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Design of All-Reflecting Aplanatic Objective

Laser beam aiming and narrow field imaging systems usually incorporate aplanatic objectives, preferably reflective. Two examples of two-reflector objective namely the Gregorian and Cassegrainian are considered. The work herein is aiming at determining the aspheric profiles of the two reflectors so as to attain aplanatism. Detailed discussion on the Cassegrainian objective and derivation of equations of its aspherics are given. The work can be extended to deal with the design of reflective objectives convenient to ultra-violet microscopy.

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1. Introduction

All-reflecting objectives have main advantage over refractive ones as they are free from chromatic aberration. They also can be made aplanatic by applying aspheric reflectors, thereby become useful tools for a variety of applications. Two-reflectors objectives, conventional or modified, are usually incorporated in some modern optical arrangements, such as laser detection and ranging, guidance systems and space imaging cameras, where mostly narrow field-of-view (FOV) is required. Attaining aplanatism in such systems results in an adequate optical performance.

The aim of this work is to derive equations to attain aplanatism for the conventional two-reflectors objectives, namely of, Cassegrainian and Gregorian types . Many works on methods and techniques to achieve aplanatism were reported. Examples are, design considerations of aplanatic telescopes [1-5], design of three and four aspheric surface configurations [6,7], design of two mirror telescopes [8-10], and designing and testing aspheric correctors surfaces [11-17]. However, the method introduced by Wassermann and Wolf [12] that, essentially, employs the sine-condition is found to be more adequate for the present investigation. For each objective configuration, two simultaneous first-order differential equations that may be integrated numerically by standard Runge-Kutta method are derived to describe the profiles of the objective's two correctors.

Cassegrainian objective type is often preferred to the Gregorian arrangement because it is inherently short. Thus, an investigation on a Cassegrainian configuration objective of particular dimensions is carried out. The investigation showed that the optical performance is likely to be diffraction-limited at the wavelength of 550 nm.

2. Geometrical Optics Consideration

For the purpose of aiming and imaging applications, the objective can be treated as a telescope. Thus, the work herein will be devoted for the derivation of equations that describe the behavior of an incoming ray from a distant object upon entering a two-reflector system. This will end up with the derivation of two simultaneous first order differential equations that describe the desired shapes of the two reflectors to ensure aplanatism. Two examples of two- reflector configuration – namely the Gregorian and Cassegrainian – will be considered here. Both arrangements are centered optical systems, in each of which, the primary and secondary reflectors are optical neighbors separated from each other by a chosen distance as depicted in Figs. (1) and (2). To facilitate ray trace calculations, two sets of Cartesian coordinates are introduced:

- First, $(Z(\eta),\eta)$ for the primary reflector with an origin at pole O, and,

- Second, $(Z'(\eta'),\eta')$ for the secondary reflector with an origin at pole O',

with their $Z(\eta)$, axes along the optical axis of the objective.

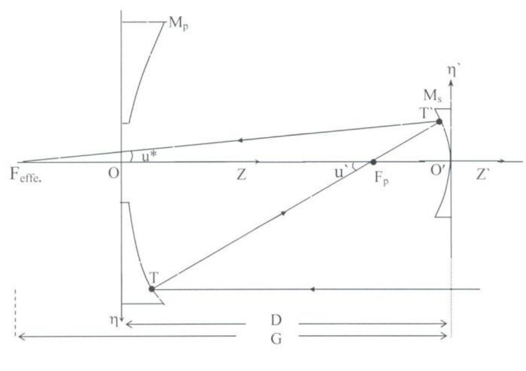


Fig. (1) A Gregorian configuration

The following dimensions may be identified:

- OO'=D is the separation between the primary and secondary reflectors poles,

- f_{effe}.O'=G is the separation between the focal point f_{effe}. (objective's effective focal point) and secondary reflector pole O'.

Figure (1) demonstrates a typical Gregorian configuration, which consists of two confocal concave reflectors. An incoming ray from a distant object point, parallel to the optic axis, u=0, hits the primary M_P at point T(Z(η),η) and bounced back to hit the secondary M_S at point T'(Z'(η'),η') after crossing the optic axis at or near the primary focal point FP. At T', the ray is reflected towards the point f_{effe}.

Figure (2), however, shows a typical Cassegrain configuration that consists of a concave primary reflector followed by a convex secondary reflector so that its focal point located at or near the focal point of the primary f_P. Again an incoming ray from a distant object point, u=0, hits the primary at T and reflected back to hit the secondary at T'. Ray trace calculations of the Cassegrain type will be treated here in detail.

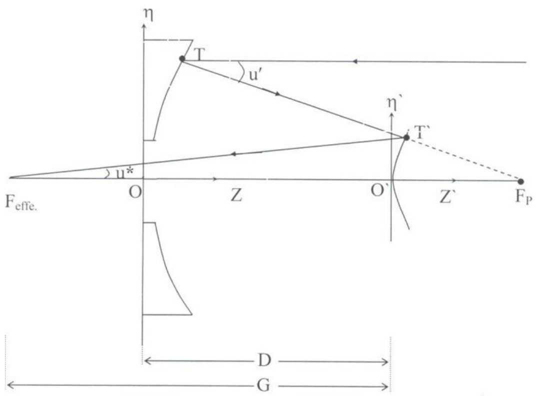


Fig. (2) A Cassegrain configuration

Referring to the (η, Z) frame, figure (3a) ,the incoming ray has the equation

$$\eta = Y \quad (1)$$

where Y is the height at which the incident ray meets the axis Oη. So that,

$$\frac{d\eta}{dY} = 1 \quad (2)$$

In the notation of Fig. (2), the condition for attaining aplanatism may be indicated by

$$\frac{Y}{sinu^*} = f_{effe.} \quad (3)$$

where $f_{effe.}$ is the effective focal length of the Cassegrain, and u^* is the angle at which ray $T'f_{effe.}$ makes with the optic axis. Thus

$$tanu^* = H\left[f_{effe.}^2 - Y^2\right]^{-1/2} \quad (4)$$

is the optical constraint that must be met to achieve aplanatism.

Referring to the (Z',η') frame, Fig. (3b), the equation of ray $T'f_{effe.}$ is

$$\eta' = Y' + Z' \tan u^* \quad (5)$$

where the height at which ray $T'F_{effe.}$ meets the axis O'η' is

$$Y' = G \tan u^* \quad (6)$$

At the primary mirror, Figs. (2) and (3a), the reflected ray TT' makes an angle u' with the axis. The tangent to the primary surface at T makes an angle β to the axis. Since $i = r$ (law of reflection), then

$$\beta = 90 - \frac{u'}{2} \quad (7)$$

and

$$\tan\beta = \frac{d\eta}{dZ} \quad (8)$$

again, it is possible to write

$$\frac{dZ}{dY} = \frac{d\eta}{dY}\frac{dZ}{d\eta}$$

or

$$\frac{dZ}{dY} = \frac{d\eta}{dY}\cot\beta$$

and when using Eq. (8)

$$\frac{dZ}{dY} = \cot\beta \quad (9)$$

where $\cot\beta = \tan\left(\frac{u'}{2}\right)$, and

$$\tan u' = \frac{\eta - \eta'}{D - Z + Z'} \quad (10)$$

At the secondary reflector, Fig. (3b), the second reflection takes place, so that the reflected ray cuts the axis at point $f_{effe.}$. The tangent to secondary reflector at T' makes an angle of inclination β' to the axis and since i'=r', then

$$\beta' = 90 - \frac{(u' - u^*)}{2} \quad (11)$$

Now

$$\tan\beta' = \frac{d\eta'}{dZ'} \quad (12)$$

also

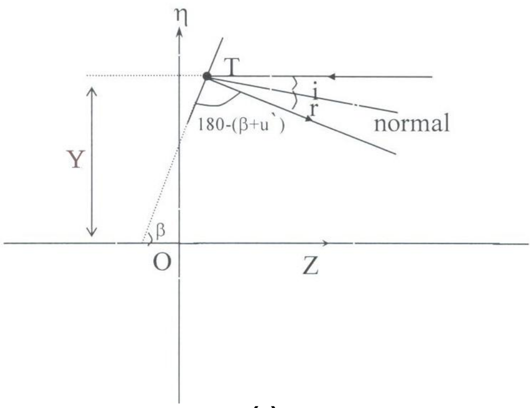
$$\frac{d\eta'}{dZ'} = \frac{d\eta'}{dY}\cdot\frac{dY}{dZ'}$$

hence

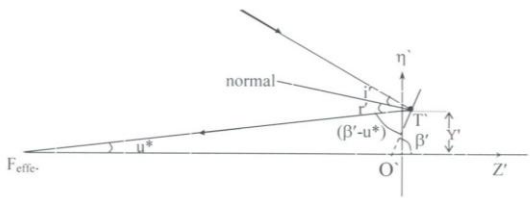
$$\frac{dZ'}{dY} = \frac{d\eta'}{dY}.\cot\beta' \quad (13)$$

By differentiating Eq. (5) with respect to Y, and substituting in Eq. (13), with β' replaced by Eq. (11), then

$$\frac{dZ'}{dY}_{(Cassegrainian)} = \frac{\left[\frac{dY'}{dY} + Z'\frac{d}{dY}tanu^*\right]\tan\left(\frac{u'-u^*}{2}\right)}{\left[1 - tanu^*\tan\left(\frac{u'-u^*}{2}\right)\right]} \quad (14)$$



(a)



(b)

Fig. (3) Ray reflection at the Cassegrainian reflectors (a) Reflection at the Cassegrainian primary reflector (b) Reflection at the Cassegrainian secondary reflector

Obviously, equations (9) and (14) permit, together with Eqs. (10), (1) and (5), a complete computation of two reflectors. For, by means of Eqs. (10), (1) and (5) η and η' from Eqs. (9) and (14) can be eliminated and thus two simultaneous first-order differential equations of the form

$\frac{dZ}{dY} = \xi_{\text{Cassegrain}}(Z, Z', Y)$, and
$\frac{dZ'}{dY} = \xi_{\text{Cassegrain}}(Z, Z', Y)$, are obtained.

These are subject to the boundary condition $Z=Z'=0$ for $Y=0$.

The same procedure adopted above can be followed to calculate the coordinates of points T and T' of the Gregorian type. Referring to Fig. (1), the incoming ray from a distance object point will hit the primary M_P at T (reference to(η,Z) frame) and have the equation, |η| = Y. The derivation will end up to,

$$\frac{dZ'}{dY}_{(Gregorian)} = \frac{\left[\frac{dY'}{dY} + Z\frac{d\,tanu^*}{dY}\right] tan\left(\frac{u'+u^*}{2}\right)}{\left[1 - tanu^*\, tan\left(\frac{u'+u^*}{2}\right)\right]} \quad (15)$$

Again, as with the Cassegrain type, after eliminating η and η', two simultaneous first-order differential equations of the form $\frac{dZ'}{dY} = \zeta_{\text{Gregorian}}(Z, Z', Y)$, and $\frac{dZ'}{dY} = \xi_{\text{Gregorian}}(Z, Z', Y)$, are obtained , which are subject to the boundary condition $Z=Z'=0$ for $Y=0$.

It is noticed that equations (14) and (15) are exactly the same. However, depending on the geometry of the system under consideration , the signs of the coordinates and the slopes of the tangent, must be taken into account.

Table (1) The exact solutions of the two aspheric reflectors (in terms of units) of the Cassegrainian objective

Z	η (=Y)	Z'	η'
8.160793E-05	0.04	1.958586E-05	8.079971E-03
3.264315E-04	8.08	7.834294E-05	1.615982E-02
7.344714E-04	0.12	1.762696E-04	2.423942E-02
1.305626E-03	0.16	3.13363E-04	3.231862E-02
1.999995E-03	0.2	4.799728E-04	3.999745E-02
2.93788E-03	0.24	7.050348E-04	4.847542E-02
3.919986E-03	0.28	9.406965E-04	5.599298E-02
5.222881E-03	0.32	1.253314E-03	6.462914E-02
6.479959E-03	0.36	1.554916E-03	7.198509E-02
7.999933E-03	0.3999999	1.919566E-03	7.997954E-02
9.874462E-03	0.4399999	2.369236E-03	8.885179E-02
0.0117514	0.4799999	2.819436E-03	9.692343E-02
1.351981E-02	0.5199999	3.243564E-03	0.103955
1.567976E-02	0.5599999	3.76154E-03	0.1119438
1.799968E-02	0.6	4.317813E-03	0.1199309
0.0208912	0.64	5.011045E-03	0.1291935
2.358415E-02	0.68	5.656572E-03	0.1372564
2.591933E-02	0.72	6.216265E-03	0.1438807
2.887918E-02	0.7600001	6.925571E-03	0.1518598
3.199897E-02	0.8000001	7.673085E-03	0.1598364
3.527875E-02	0.8400001	8.458798E-03	0.1678107
3.871854E-02	0.8800001	9.282688E-03	0.1757825
4.316878E-02	0.9200001	1.034837E-02	0.1855834
0.047004	0.9600002	1.126658E-02	0.1936286
5.100238E-02	1	1.222364E-02	0.2016707
5.516396E-02	1.04	1.321955E-02	0.2097098
5.831663E-02	1.08	1.397386E-02	0.2155976
0.0627161	1.12	1.502627E-02	0.2235513
6.727549E-02	1.16	1.611667E-02	0.2315017
7.199484E-02	1.2	1.724504E-02	0.2394483
7.841908E-02	1.24	1.878059E-02	0.2498528
8.355956E-02	1.28	2.000889E-02	0.2578701
8.711237E-02	1.32	2.085763E-02	0.263266
0.0943299	1.36	2.258134E-02	0.2738929
9.995975E-02	1.4	0.0239254	0.2818977
0.1036693	1.44	2.481079E-02	0.2870475
0.1117088	1.48	2.672905E-02	0.2978945
0.1155067	1.52	2.763494E-02	0.3028803
0.1241102	1.56	2.968643E-02	0.3138725
0.1279835	1.6	3.060971E-02	0.3186944
0.1371641	1.64	3.279727E-02	0.3298314
0.1411	1.679999	0.0337348	0.3344893
0.147898	1.719999	3.535362E-02	0.3423789
0.157968	1.759999	3.775046E-02	0.3537306
0.1619737	1.799999	3.870357E-02	0.3581429
0.1726525	1.839999	4.124338E-02	0.3696364
0.1766888	1.879999	4.220294E-02	0.3738852
0.1879892	1.919999	4.488844E-02	0.3855191
0.1920431	1.959999	0.0458514	0.3896046
0.2039782	1.999999	4.868532E-02	0.4013783

1. Determination of the Two Cassegrainian Reflector's Profiles

Equations (9) and (14) are integrated numerically by using Runge-Kutta method. As an example, a

Cassegrainian configuration of 1.25 primary focal length ratio and 6.25 objective's focal ratio and of initial parameters, given in terms of units, D=4, G=5 and $f_{effe.}$=25, is examined. A computer program has been written, so that it prints out values of Z and Z' as well as η and η' for a range of parameter Y (step size ΔY =0.04 units) in tabulated form. The output (see Table 1) shows the exact solutions of the two aspheric reflectors. For the purpose of ray tracing and evaluating the optical performance, the two calculated aspherics are imitated by sixteenth power polynomials. Preliminary investigation showed that the optical performance of the system at the wavelength of 550 nm is likely to be diffraction-limited for off-axis angles up to three arcmins.

2. Conclusion

This research is aimed at attaining aplanatism to be utilized in any centered two-reflector objective system. The work under consideration has specifically been focused on deriving formulae for the design of the two aspheric profiles of the Gregorian and Cassegrainian objective types for use in aiming and imaging. These formulae may be utilized by optical designers as a useful and simple mean for generating two-reflector aplanatic configurations of different dimensions convenient for certain application. As an example, a rather fast aplanatic Cassegrainian objective is investigated. Ray trace calculations showed that this type yields diffraction-limited quality up to three arcmins field-of-view (FOV) at 550nm wavelength. Further optimization of the optical performance of the system may be achieved simply by varying the geometric dimensions.

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