desired output  $y_d(t)$  there is a unique input  $w_d(t)$  that excites the system and yields the output  $y_d(t)$ . On the basis of this framework, the problem is to find a relatively simple recursive law ( see fig.3)

$$w_{k+1}(t) = F(w_k(t), \Delta y(t)) \dots(2)$$

It is desirable to know that there is a certain function norm  $\|\Delta y(t)\|$  such that  $\|\Delta y_{k+1}(t)\| \leq \|\Delta y_k(t)\|$  for all  $k = 1,2,\dots$

The goal of this paper is to introduce a quasi natural potential in robot dynamics which induces a type of sinusoidal position feedback with saturation in servo – loops,

which do the original robot dynamics incorporated with these servo – loops satisfy the learn ability of skilled motion for robotic manipulators, with an internal model as a based controller. The proposed feedback with a form :

$-(\mu \Delta \dot{q} + \psi \sin(\Delta q))$  with positive definite gain matrices  $\mu$  and  $\psi$  where

$$\sin(\Delta q) = (\sin \Delta q_1, \dots, \sin \Delta q_n)^T$$

Leads to the learn ability of the skilled motion for robot manipulator with respect to the residual input torque  $\Delta w$  and the output:

$$\Delta y = \Delta \dot{q} + \beta \sin(\Delta q), \text{ with constant } \beta > 0.$$

### 3. LEARNING CONTROL FOR LINEAR TIME VARIING SYSTEM WITH INTERNAL MODE

To explain the essence of the proposed learning control method, consider a linear time varying system [M.W.Spong,1987, T.Y, Abdulaa2000]

$$H(t)\ddot{q}(t) + S(t)\dot{q}(t) + P(t)q(t) = u(t) \dots(3)$$

where  $u(t)$  is  $m \times 1$  input vector. Further  $q(t)$  is  $n \times 1$  state vector. The exact values of the coefficient matrices  $H(t)$ ,  $S(t)$  and  $P(t)$  are not needed to be known.  $y(t)$  is  $l \times 1$  output vector, it is assumed that the output  $y(t)$  is the velocity signal  $\dot{q}(t)$ . Suppose that for this system a desired output  $y_d(t)$  is given, which is continuously differentiable on  $[0, T]$ . Since the coefficient matrices  $H(t)$ ,  $S(t)$  and  $P(t)$  are unknown, the input that generates the

desired output can not be obtained through calculation. So one may introduce a kind of learning scheme to realize the desired output. In fig .2 , if the learning control part  $L = 0$  .(without learning ) then

$$\mathbf{u}_e = \mathbf{u}_m - \mathbf{u}$$

and by feed forward control

$$\mathbf{u} = \mathbf{u}_{ref} - \mathbf{u}_e \quad \dots (4)$$

where  $\mathbf{u}_{ref}$  is the reference input. Then

$$\mathbf{u}_m = \mathbf{u}_{ref} \quad \dots (5)$$

$$\text{and } \mathbf{u}_e = \mathbf{u}_m - (\mathbf{u}_{ref} - \mathbf{u}_e)$$

From fig. 2 the plant input  $\mathbf{u}$  is composed of two parts, a model based controller plus a learning control part  $L$  , the control input becomes as

$$\mathbf{u} = \mathbf{u}_{ref} - \mathbf{u}_e + L \quad \dots \dots (6)$$

let us now consider a type of servo – loop defined by

$$L = \mathbf{w} - \boldsymbol{\mu} \Delta \dot{\mathbf{q}} - \boldsymbol{\psi} \sin(\Delta \mathbf{q}) \quad \dots (7)$$

where  $\boldsymbol{\mu}$  and  $\boldsymbol{\psi}$  are positive definite diagonal matrices whose diagonal elements signify position and velocity gains, respectively .Substituting eq.7 into eq.6 yields

$$\mathbf{u} = \mathbf{u}_{ref} - \mathbf{u}_e + \mathbf{w} - \boldsymbol{\mu} \Delta \dot{\mathbf{q}} - \boldsymbol{\psi} \sin(\Delta \mathbf{q}) \quad \dots (8) \hat{\mathbf{H}}(t)\ddot{\mathbf{q}}(t) + \hat{\mathbf{S}}(t)\dot{\mathbf{q}}(t) + \hat{\mathbf{P}}(t)\mathbf{q}(t) = \mathbf{u}_m(t) \quad \dots (12)$$

The learning part  $w$  is updated according to [S.Arimoto and T. Naniwa ,1995] ( see fig. 4)

$$\mathbf{w}_{k+1}(t) = \mathbf{w}_k(t) - \boldsymbol{\gamma} \Delta \mathbf{d}_k(t) \quad \dots (9)$$

where  $\Delta \mathbf{d}_k(t)$  is a function of the output  $\Delta \mathbf{y}_k(t)$  , such as  $\Delta \mathbf{d}_k = \Delta \mathbf{y}$  then  $\Delta \mathbf{d}_k = \Delta \dot{\mathbf{q}}_k + \boldsymbol{\beta} \sin(\Delta \mathbf{q}_k) \quad \dots (10)$

If the output vector  $\mathbf{y}$  stands for angular velocities, i.e.  $\Delta \mathbf{y} = \dot{\mathbf{q}}$  . we assume that the

gain matrix  $\lambda$  is positive definite or diagonal with positive diagonal elements. Now, let us pose a question as to what kind of characterizations of robot dynamics is crucial in assurance of robot 's learn ability ( the existence of a function norm  $\| \cdot \|$  ) such that

$$\| \Delta \mathbf{d}_k \| \rightarrow 0 \quad \text{as } k \rightarrow \infty .$$

The transfer function matrix of the filter is  $(\boldsymbol{\tau}_m \mathbf{S} + \mathbf{I})^{-1}$  , where  $\boldsymbol{\tau}_m$  is

(  $n \times n$  ) constant diagonal matrix. The filter time constants ( i .e. the diagonal elements of  $\boldsymbol{\tau}_m$  ) are chosen as a compromise between the speed of response and the robustness [M.Morari, and E. Zafiriou ,1989,] . A feedback loop is obtained by considering  $\mathbf{u}_{ref}$  and  $\mathbf{u}_m$  as the filter inputs and  $\mathbf{u}_e$  as it 's output. Using concept from block diagram algebra , the relation between the input and the output is converted as shown in fig. 3.

From fig. 3 the perturbation signal is

$$\mathbf{u}_e = \boldsymbol{\tau}_m^{-1} \int_0^T (\mathbf{u}_m - \mathbf{u}_{ref}) dt \quad \dots (11)$$

A model is selected as

And

$$\mathbf{u}_{ref} = \hat{\mathbf{H}}(t)\ddot{\mathbf{q}}_d(t) + \hat{\mathbf{S}}(t)\dot{\mathbf{q}}_d(t) + \hat{\mathbf{P}}(t)\mathbf{q}_d(t) \quad \dots \dots \dots (13)$$

where  $\hat{\mathbf{H}}$  and  $\hat{\mathbf{S}}$  are constant diagonal matrices that are positive definite.

From fig.3 and considering eq . 12 and eq . 13 it can be seen that only signals up to the first derivative of displacement ( i.e. velocity)

are needed. Since the position and velocity signals are available , the configuration of fig.3 is suitable for implementation. The filter avoid the necessity for acceleration signal.

Return to the problem of characterization of robot dynamics. We assume that a desired trajectory  $q_d(t)$  for  $t \in [0,T]$  is given a priori and twice continuously differentiable . Then , there exists a desired input  $w_d (= u_d)$  and hence the motion of the robot arm at the  $k$  th exercise is subject to the following residual dynamics and by substituting eq. 8 and eq. 12 into eq. (3) results in

$$\begin{aligned}
 & H(t)\ddot{q}(t) + (S(t) + \hat{H}(t)\tau_m^{-1})\dot{q}(t) \\
 & \quad + (P(t) + \hat{S}(t)\tau_m^{-1})q(t) \\
 = & \hat{H}(t)\ddot{q}_d(t) + (\hat{S}(t) + \hat{H}(t)\tau_m^{-1})\dot{q}_d(t) \\
 & \quad + \hat{S}(t)\tau_m^{-1}q_d + w - \mu\Delta\dot{q} \\
 & \quad - \psi \sin(\Delta q) \dots (14)
 \end{aligned}$$

Define  $\Delta w = w_k - w_d$  and  $z_k = q_k - q_d$

where  $w_d$  is the control input which generates the desired trajectory  $y_d$  and then eq.(14) can be written as :

$$\begin{aligned}
 & H(t)\ddot{z}_k(t) + (S(t) + \hat{H}(t)\tau_m^{-1} + \mu)\dot{z}_k(t) \\
 & \quad + (P(t) + \hat{S}(t)\tau_m^{-1})z_k(t) \\
 & \quad + \psi \sin(z_k(t)) = \Delta w_k \dots (15)
 \end{aligned}$$

If  $\tau_m$  is chosen sufficiently small , then eq.15 can be approximated by :

$$\begin{aligned}
 & H(t)\ddot{z}_k(t) + (\hat{H}(t)\tau_m^{-1} + \mu)\dot{z}_k(t) + (\hat{S}(t)\tau_m^{-1})z_k(t) \\
 & \quad + \psi \sin(z_k(t)) = \Delta w_k \dots (16)
 \end{aligned}$$

Assume, that  $z_K$  and  $\dot{z}_K$  are bounded . The bounded ness of  $z_k(t)$  and  $\dot{z}_k(t)$  is assured , since the internal model forces  $w_o(t)$  to be close to  $w_d(t)$

. To show the convergence of the learning control algorithm , the following theorem may be stated.

**4.THEOREM:**

Let the system described by eq.(17) satisfies the conditions b1-b6 and uses the control law in eq. 9 , if the condition  $\gamma \geq 2(\tau_m^{-1}\hat{H})$  is satisfied then the trajectory  $(q_k , \dot{q}_k)$  converges to the desired one  $(q_d , \dot{q}_d)$  in  $t \in [0 , T]$  , as  $k \rightarrow \infty$  , provided that the bounded ness of  $z_k(t)$  (t) and  $\dot{z}_k(t)$  for all k is satisfied.

**5.Convergence proof:**

To gain an insight into the problem , we note that there is an ideal input  $w_d$  that realize the prescribed output  $y_d(t)$  according to the postulate b6, though it is implicitly assumed that such an input  $w_d$  can not be calculated in ordinary cases. Then for a given desired output  $y_d(t)$  ,  $0 \leq t \leq T$  , the iterative control law of eq.9 guarantees that for each  $t \in [0 , T]$  ,  $y_k(t) \rightarrow y_d(t)$  as  $k \rightarrow \infty$  ,  $0 \leq t \leq T$  is chosen to be continuous and  $q_k(0) = q_0$  , for all  $k = 1 , 2 , \dots$

To show the convergence proof of the learning control algorithm in eq.9 , it is assumed that the gain matix  $\gamma$  is symmetric , positive definite and satisfies the condition

$$\gamma \geq 2(\tau_m^{-1}\hat{H}) \dots (17)$$

Subtracting  $w_d$  from both sides of Eq.(9) yields

$$\Delta w_{k+1}(t) = \Delta w_k(t) - \gamma \Delta d_k(t) \dots (18)$$

where

$$\Delta w_k(t) = w_k(t) - w_d(t) \dots (19)$$

Inner products of both sides of Eq.(18) via the positive definite symmetric matrix  $\gamma^{-1}$  are expressed as

$$\begin{aligned} \Delta \mathbf{w}_{k+1}(t)^T \boldsymbol{\gamma}^{-1} \Delta \mathbf{w}_{k+1}(t) & \\ &= (\Delta \mathbf{w}_k - \boldsymbol{\gamma} \Delta \mathbf{d}_k(t))^T \boldsymbol{\gamma}^{-1} (\Delta \mathbf{w}_k(t) \\ &- \boldsymbol{\gamma} \Delta \mathbf{d}_k(t)) \dots (20) \end{aligned}$$

Integration of both sides of this equation over the time interval  $[0, T]$  leads to and substituting eq.10 into eq.20 yields

$$\begin{aligned} \int_0^T e^{-\lambda t} (\Delta \mathbf{w}_{k+1}(t)^T \boldsymbol{\gamma}^{-1} \Delta \mathbf{w}_{k+1}(t)) dt & \\ &= \int_0^T e^{-\lambda t} [(\Delta \mathbf{w}_k(t) - \boldsymbol{\gamma} \dot{\mathbf{z}}_k \\ &- \boldsymbol{\psi} \sin(\Delta \mathbf{z}_k(t)))^T \boldsymbol{\gamma}^{-1} (\Delta \mathbf{w}_k(t) - \boldsymbol{\gamma} \dot{\mathbf{z}}_k \\ &- \boldsymbol{\psi} \sin(\Delta \mathbf{z}_k(t)))] dt \dots (21) \end{aligned}$$

$$\begin{aligned} \int_0^T e^{-\lambda t} (\Delta \mathbf{w}_{k+1} \boldsymbol{\gamma}^{-1} \mathbf{w}_{k+1}) dt & \\ &= \int_0^T e^{-\lambda t} [(\Delta \mathbf{w}_k^T \boldsymbol{\gamma}^{-1} \Delta \mathbf{w}_k)] dt \\ &+ \int_0^T e^{-\lambda t} [\Delta \dot{\mathbf{z}}_k^T \boldsymbol{\gamma} \Delta \dot{\mathbf{z}}_k] dt \\ &- 2 \int_0^T e^{-\lambda t} [(\Delta \dot{\mathbf{z}}_k^T \Delta \mathbf{w}_k)] dt \\ &- 2 \int_0^T e^{-\lambda t} [\boldsymbol{\psi} \Delta \mathbf{w}_k^T \sin(\mathbf{z}_k)] dt \\ &+ 2 \int_0^T e^{-\lambda t} \boldsymbol{\psi}^2 \boldsymbol{\gamma}^{-1} (\mathbf{1} \\ &- \cos(\mathbf{z}_k^T(t))) dt \dots (22) \end{aligned}$$

Noting that  $[\mathbf{1} - \cos(\theta) \geq \frac{1}{2} \sin(\theta)^2]$ ,

Now

$$\begin{aligned} \int_0^T e^{-\lambda t} [(\Delta \dot{\mathbf{z}}_k^T \Delta \mathbf{w}_k)] dt & \\ &= \int_0^T e^{-\lambda t} [\Delta \dot{\mathbf{z}}_k^T [\mathbf{H}(t) \dot{\mathbf{z}}_k(t) \\ &+ (\hat{\mathbf{H}}(t) \boldsymbol{\tau}_m^{-1} + \boldsymbol{\mu}) \dot{\mathbf{z}}_k(t) + (\hat{\mathbf{S}}(t) \boldsymbol{\tau}_m^{-1}) \mathbf{z}_k \\ &+ \boldsymbol{\psi} \sin(\mathbf{z}_k)]] dt \dots (23) \end{aligned}$$

by using eq.16 the last equation is obtained.

Then

$$\begin{aligned} \frac{1}{2} \int_0^T \frac{d}{dt} \left[ e^{-\lambda t} [\dot{\mathbf{z}}_k^T \mathbf{H} \dot{\mathbf{z}}_k + \mathbf{z}_k^T (t) \hat{\mathbf{S}} \boldsymbol{\tau}_m^{-1} \mathbf{z}_k] dt \right. & \\ &+ \int_0^T e^{-\lambda t} \dot{\mathbf{z}}_k \hat{\mathbf{H}} \boldsymbol{\tau}_m^{-1} \dot{\mathbf{z}}_k dt \\ &+ \int_0^T e^{-\lambda t} \dot{\mathbf{z}}_k^T \boldsymbol{\gamma} \dot{\mathbf{z}}_k dt \\ &+ \int_0^T e^{-\lambda t} \dot{\mathbf{z}}_k^T \boldsymbol{\psi} \sin(\mathbf{z}_k) dt \\ &- \frac{1}{2} \int_0^T e^{-\lambda t} \dot{\mathbf{z}}_k^T \dot{\mathbf{H}} \dot{\mathbf{z}}_k dt \\ &+ \frac{1}{2} \int_0^T \lambda e^{-\lambda t} [\dot{\mathbf{z}}_k^T \dot{\mathbf{H}} \dot{\mathbf{z}}_k \\ &+ \mathbf{z}_k^T \hat{\mathbf{S}} \boldsymbol{\tau}_m^{-1} \mathbf{z}_k] dt \dots (24) & \\ &= \frac{1}{2} \left[ e^{-\lambda t} [\dot{\mathbf{z}}_k^T \mathbf{H} \dot{\mathbf{z}}_k + \mathbf{z}_k^T \hat{\mathbf{S}} \boldsymbol{\tau}_m^{-1} \mathbf{z}_k] dt + \right. & \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} \int_0^T e^{-\lambda t} \dot{z}_k^T [2\hat{H}\tau_m^{-1} + \lambda H - H + 2\gamma] \dot{z}_k dt + \\ & \int_0^T e^{-\lambda t} \dot{z}_k^T \psi \sin(z_k) dt - \\ & \frac{1}{2} \int_0^T e^{-\lambda t} z_k^T (\lambda \hat{S} \tau_m^{-1}) z_k dt \quad \dots (25) \end{aligned}$$

Note that the assumption on complete initialization is used in the derivation of the last result. Substitute eq.25 into eq.22 yields:

$$\begin{aligned} & \int_0^T e^{-\lambda t} (\Delta w_{k+1}^T \gamma^{-1} \Delta w_{k+1}) dt \\ & = \int_0^T e^{-\lambda t} [(\Delta w_k^T \gamma^{-1} w_k)] dt \\ & - 2 \int_0^T e^{-\lambda t} [\dot{z}_k^T \psi \sin(z_k)] dt \\ & - e^{-\lambda t} [\dot{z}_k^T H \dot{z}_k + z_k^T \hat{S} \tau_m^{-1} z_k] \\ & - \int_0^T e^{-\lambda t} z_k^T (\lambda \hat{S} \tau_m^{-1}) z_k dt \\ & - \int_0^T e^{-\lambda t} \dot{z}_k^T [2\hat{H}\tau_m^{-1} + \lambda H - \dot{H} + 2\gamma \\ & - \gamma] \dot{z}_k dt \\ & - 2 \int_0^T e^{-\lambda t} [\psi \Delta w_k^T \sin(z_k)] dt \\ & + 2 \int_0^T e^{-\lambda t} \psi^2 \gamma^{-1} (1 \\ & - \cos(z_k^T(t))) dt \quad \dots (26) \end{aligned}$$

And

$$\begin{aligned} & 2 \int_0^T e^{-\lambda t} \psi^2 \gamma^{-1} (1 - \cos(z_k^T(t))) dt \\ & = -m_1 - \int_0^T e^{-\lambda t} m_2 \cos(z_k^T(t)) dt \end{aligned}$$

where  $m_1$  is positive gain and  $m_2 = 2\psi^2\gamma^{-1}$  are positive diagonal gain .

Since  $H, \hat{S}$  and  $\hat{H}$  are positive definite , then if  $\lambda > 0$  is chosen sufficiently large so that

$$[2\hat{H}\tau_m^{-1} + \lambda H - \dot{H} + \gamma] > 0 \quad \dots (27)$$

And the learning gain satisfies the following condition

$$2\hat{H}\tau_m^{-1} \leq \gamma \quad \dots (28)$$

Then

$$\begin{aligned} & \int_0^T e^{-\lambda t} (\Delta w_{k+1}^T \gamma^{-1} \Delta w_{k+1}) dt \\ & \leq \int_0^T e^{-\lambda t} [(\Delta w_k^T \gamma^{-1} \Delta w_k)] dt \quad \dots (29) \end{aligned}$$

The result in eq.29 can be expressed as

$$\|\Delta w_{k+1}\|^2 \leq \|\Delta w_k\|^2 \quad ,$$

where  $\| \cdot \|$  is defined as

$$\begin{aligned} \|\Delta w_k(t)\| & = \\ & \left\{ \int_0^T \Delta w_k(t) \gamma^{-1} \Delta w_k^T(t) dt \right\}^{-1/2} \dots (30) \end{aligned}$$

The result in eq.30 means that the sequence  $\|\Delta w_k(t)\|^2$  is monotonously decreasing as long as  $z_k$  and  $\dot{z}_k$  do not vanish . Since  $\|\Delta w_k(t)\|$  is bounded , the monotonous decrease of  $\|\Delta w_k(t)\|$  implies the convergence , which proves that  $\dot{z}_k \rightarrow 0$  and  $z_k \rightarrow 0$  as  $k \rightarrow \infty$  , which means the that

$$\lim_{k \rightarrow \infty} \|\Delta d_k(t)\| = 0$$

under the set of postulate b1 – b6 the learning update law (see fig.5)

$$\begin{aligned} w_{k+1}(t) &= w_k(t) \\ &- \gamma \{ \Delta \dot{q}_k(t) + \beta \sin(\Delta q_k(t)) \} \end{aligned} \quad \dots (31)$$

satisfies the learnability and gives rise to the convergence of trajectory tracking, that is ,  $q_k(t) \rightarrow q_d(t)$  in the sense of uniform norm as  $k \rightarrow \infty$

### 6.SIMULATION STUDY

To show the effectiveness of the proposed learning control scheme, a numerical example is considered. The proposed method is compared with an algorithm suggested in [T.Y, Abdulaa2000,S. Arimoto , 1990] which uses a learning control law with proportional plus derivative (PD) feedback.

Consider a linear time varying system described by as :

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -(2+5t) & -(3+2t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [ 0 \quad 1 ] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

with  $x_1(0) = 0$  ,  $x_2(0) = 0$

The desired output is given by as shown in fig.6:

$$y_d(t) = 12 t^2 (1 - t) \quad , t \in [ 0, 1 ] \text{ sec}$$

The internal model is selected using mean values of the system parameter variations as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3.5 & -2.5 \end{bmatrix}$$

Computer simulation is carried out for the proposed algorithm. The simulation is conducted by means of the fourth order Runge – Kutta method. The output for different trials is compared with the output for the learning control algorithm with PD feedback suggested in [18], as

$$u = w - k_1(x - x_d) - k_2\dot{x}$$

where  $k_1$  and  $k_2$  are constant gain matrices as. Results of simulation using the proposed learning control method with internal model for different cases are shown in figs. 7, 8 , 9 and 10 Notice that the proposed learning control algorithm converges faster than that of PD feedback method with learning.

These figures demonstrate that the error is reduced when the operation is repeated. Figs. 7 and 8 show results of simulation for the ideal case , while figs. 9 and 10 show results for the case when there is an error in initial setting ,such that  $x_1(0) = 0.001$  ,

$x_2(0) = 0.001$ . From the computer simulation , we can observe that the proposed method is less sensitive to the error in initial settings than that of the PD feedback method with learning.

### 7.CONCLUSIONS

In this paper the internal model concept is used for robot applications , and enhanced by



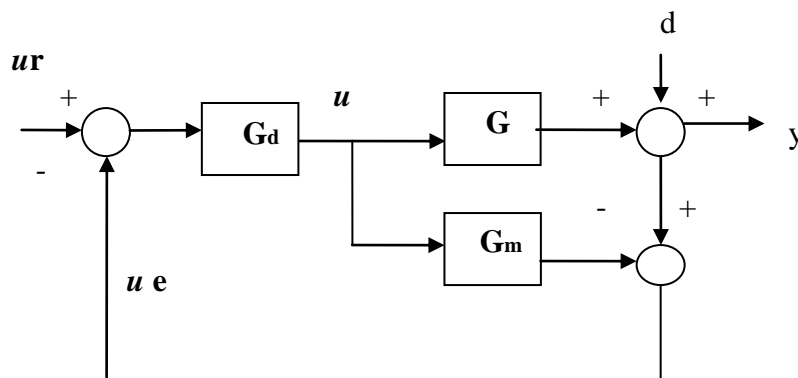
learning capabilities. It is used for motion tracking of robot manipulator.

A quasi – natural potential in robot dynamics is used in the research, which gives a type of sinusoidal position feedback with saturation in servo – loops. By means of saturated proportional and differential feedback of the quasi – natural potential the learnability for robotic manipulators is discussed. The learning control concept is added to the internal model structure. The proposed control consists of an internal model based controller plus a feed forward learning control part. The learning control part uses velocity error signal in updating the feed forward part. The proposed scheme is applied for different applications.

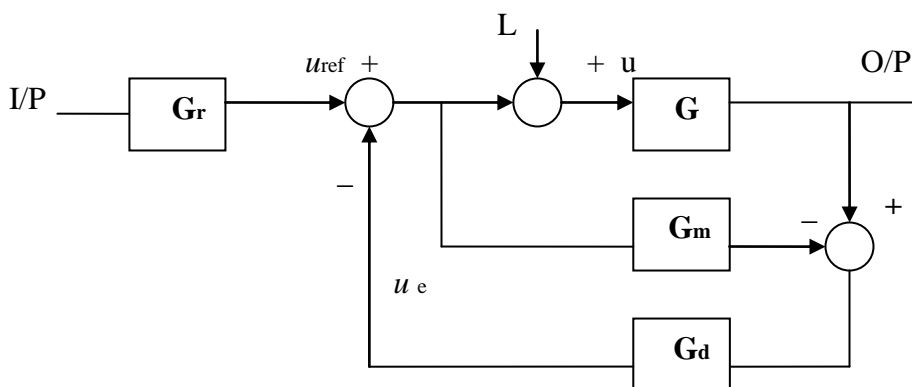
## 8. REFERENCES

1. Abdelhamid Tayebi, July 2004, “Adaptive iterative learning control for robot manipulators”, *Automatica*, vol 40, issue 7, p.p 1195 – 1203.
2. C.C. Cheah, D.wang, Y.C.Oh ,1995," Learning control of motion and force for constrained robotic manipulators", *Int. J. of Robotic and Auto*, vol, 10, no. 3, p.p 79 – 87.
3. De Luca and G.Ulivi, 1992 ,“Iterative learning control for robots with elastic joints”, In proc. IEEE Int. Conf. On Robotics and Automation, France , pp . 1920-1926.
4. D.M. Dutton and G. V. Conroy,1996, " A review of machine learning", *The knowledge Engineering Review*, vol , 12, No. 4 , p.p 341 – 367.
5. K.E. Avrachen kov and R.W. Longman , 2003, ”Iterative learning control for over - determined , under- determined , and ILL – conditioned systems” . *Int. J.Appl. Math. comput. Sci.*, Vol. 13 , No. 1, 113-122.
6. K. M. Omran ,1996 ,”On the learning control of robot manipulators” M. Sc. thesis , Basrah University .
7. M.Morari, and E. Zafiriou ,1989, “ Robust process control”, Prentice – Hull International Edition.
8. M.W.Spong, 1987,"Modeling and control of Elastic joint Manipulators", *J. Dynamic systems, Measurement and Control*, vol 109, PP 310-319.
9. P. Lucibello, S. Panzieri and F. Pascucci, June, 2003 ,“Suboptimal output regulation of robotic manipulators by iterative learning”, *Int. J. Con. On Advanced Robotic*, PP. 24-30.
10. Rafael Kelly, July 2005 ,” Learning control of robot manipulators by interactive simulation”, vol 23, Issue 4, p.p 515 – 520.
11. S.Arimoto, S.S.kawamura and F.Miyazaki, March ,1984, "Bettering operation of robots by learning " *J.Robotics System*, vol. 1,NO.2, pp. 123-140.
12. S. Arimoto and T. Naniwa ,1995, " A quasi – natural potential that gives rise to learnability And A daptability in robotic

- systems", J. of systems Engineering , no.1, vol. 5, p. p 163 – 173.
13. S. Arimoto , 1990, " Learning control theory for robotic motion", Int. J. of Adaptive Control and signal processing, vol. 4, no.6, pp.543 – 564.
  14. S.P Chan, 1995, “ Robust sliding model control of robot manipulators using internal model”, International Journal of robotics and Automation, vol. 10 , pp. 63 -69, No.8.
  15. Tom Oomen, Jeroen van de Wijdevev, Okko Bosgra, April 2009, “Suppressing intersample behavior in iterative learning control” , Automatica, Volume 45, Issue 4, p.p 981 – 988.
  16. T, Y, Abdalaa, May, 2000, "Robust learning control of robot manipulator internal model" ,PH.D Thesis , Basrah University.
  17. W.E. Dixon and J.chen , September ,2003,“Acomposite Energy Function-Based learning control Approach for nonlinear systems with time varing parametric uncertainties”, IEEE, Transactions on Automatic control, Vol 48, No. 9 .
  18. Z, Qu , J . Dorey , D.Dawison and R.W.Johnson,1993, " Learning control of robot motion", J. of Robotic system, vol 10, no.1 , p.p 123-140.



**Fig. 1: Internal model structure**



**Fig. 2 : A two – degree of freedom control structure with internal model and learning**

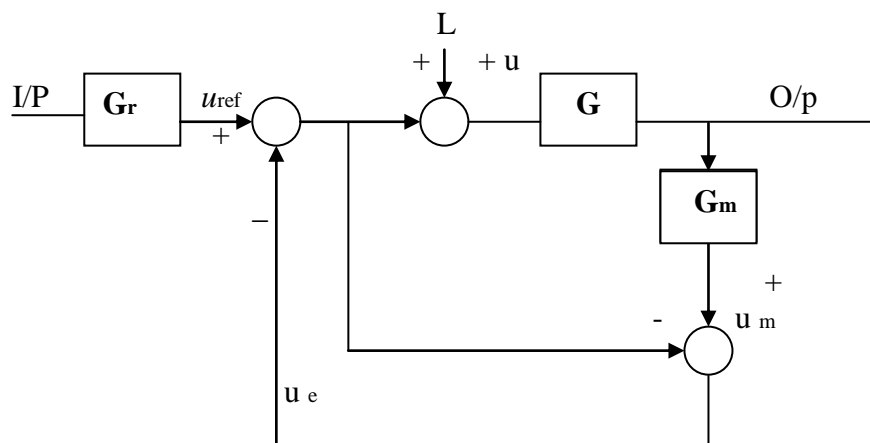


Fig. 3: Equivalent block diagram to fig.1

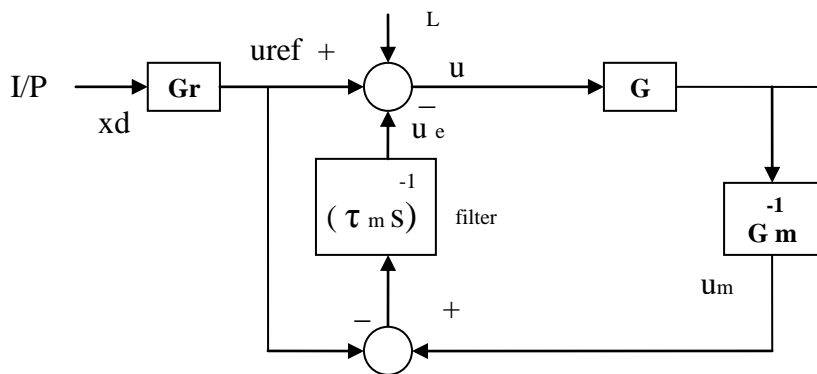


Fig. 4 :Internal model structure with learning control and using filter

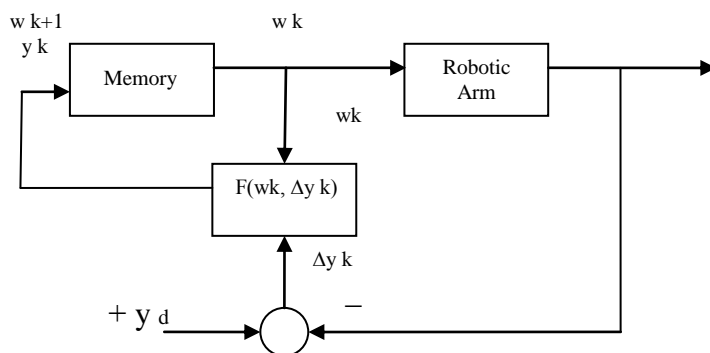


Fig. 5 : Iterative Learning control

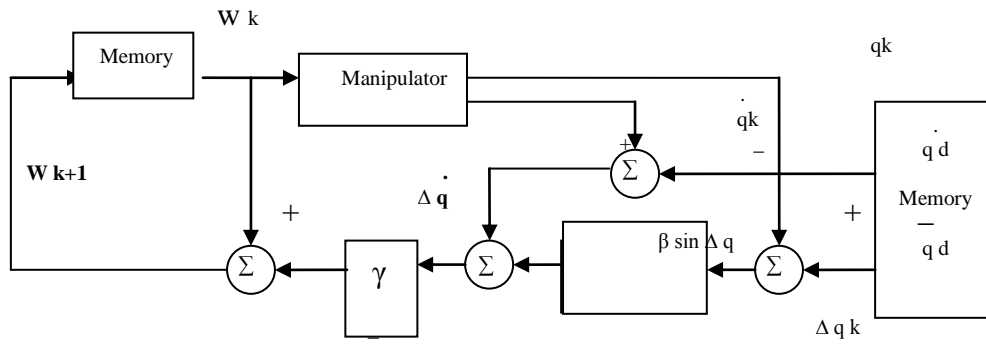


Fig. 6. Learning control law,  $w_{k+1}(t) = w_k(t) - \gamma [\Delta q_k + \beta \sin \Delta q_k]$

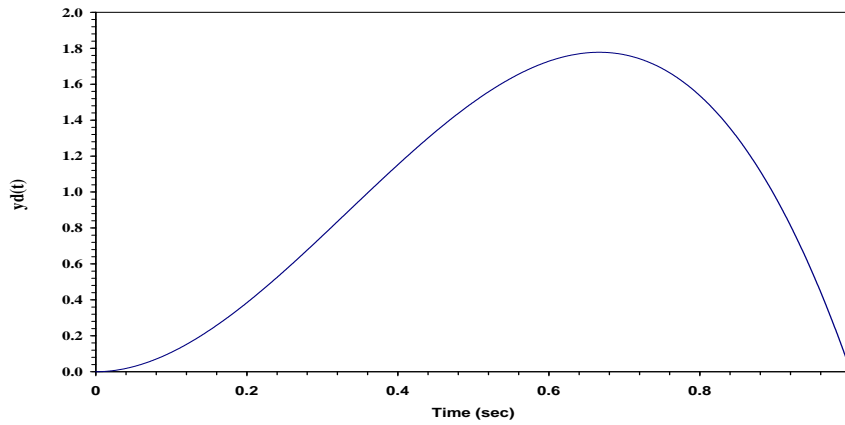


Fig. 7 The desired output of the plant

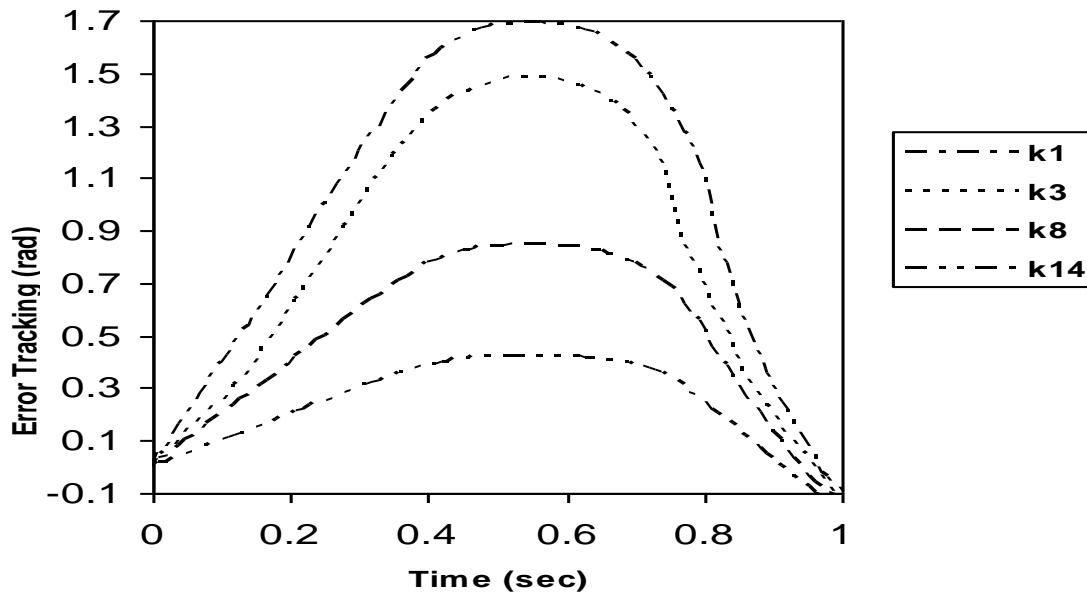


Fig. 8 : Tracking Error using PD

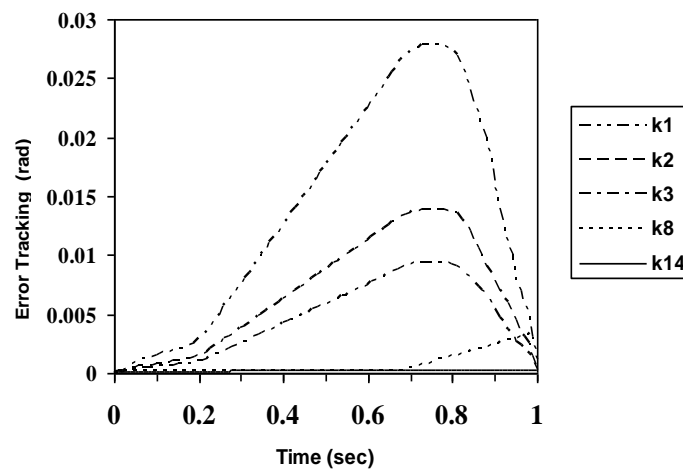


Fig. 9: Tracking Error using the proposed method

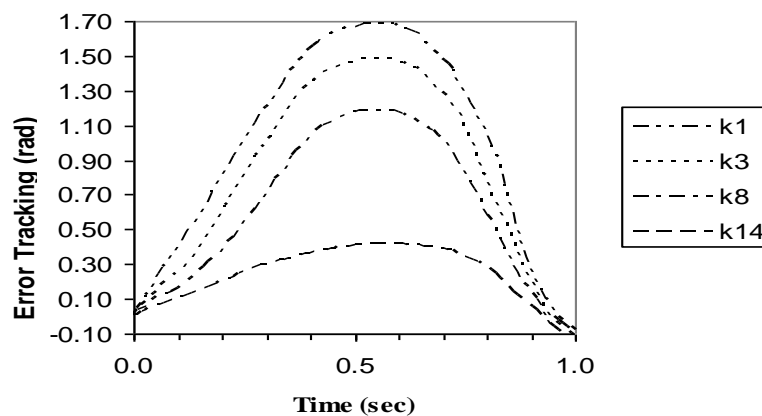


Fig. 10 Tracking Error using PD with error in initial settings

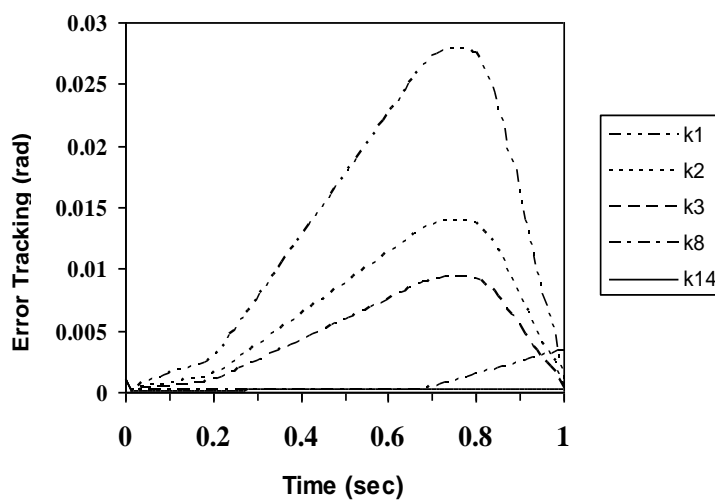


Fig. 11: Tracking Error using the proposed method with error in initial settings

## قابلية تعلم الحركة الماهرة لانظمة الروبوت باستخدام النموذج الداخلي

### الخلاصة

تم في هذا البحث دراسة طريقة مقترحة للتحكم المتعلم باستخدام النموذج الداخلي ويمكن استخدامها كطريقة تحكم تنابعية في التحكم بمعالجات الروبوت . ان الطريقة المقترحة للتحكم المتعلم تتألف من مسيطر النموذج الداخلي كمتحكم اساس مضاف اليه مسيطر التحكم المتعلم كمسيطر بالاتجاه الامامي وتم استخدامه في السيطرة على احد الانظمة المتغيرة بصورة خطية مع الزمن . ان تركيب نظام السيطرة المقترح وهو من الدرجة الثانية يتألف من مسيطر داخلي للضوضاء في النظام ويستخدم نظام سيطرة خارجي لتتبع المسار. وقد تم في هذا البحث استخدام نوع جديد من التغذية الراجعة للموقع يعتمد على الدالة المثلثية. وقد تم في الطريقة المقترحة تحسين الاداء في المحاولة الجديدة اعتمادا على بيانات الاشتغال الناتجة في المحاولة السابقة. يتضمن البحث وضع اثبات رياضي لصحة الطريقة المقترحة . بالاضافة الى وجود امثلة توضح فعالية هذه الطريقة. كما تم دراسة الحساسية لتأثير وجود خطأ في العودة إلى الوضع الابتدائي من خلال المحاكاة بالحاسبة . ان النتائج تبين الأداء الجيد للطريقة المقترحة.