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Turbulent Prandtl Number and its Use in Prediction of Heat Transfer Coefficient for Liquids

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Abstract:

A theoretical study is performed to determine the turbulent Prandtl number (Pr_t) for liquids of wide range of molecular Prandtl number (Pr=1 to 600) under turbulent flow conditions of Reynolds number range 10000- 100000 by analysis of experimental momentum and heat transfer data of other authors. A semi empirical correlation for Pr_t is obtained and employed to predict the heat transfer coefficient for the investigated range of Re and molecular Prandtl number (Pr). Also an expression for momentum eddy diffusivity is developed. The results revealed that the Pr_t is less than 0.7 and is function of both Re and Pr according to the following relation:

Prt=6.374Re-0.238 Pr-0.161

The capability of many previously proposed models of Prt in predicting the heat transfer coefficient is examined. Cebeci [1973] model is found to give good accuracy when used with the momentum eddy diffusivity developed in the present analysis. The thickness of thermal sublayer decreases with Reynolds number and molecular Prandtl number.

Keywords: *Turbulent Flow, Heat Transfer Coefficient.*

1. Introduction

Despite many years of intensive research into turbulent diffusion, it is still poorly understood and can only be rather crudely predicted in many cases [Philip and Webester 2003]. Because of highly complex turbulent flow mechanism, the prediction of the transport rates necessarily involves the formulation of conceptual models which embody many simplifying assumptions [Gutfinger 1975]. In the momentum problem the eddy viscosity remains unknown while the eddy conductivity is unspecified in the case of heat transfer. The classical approach for obtaining the transport mechanism for the heat transfer problem follows the laminar approach, namely, the momentum and thermal transport mechanisms are related by a factor, the Prandtl number, hence combining the molecular and eddy viscosities one obtain the Boussinesq relation for shear stress:

$$\frac{\tau}{\rho} = (\nu + \varepsilon_m)\frac{du}{dy} \qquad (1)$$

and the analogous relation for heat flux:

$$\frac{q}{\rho c_p} = (\alpha + \varepsilon_h)\frac{dT}{dy} \qquad (2)$$

The turbulent Prandtl number is the ratio between the momentum and thermal eddy diffusivities, i.e., Prt=εm/ εh. Thus Eq.(2) can be written as:

$$\frac{q}{\rho c_p} = (\alpha + \varepsilon_m/Pr_t)\frac{dT}{dy} \qquad (3)$$

Thus if one knows the eddy diffusivity and turbulent Prandtl number, Prt, the heat transfer problem can be solved [Simpson et. al. 1970]. A number of experimental and theoretical investigations have been devoted to obtain the eddy diffusivity but these for turbulent Prandtl number are less

As the complex nature of the turbulent transport process is not yet well understood, the development of models for the prediction of Prt requires the introduction of many tenuous assumptions regarding the behavior of turbulence. The validity of these assumptions may be verified only indirectly by comparing the predicated Prt values with the measured ones. There are basically two routes by which Prt may be evaluated experimentally [Gutfinger 1975]:

1. Utilizing experimental time-average velocity and temperature (or concentration) profiles together with the integrated Reynolds equations. From equations (1) and (2)

$$Pr_t = \frac{\frac{\tau}{\rho} / \frac{du}{dy} - 1}{\frac{q}{\rho C_p} / \frac{dT}{dy} - 1} \qquad (4)$$

Thus, if the momentum and heat flux variation with the distance from the wall are known, ε m, ε h , and hence Prt can be evaluated.

2. Direct measurement of the Reynolds transport terms ($\overline{u'\upsilon'}$, $\overline{T'\upsilon'}$) and use of the definition of the eddy diffusivity:

$$\frac{\tau_t}{\rho} = \overline{u'\upsilon'} = \varepsilon_m \frac{du}{dy}$$

$$\frac{q_t}{\rho} = \overline{T'\upsilon'} = \varepsilon_h \frac{dT}{dy}$$

Hence,

$$\varepsilon_m = \frac{\overline{u'\,\upsilon'}}{du\ /\ dy} \quad \text{and} \quad \varepsilon_h = \frac{\overline{T'\upsilon'}}{dT/dy}$$

or $$Pr_t = \frac{\overline{u'\upsilon'}}{\overline{T'\upsilon'}} \frac{dT}{dy} \frac{dy}{du}$$

Thus, Prt can be evaluated from the experimentally determined velocity and temperature fields, and the turbulent fluctuating terms $\overline{u'\upsilon'}$ and $\overline{T'\upsilon'}$.

The characteristic feature of all the reported experimentally determined Prt is the great scatter of the data. The results of different investigations lack in agreement and are often conflicting. This situation arises directly from the necessity to differentiate the two measured profiles and determine the shear stress and heat flux profiles for the calculation of Prt. The differentiation procedure greatly amplifies the uncertainties associated with the experimentally measured velocity and temperature profile. These difficulties arise from the inaccessibility of probes to the thin region adjacent to the wall in which the velocity and temperature gradients concentrate, particularly at high molecular Pr [Gutfinger 1975].

Experimentally determined turbulent prandtl numbers have been surveyed by Kestin and Richardson (1963) and then by Blom and de Veres (1968). The results of surveys together with the more recent data of Bolm (1970), Simpson et al. (1970), Sleicher et al. (1973), and Quarmby and Quirk (1969,1972,1974) and Gutfinger (1975) are summarized in Table 1. Although the results are greatly different, various investigators have concluded, on the basis of their own results, that Prt is varyingly affected by the molecular Prandtl number, the flow Reynolds number, and the distance from the wall. The suggested trends of Prt with these parameters for the case of liquid metals (Pr<0.1) differ from those for ordinary fluids (Pr>0.7). For liquid metals Prt is greater than unity and decreases with increasing Reynolds number and the distance away from the wall (Carr and Balzhiser, 1967). This effect is opposite to that for air (Sleicher, 1958), where Prt is less than one, and increases towards unity with increase in Re and distance from the wall. Although the above trends have provided a basis for the formulation of models for the prediction of Prt to be considered in predicting heat transfer coefficient, the picture is by no means conclusive, and many studies have produced results which exhibit a different behavior. The value of Pr=0.7 to 1 is of practical importance because it represents most gases. Also the value of Pr=7 represents water and light liquids near room temperature [Kays 1993]. Quarmby and Quirk (1974) have concluded, on the basis of an extensive study of turbulent flow in a plain circular tube with Prandtl numbers varying from 0.7 to 1200 and Reynolds numbers ranging from 5230 to 23550, that Prt is a simple function of the nondimensional distance from the wall, independent of both the Prandtl number and Reynolds number. They found that Prt varies smoothly from 0.5 at the wall to unity at the tube center. Brienkworth and Smith (1969) (Pr=6) and Eckelman and Hanratty (1972) (Pr=0.7) suggest on the other hand, that Prt is constant and equal to approximately unity over the whole of the boundary layer. Yet a different trend was reported by Simpson et al. (1970) who found that for boundary layer flows of air (Pr=0.77), the local value of Prt near the wall is greater than unity, displaying a maximum value of approximately 1.4 at the wall and decreasing to approximately unity in the outer region. Shlanchyauskas et al. (1974) for Pr>1 found that Prt is constant and equal to 0.75. In view of the contradictory experimental data it is significant to note the analysis of Deissler (1963) based on the statistical nature of the turbulent process. Deissler suggests that Prt approaches unity as the velocity gradient increases, regardless of the molecular Prandtl number of the fluid. Deissler also suggests that the effect of Pr is much greater at low values of Pr (i.e., liquid metals) than at higher ones. This is consistent with the results of both Sliecher (1958) and Quarmby and Quirk (1974). Miyak [1992] and Gurniki et. al [2000] stated that the Prt is between 0.33 and 0.5. Aravinth [2000], developed a resistance-in-series model to quantitatively predict the heat and mass transfer processes for turbulent fluid flow through tubes and circular conduits under uniform wall temperature condition. Kays [1994] examined the available experimental data on Prt for the two dimension boundary layer in circular tube and flat plate. Churchill [2002] analyzed the viscosity and eddy conductivity in fully developed turbulent pipe flow and redefined the Prt directly in terms of the time-averaged fluctuations and stated that it remains an essential parametric variable. Toorman (2003) showed that Prt vary from 1 to 10 in the near wall region and fluctuates around 1 in the fully turbulent region. Crimaldi et al. [2006] measured the distribution of Prt in the laboratory boundary layer and developed an analytical model for Prt and found that it is significantly larger than unity even at large distance from the wall.

The above comments are sufficient to indicate the general inconsistencies and lack of agreement which are characteristic of much of the published work. These inconsistencies, and low accuracy of the available measurement, have hindered the development of a satisfactory empirical theory. This in turn has necessitated the formulation of semi-empirical theories in an attempt to both rationalize the existing data and to provide a starting point for heat and mass transfer calculations. In the present work it is aimed to obtain a model for average Prt from available experimental data for wide range of Reynolds and molecular Prandtl numbers. Also it is aimed to examine the capability of many previously proposed models for Prt to predict heat transfer coefficient by comparing them with the experimentally determined heat transfer coefficient.

Table (2) Models for Turbulent PRANDTL Number for Various Authors.

Equation for Pr_t	Author
$\frac{1}{Pr_t} = n \, Re \, Pr \left[1 - \exp\left(-n \, Re \, Pr\right)\right]$ n=experimental constant=0.000153	Diessler (1952)
$\frac{1}{Pr_t} = \frac{1 + 135 \, Re^{-0.45} \exp\left(-\eta^{0.25}\right)}{2 + 57 \, Re^{-0.46} \, Pr^{0.58} \exp\left(-\eta^{0.25}\right)}$ 0.6<Pr<15 $\eta = \frac{y}{R}$, R=pipe radius	Azer and Chao (1961)
$\frac{1}{Pr_t}$ =0.014Re$^{0.45}$Pr$^{0.2}$[1-exp(-1/{0.014 Re$^{0.45}$Pr$^{0.2}$}]	Aoki(1963)
Pr_t =1.0 low turbulence intensity $Pr_t = \sqrt{Pr}$ high turbulence intensity	Marchello and Toor (1963)
$Pr_t = \frac{2 + 6 \, Pr}{9 \, Pr}$ low turbulence intensity $Pr_t = \frac{2 + 9 \, Pr}{3 + 9 \, Pr}$ high turbulence intensity	Tyldesley and Silver (1968)
$\frac{1}{Pr_t}$ =0.91+0.13Pr$^{0.545}$ 0.7<Pr<100	Graber (1970)
$Pr_t = Pr \frac{\exp\left(y^+ \sqrt{\frac{f}{2}}\right) - 1}{\exp\left(\sqrt{Pr} \, y^+ \sqrt{\frac{f}{2}}\right) - 1}$	Thomas and Rajagopal (1973)
$Pr_t = \frac{1 - \exp(-y^+ / A^+)}{1 - \exp(-y^+ / B^+)}$ A^+ is constant =26 $B^+ = \frac{1}{\sqrt{Pr}} \sum_1^5 C_i (\log(Pr))^{i-1}$ C1=34.96, C2=28.97,C3=33.95, C4=6.33, C5=-1.186	Cebeci (1973)
$Pr_t = \frac{0.0014\{1 - \exp(-Re^{1/2}/2)\}}{0.00124 \, Pr^{-0.112}}$	Rosen and Tragrdh (1995)

2. Theoretical Aspects

The shear stress due to molecular and turbulent transport of momentum is given by Eq.(1)

since $u^+ = u/u^*$, $y^+ = yu^*/\nu$, and $\sqrt{\frac{\tau_w}{\rho}} = u^*$

Eq.(1) becomes, $\frac{\tau}{\rho u^*} = (1+\frac{\varepsilon m}{\nu})\frac{d(u^+u^*)}{d(\frac{yu^*}{\nu})}$

or $\frac{\tau}{\rho u^{*2}} = (1+\frac{\varepsilon m}{\nu})\frac{du^+}{dy+}$; hence:

$\frac{\tau}{\tau_w} = (1+\frac{\varepsilon m}{\nu})\frac{du^+}{dy+}$	5

Since $\frac{\tau}{\tau_w} = \frac{r}{R}$ where R=radius and r=distance from the center=R-y; hence

$\frac{\tau}{\tau_w} = \frac{R-y}{R} = \frac{R^+ - y^+}{R^+} = (1-\frac{y^+}{R^+})$	6

Eq.(5) becomes:

$(1-\frac{y^+}{R+}) = (1+\frac{\varepsilon m}{\nu})\frac{du^+}{dy^+}$	7

where $R+=(Ru^*/\nu)=\sqrt{\frac{f}{8}}\,Re$

thus:

$du^+ = \frac{(1-\frac{y^+}{R+})}{1+\frac{\varepsilon m}{\nu}}dy^+$	8

Equation (8) can be integrated to the limits that at y+=0, u+=0 and at y+=y1+, u+=ub. Where u+b is the dimensionless average bulk fluid velocity(ub/u*) and y1+ is the distance from the wall beyond which the velocity becomes equal to ub. Therefore Eq. (8) becomes

$u_b^+ = \int_0^{y_1^+} \frac{(1-\frac{y^+}{R^+})}{1+\frac{\varepsilon m}{\nu}}dy^+$	9

since $u_b^+ = \frac{u_b}{u^*} = \frac{u_b}{u_b\sqrt{\frac{f}{2}}} = \sqrt{\frac{2}{f}}$,

substitution in Eq. (9) yields

$\sqrt{\frac{2}{f}} = \int_0^{y_1^+} \frac{(1-\frac{y^+}{R^+})}{1+\frac{\varepsilon m}{\nu}}dy^+$	10

Since;

$q_w = \rho C_p(\alpha + \varepsilon_h)\frac{dT}{dy}$	11

$q_w = \rho C_p(\frac{\alpha}{\nu} + \frac{\varepsilon_h}{\nu})\frac{dT}{d(y/\nu)}$	12

Hence:

$q_w = \rho C_p u^*(1/Pr + \frac{\varepsilon_h}{\nu})\frac{dT}{dy+}$	13

$\int_0^{y+} \frac{dy+}{1/Pr + \varepsilon_h/\nu} = \frac{u^*\rho Cp}{q_w}\int_{Tw}^{T} dT$	14

hence:

$(T-T_W) = (q_w/u^*\rho C_p)\int_0^{y+}\frac{dy+}{1/Pr+\varepsilon_h/\nu}$	15

Eq.(15) gives the temperature at any point (y+). If the temperature profile is written in dimensionless form, i.e.:

T+ =(T-Tw)/(Tb-Tw)

If Eq.(14) is integrated from the wall(y+=0 and T=Tw) to the center of turbulent core (y+ =R+ and T=Tb), one obtain

$(T_b-T_w) = (q_w/u^*\rho C_p)\int_0^{R+}\frac{dy^+}{1/Pr+\varepsilon_h/\nu}$	16

dividing Eq. (15) by Eq. (16) gives:

$T^+ = \frac{T-T_W}{T_b-T_W} = \frac{\int_0^{y+}\frac{dy+}{1/Pr+\varepsilon_h/\nu}}{\int_0^{R+}\frac{dy+}{1/Pr+\varepsilon_h/\nu}}$	17

Since Q=hA(Tb –TW)

h=q/(Tb-Tw)

Sub. in Eq.(16)

$u^*\rho Cp/h = \int_0^{R+}\frac{dy+}{1/Pr+\varepsilon_h/\nu}$

Since Nu=hd/k

$u^{*}\rho C_p d/Nu\ k=\int_0^{R+}\frac{dy+}{1/Pr+\varepsilon_h/\nu}$	**18**

$u^{*}=ub\sqrt{f/2}$ =(Re μ/ ρd) $\sqrt{f/2}$

hence from Eq.(18):

$Nu=\frac{\sqrt{f/2}RePr}{\int_0^{R+}\frac{dy+}{1/Pr+\varepsilon_h/\nu}}$	**19**

Since Prt=εm/ εh, thus Eq.(19) becomes:

$Nu=\frac{\sqrt{f/2}RePr}{\int_0^{R+}\frac{dy+}{1/Pr+(\frac{\varepsilon_m}{\nu})/Pr_t}}$	**20**

Accordingly, if the variation of εm/ν with y+ and Prt are known, the value of Nu can be estimated from Eq.(20).

3. New Momentum Eddy Diffusivity Model

The eddy diffusivity behavior in the viscous sublayer, damped turbulence layer, and turbulent core affect greatly the rate of heat (or mass) transport between the wall and bulk. Previous studies [Von Karman 1939, Lin et. al. 1953, Deissler 1952, Wasan and Wilk1964, Rosen and Tragradh 1995, Meignen and Berthoud 1998, Lam 1988, Gurniki et. al 2000, and Wang and Nesic 2003] showed that the eddy diffusivity is function of many variables such as the distance from the wall (y+), Re, and Pr. Most studies [Wasan and Wilke 1964, Townsend 1961, Hughmark 1969, Shaw and Hanraty 1964, Escobedo et. al. 1995, Papavassiliou 1997] showed that in the near wall region (the region of major importance in the property transport) εm /ν proportional to y+3, i.e, εm /ν=A y+3.

In the present analysis the region of interest is divided into three zones for momentum eddy diffusivity variation:

a- Near wall region:

εm /ν=A y+3 0<y+<y1+	**21a**

where y1+ is the distance beyond which the eddy diffusivity becomes linear with y+ rather than y+3 variation.

b- Region with linear variation of eddy diffusivity:

Many studies [Sleicher et al.1958, Levich 1962, Mizushina et. al 1971] showed that in the transition region the eddy diffusivity vary linearly with y+, i.e.,

εm /ν=B y+ y1+< y+ <y2+	**21b**

Best fitting of experimental results for εm /ν obtained by Sleicher et. al. [1958] indicated that outside the viscous sublayer B=0.45. Levich [1962] showed that B=0.4.

c- Near turbulent core region:

The large eddies in the core region and the small variation in turbulent intensity in the central region makes the eddy viscosity constant. Therefore in the present analysis the momentum eddy diffusivity for central region will be considered constant as did by Mizushina et. al. [1971] and Hinze [1975]:

εm /ν=0.07 R+ y+>y2+	**21c**

Now, for predicting momentum eddy diffusivity expression near the wall, Eq. (21a) is inserted in Eq.(10) and the experimental value of friction factor is substituted in Eq.(10). The well known Blasius correlation for friction factor will be adopted, i.e.:

f=0.079Re-0.25	**22**

Hence Eq.(10) becomes

$5.032Re^{0.5}=\int_0^{y_1^+}\frac{(1-\frac{y^+}{R^+})}{1+Ay^{+3}}dy^+$	**23**

The upper limit of integration in Eq.(23) is obtained by equating equations (21a) and (21b), i.e.:

A y1+3= 0.45 y1+

or $y_1^+=\sqrt{\frac{0.45}{A}}$

Thus Eq. (23) becomes

$5.032Re^{0.5}=\int_0^{(0.45/A)^{0.5}}\frac{(1-\frac{y^+}{R^+})}{1+Ay^{+3}}dy^+$	**24**

Substitution of Re in Eq.(24) and performing the integration numerically the value of A can be obtained by trial and error for each value of Re. Fig. (1) shows a plot of A vs Re as obtained from Eq.(24). From best fit method:

A=0.0064Re-0.322	**25a**

Substitution of A in Eq.(21a) gives the expression of eddy diffusivity near the wall.

Therefore,

y1+= $\sqrt{\frac{0.45}{A}}$ **=8.4Re0.161**	**25b**

y2+ is obtained from equating Eqs. (21b) and Eq.(21c), i.e.,

y2+=0.156R+	**25c**

Now to investigate whether Prt is unity over the whole range of Re and Pr the expressions of momentum eddy diffusivity developed in Eqs. (21a, 21b, 21c) is substituted in Eq.(19), i.e., εh=εm (or Prt=1) to estimate Nu and compare with experimental Nu. Thus Eq. (19) becomes:

$$Nu=\frac{\sqrt{f/2}\,\mathrm{Re}\,\mathrm{Pr}}{\int_0^{y_1^+}\frac{dy+}{1/\mathrm{Pr}+0.0064\mathrm{Re}^{-0.322}y^{+3}}+\int_{y^+_1}^{y_2^+}\frac{dy+}{1/\mathrm{Pr}+0.45y^+}+\int_{y_2^+}^{R^+}\frac{dy+}{1/\mathrm{Pr}+0.07R^+}} \quad (26)$$

Insertion of Re and Pr with f from Eq.(22) and performing the integration numerically, Nu can be obtained. The results are plotted in Fig. 2 as compared with the experimental Nu of Friend and Mitzner [1958]. Friend and Mitzner performed an experimental heat transfer study for wide range of Pr and obtain the following relation for 0.5 > Pr> 800:

$$Nu=\frac{\frac{f}{2}\mathrm{Re}\,\mathrm{Pr}}{1.2+11.8\sqrt{f/2}\,(\mathrm{Pr}-1)\,\mathrm{Pr}^{-1/3}} \quad (27)$$

Fig. 2 reveals that the Nu obtained from Eq.(26) is different from that obtained from experimental results indicating that the turbulent Prandtl number is not unity, i.e., εh and εm are not equal. They also indicate that at a particular Pr, the higher the Re is the higher the difference between Nu from Eqs.(26) and (27). Also at a particular Re the higher the Pr is the higher the difference.

4. Examination of the capability of proposed Pr_t models to predict nu value

Figure 2 indicates that Prt is not unity. To investigate which models of Prt presented in Table 1 gives accurate results, these models are substituted into equation (20) with εm/ν taken from Eqs. (21). Thus Eq.(20) becomes:

$$Nu=\frac{\sqrt{f/2}\,\mathrm{Re}\,\mathrm{Pr}}{\int_0^{y_1^+}\frac{dy+}{1/\mathrm{Pr}+(0.0064\mathrm{Re}^{-0.322}y^{+3}/\mathrm{Pr}_t)}+\int_{y^+_1}^{y_2^+}\frac{dy+}{1/\mathrm{Pr}+(0.45y^+/\mathrm{Pr}_t)}+\int_{y_2^+}^{R^+}\frac{dy+}{1/\mathrm{Pr}+(0.07R^+/\mathrm{Pr}_t)}} \quad (28)$$

Now inserting of various models from Table 1 in Eq. (28) and performing the integration numerically, with y1+ and y2+ from Eqs.(25b) and (25c) and f from Eq. (22), the Nu value can be obtained for each value of Re and Pr. Fig. 3 shows comparison between Nu predicted from various models (some models are avoided to prevent confusion) with experimental Nu of Friend and Mitzner (Eq. (27)). The figure shows that most models exhibit considerable deviation. The model of Cebeci [1973] exhibits good agreement for high Pr values.

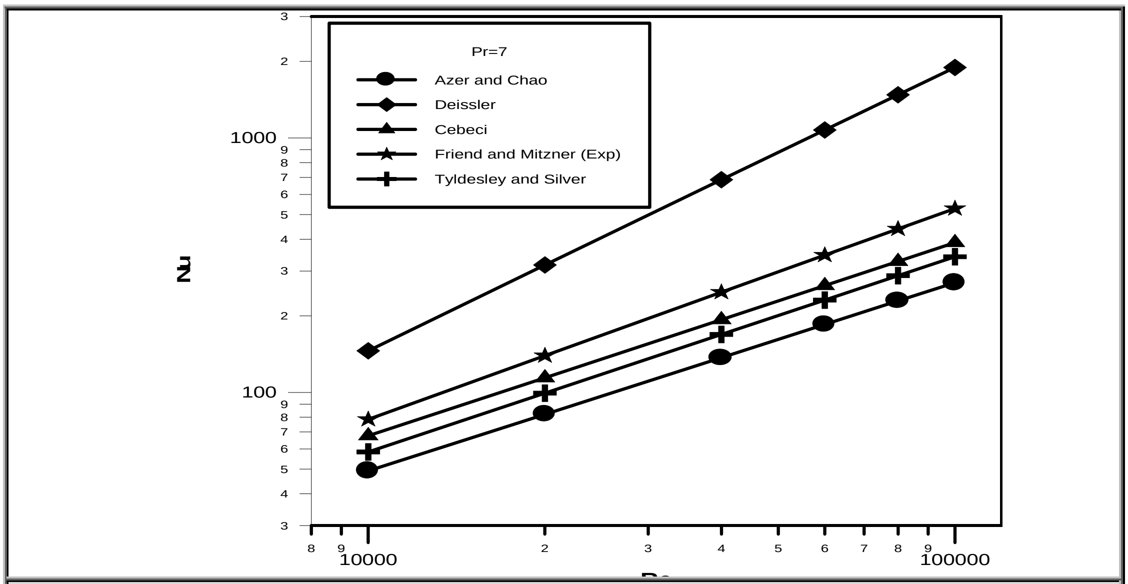

Figure (3) Comparison of Nu Obtained from Various Pr_t Models and Experimental Nu for Pr=7.

5. More accurate model for turbulent PRANDTL number

To obtain more accurate expression for Prt from experimental results of Friend and Mitzner, Nu is estimated from Eq. (27) of Friend and Mitzner and substituted in Eq. (28). Performing the integration of the denominator numerically, Prt is obtained by trial and error for each value of Re and Pr. In present analysis the variation of Prt with the radial distance is ignored, i.e., the average turbulent Prandtl number is determined. Using statistical method the following relation is obtained:

Prt=6.374Re-0.238 Pr-0.161 C.C=0.98	**29**

The results shows that for liquids of Pr > 1 the Prt is less than one indicating that the thermal eddy diffusion is larger than momentum eddy diffusion. Also Prt decreases with increasing Re and Pr indicating that the increase in thermal eddy diffusion is larger than that in momentum eddy diffusion. Many studies [Deissler 1952, Jenkis 1951, Azer and Chao 1961] showed that for liquid metals (Pr< 0.1) the Prt is higher than one. Also it is to be mentioned that Miyak [1992] and Gurniki et. al [2000] found that the Prt is between 0.33 and 0.5.

6. Velocity Profile in the Near Wall Region

The velocity profile in the near wall region is obtained by substituting Eq. (21a) into Eq.(8) and integrating both sides, i.e.:

$$u^+ = \int_0^{y^+} \frac{(1-\frac{y^+}{R^+})}{1+0.0064\mathrm{Re}^{-0.322}\, y^{+3}} dy^+ \qquad \mathbf{30}$$

where y+ varies from 0 to y1+. By performing the integral u+ is obtained for each y+. Fig. 4 shows the velocity profile at various Re as compared with other authors. The figure shows that the velocity profile obtained from present analysis is in good agreement with previous experimental and theoretical studies. Also the figure reveals that very close to the wall the velocity profile is linear and Re has no effect on u+.

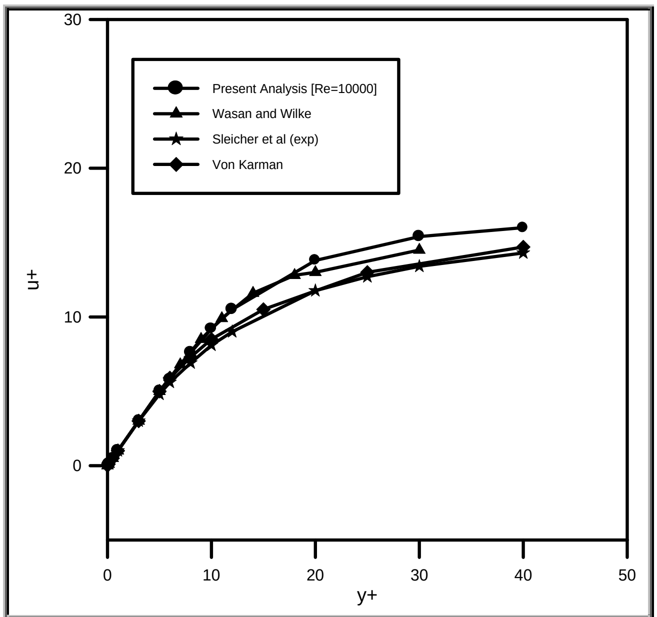

Figure (4) Velocity Profile as Compared with other Workers

7. Temperature Profile

To obtain temperature profile Eq.(17) is used with εh= εm/Prt and Prt from Eq. (29) and the expression of εm/ν from Eqs.(21) for each region of y+. Thus Eq. (17) becomes:

$$T^+ = \frac{\displaystyle\int_0^{y+} \frac{dy^+}{1/\Pr + (\varepsilon_m/\nu)/6.374\,\mathrm{Re}^{-0.238}\,\Pr^{-0.161}}}{\displaystyle\int_0^{y_1^+} \frac{dy^+}{1/\Pr + \frac{0.0064\,\mathrm{Re}^{-0.322}\,y^{+3}}{6.374\,\mathrm{Re}^{-0.238}\,\Pr^{-0.161}}} + \int_{y_1^+}^{y_2^+} \frac{dy^+}{1/\Pr + \frac{0.45\,y^+}{6.374\,\mathrm{Re}^{-0.238}\,\Pr^{-0.161}}} + \int_{y_2^+}^{R^+} \frac{dy^+}{1/\Pr + \frac{0.07\,R^+}{6.374\,\mathrm{Re}^{-0.238}\,\Pr^{-0.161}}}} \quad (31)$$

The denominator of Eq.(31) is constant because the limits of integration are constants while the integral in the numerator varies with y+, i.e., for each value of y+ there is a value of numerator and consequently a value of T+. The expression of εm/ν in the numerator is taken from Eqs.(21a) according to y+, the limit of integral. The values of T+ that are obtained from Eq. (31) are shown graphically in Fig. 5 for Re=40000. The same trend in Fig. 5 is for other values of Re (10000 and 10000). The figure shows the effect of Pr on temperature profile. In a turbulent boundary layer, the gradient is very steep near the wall and weaker farther from the wall where the eddies are larger and turbulent mixing is more efficient [Lienhard 2001]. As the Pr increases the temperature profile close to the wall becomes more flat (the slope increases) indicating that the dimensionless thickness of the of thermal sublayer (δ+T) decreases. Fig. 6 reveals that increasing Re leads to slight increase in δ+T. Using statistical methods the following relation is obtained for δ+T:

$$\delta^+_T = 8.635\,\mathrm{Re}^{-0.067}\,\Pr^{-0.245};\ \mathrm{C.C}=0.99 \quad (32)$$

or

$$\delta_T/d = 8.635\sqrt{\frac{2}{f}}\ \mathrm{Re}^{-1.067}\,\Pr^{-0.245} \quad (33)$$

Hence the thickness of thermal sublayer decreases with Re and Pr. Levich [1962] proposed the following relation for the ratio of viscous sublayer to thermal sublayer:

$$\frac{\delta_b}{\delta_T} = \Pr^{1/3} \quad (34)$$

Hence the thickness of viscous sublayer is

$$\delta_b/d = 8.635\sqrt{\frac{2}{f}}\ \mathrm{Re}^{-1.067}\,\Pr^{0.088} \quad (35)$$

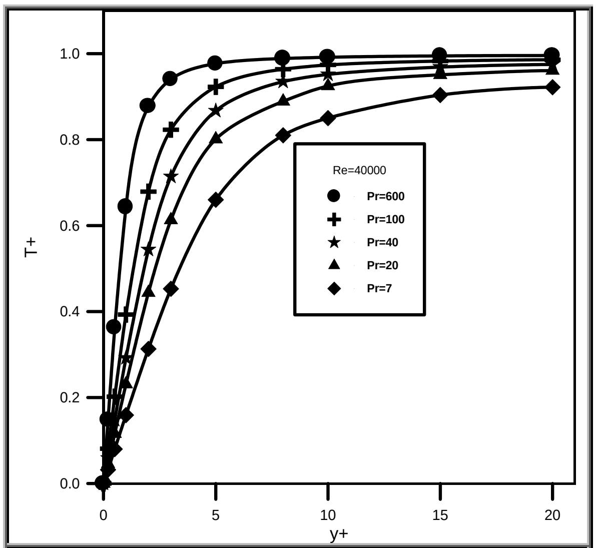

Figure (5) Temperature Profile Obtained from the Models of Eddy Diffusivity and Pr_t of Present Analysis at Various Pr and Re=40000.

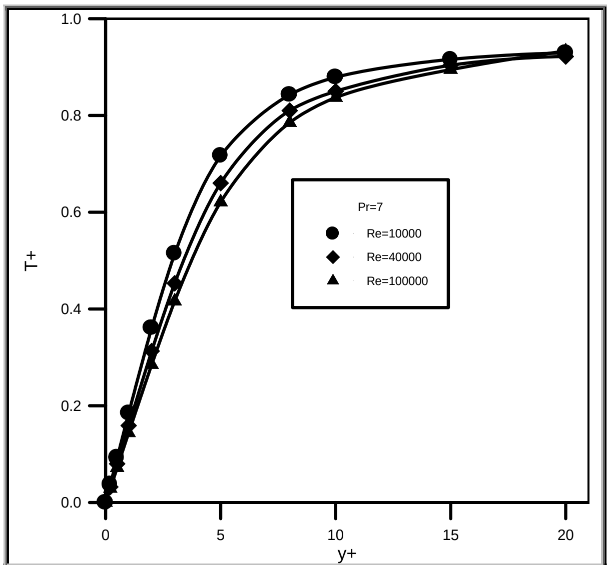

Figure (6) Temperature Profile Obtained from the Models of Eddy Diffusivity and Prt of Present Analysis at Various Re and Pr=7.

Predicting Heat Transfer Coefficient

The expressions of momentum eddy diffusivity and turbulent Prandtl number developed in, Eqs. (21) and Eq. (29) respectively are substituted in Eq.(20) to obtain Nu. Figs. 7 to 9 show a comparison between Nu predicted from present analysis with other experimental works. It is evident that Nu predicted from present analysis is in good agreement with other experimental works for the whole range of Re and Pr.

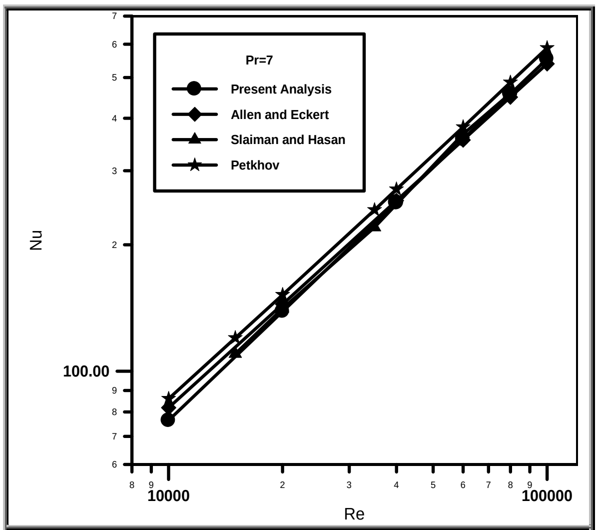

Figure (7) Comparison between Nu Obtained from Present Analysis with Other Works at Pr=7.

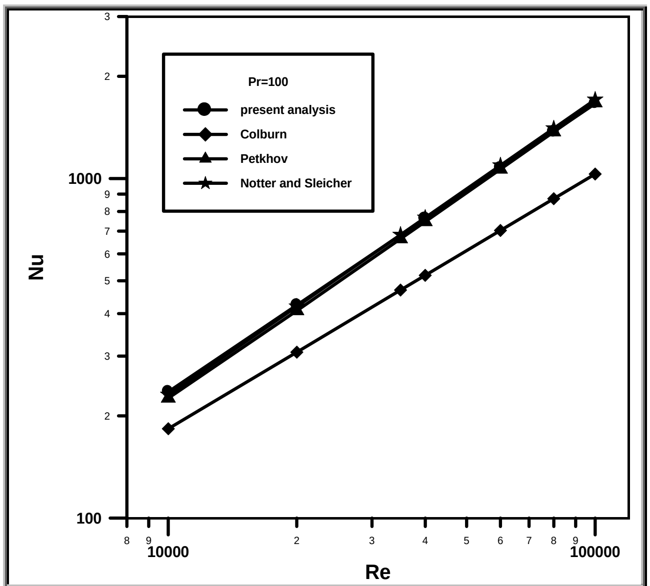

Figure (8) Comparison between Nu Obtained from Present Analysis with Other Works at Pr=100.

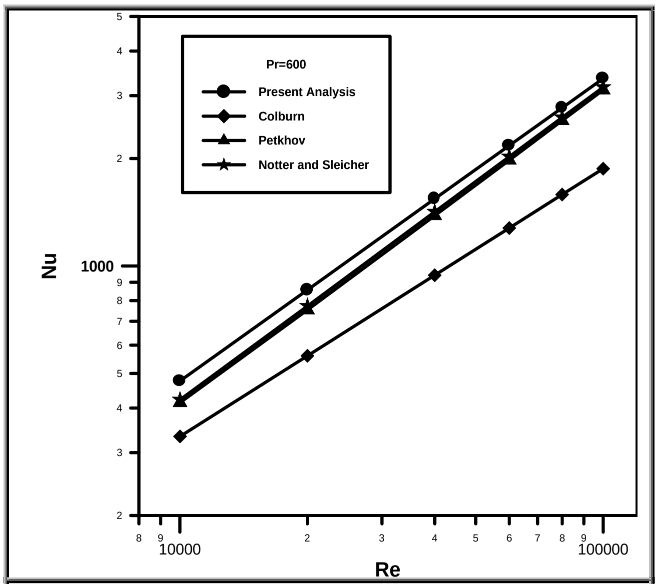

Figure (9) Comparison between Nu Obtained from Present Analysis with Other Works at Pr=600..

CONCLUSIONS

The following points are concluded for the investigated range of Re and Pr:

1- Turbulent Prandtl number plays an important role in determining the value of heat transfer coefficient and the assumption of unity turbulent Prandtl number is very poor for fluids of Pr>1, i.e., the momentum eddy diffusivity is not equal to thermal eddy diffusivity.

2- The Turbulent Prandtl number depends on both Reynolds and molecular Prandtl numbers and its value is generally less than 0.7.

3- The models of turbulent Prandtl number and momentum eddy diffusivity developed in present analysis give more accurate results in predicting the heat transfer coefficient than previously proposed models.

4- The thickness of thermal sublayer is function of Re and Pr while the thickness of viscous sublayer is strongly affected by Re and slightly by Pr.

REFERENCES

1. Allen R. W. and E. R. G. Echert,(1969), "Friction and heat transfer measurements to turbulent pipe flow of water" J. Heat Transfer, August, P.301.
2. Aoki S. 1963 "Consideration on the heat transfer in liquid metals", Bull, Tokyo, Inst. Tech., 54, 63-73.
3. Aravinth, S., 2000, Prediction of heat and mass transfer for fully developed turbulent Fluid Flow through tubes, International Journal of Heat and Mass Transfer, (43), pp.1399-1408
4. Azer N. Z., and B. T. Chao, 1961, Turbulent Heat Transfer in Liquid Metals- Fully Developed Pipe Flow with Constant Wall Temperature, Int. J. Heat Mass Transfer 3,77-83.
5. Blom, J., (1970) “An Experimental Determination of the Turbulent Prandtl Number in a Developing

Temperature Boundary Layer, presented at The Fourth International Heat Transfer Conf., Paris, Versailles.
6. Blom, J., and deVries, D. A., (1968) “On the Values of the Turbulent Prandtl Number,” Third All – Union Heat and Mass Transfer Conference, Paper No. 1.8, Minsk.
7. Brinkworth, B. T., and Smith, P. C., (1969) “Velocity Distribution in the Core of Turbulent Pipe Flow,” Chem. Eng. Sci., 24, 787.
8. Carr, A. D., and Balzhiser, R. E., (1967) “Temperature Profiles and Eddy Diffusivities in Liquid Metals, ” Birt. Chem. Eng. 12, 53.
9. Cebeci, T., (1973) A Model for Eddy Conductivity and Turbulent Prandtl Number, ASME J. Heat Mass Transfer, 95, 227.
10. Churchill S. W., (2002), "A Reinterpretation of the Turbulent Prandtl Number", Ind. Eng. Chem. Res., 41 (25), 6393-6401, 2002.
11. Crimaldi J. P., J.R. Koseff, and S. G. Monismith, (2006)" A mixing-length formulation for the turbulent Prandtl number in wall-bounded flows with bed roughness and elevated scalar sources"; Physics of Fluids, 18, 095102, September.
12. Danckwerts, P.V., (1951) "Significance of Liquid Film Coefficient in Gas Absorption"; Ind. And Eng. Chem., 43, 1460.
13. Deissler R. G., 1952 "Analysis of Fully Developed Turbulent Heat Transfer at Low Peclet Number in Smooth Tubes with Application to Liquid Metals, NACA RM E 52F05.
14. Deissler, G., (1955), "Analysis of tubulent heat transfer, mass transfer and friction in smooth tubes at high Pranadtl and Schmidt numbers", NASA technical report
15. Deissler, R. G., .(1952). “Turbulent Heat Transfer and Temperature Fluctuations in a Field with Uniform Velocity and Temperature Gradients,” Int. J. Heat Mass Transfer, 6, 257
16. Eckelman,L. D., and Hanratty , T. J., (1972) “Interpretation of Measured Variations of the Eddy Conductivity, ” Int. J. Heat Mass Transfer, 15, 2231.
17. Escobedo J. and G.A. Mansoori and U. L. Chicago, (1995) Solid Particle Deposition During Turbulent Flow Production Operations, Society of Petroleum Eng., SPE 29488, Oklahama, April.
18. Friend, W.L. and A. B. Metzner, , (1958) AICHE. J., Vol. 4, No. 4, P. 393,.
19. Graber H., 1970, Der Warmeubergang in Glatten Rohren, zwischen parallelen, in Ringaspalten and langs Rohrbudlen bei exponentiller Warmefluissverteiiung in erzwungener laminarer oder turbulenter Stromung, Int. J. Heat Mass Transfer, 13, 1645-1703.
20. Gurniki F,, K. Fukagata, S. Zahrai, and F.H. Bark, (2000)" LES of turbulent Channel Flow of Binary Electrolyte", J. Appl. Electrochem., 30, 1335-1343.
21. Gutfinger G., 1975 "Topics in Transport Phenomena", Mech. Eng. Dept., Israel Institute of Technology, Israel, Hemisphere Publishing Corporation, John Wiley and Sons, Washington,.
22. Hinze J.O., 1975 Turbulance Phenomena, 2nd Edition., McGraw-Hill, USA.
23. Hasan, 2003, « Heat, Mass, and Momentum Analogies to Estimate Corrosion Rate under Turbulent Flow Conditions », Ph.D. Thesis, Chem.Eng. Dept, Nahrain University, Baghdad, 2003.
24. Jenkis R., (1951) Variation of Eddy Conductivity with Prandtl Modulus and it is Use in Prediction of turbulent heat Transfer Coefficient, Heat Transfer and Fluid Mechanics Institute , Stanford Univ. Press., pp. 147-158.
25. Kays M., Convective Heat and Mass Transfer, 3nd Edition, McGraw-Hill, United State, 1993
26. Kestin, J., and Rechardson, P. D., (1967) “Heat Transfer Across Turbulent, Incompressible Boundary Layers,” Int. J. Heat Mass Transfer, 6, 147(1967).
27. Lam C.Y., (1988) An Eddy Viscosity Turbulence Model, Int. Com. Heat Mass Transfer, Vol.15, pp. 355-363.
28. Levich W.G., (1962) Physicchemical Hydrodynamis, New Jersey, Prentice Hall, pp.689,1962.
29. Lienhard J. H., (2001), "Heat Transfer Text Book", 3rd Ed., Pholgiston, Mech. Eng. Dep., Houston, U.S.A.
30. Lin, C.S., R.W Moulton, and G. L. Putnam, (1953) "Mass Transfer between Solid Wall and Fluid Streams", Ind. Eng. Chem. 43,636.
31. Marchello J.M. and H. L. Toor, (1963), "A Mixing Model for Transfer Near a Boundary", Ind. Eng. Chem. Fund., 2, 1, 8.
32. Meignen R. and G. Berthoud, 1998 Int. J. Heat Mass Transfer 41, 3373.
33. Miyake Y., (1992), Computational Fluid Dynamics, Ed. M. Yasuhara and H. Daiguji, Chap. 10, Univ.Tokyo Press, pp.223 .
34. Mizushina T. and F. Ogino, (1970) "Eddy viscosity and universal Velocity Profile in Turbulent Flow in a Straight Pipe" J. of Chem. Eng. of Japan, 3, 166.
35. Mizushina T., F. Ogino, Y. Oka and H. Fukuda, (1971) Int. J. Heat Mass Trans.,Vol. 14, P. 1705.
36. Notter, R.H. and C. A. Sleicher (1972)" A solution of Turbulent Graetz Problem", Chem Eng. Sci., 27,2073-2093.
37. Papavassiliou D.V. and J. Thomas Hanratty, (1997)Int. J. Heat Mass Transfer 40, pp. 1303,.
38. Petukhov B. S., (1970) Heat Transfer and Friction in Turbulent Pipe Flow with Variable Physical Properties, Advances in Heat Trans., Vol. 6, P. 503,.
39. Philip J.W. and D. R. Webster, (2003)" Turbulent Diffusion ", School of Civil and Environmental Eng., Georgia Institute of Technology, Atlanta, Georgia,30332-0355..
40. Quarmby, A. and R. K. Annad, (1969), "Axisymmetric Turbulent Mass Transfer in a Circular Tube", J. Fluid Mechanics, 38, (3), 433.

41. Quarmby, A., and Quirk, R. K., (1974) “Aximmetric and Non – Aximmetric Turbulent in a Plain Circular Tube at High Schmidt Number,”Int. J. Heat Mass Transfer, 17,143.
42. Quarmby, A., and R. K Quirk, (1972), "Measurement of Radial and Tangential Eddy Diffusivity of Heat and Mass Transfer in Turbulent Flow in a Plan Tube" Int. J. Heat Mass Transfer, 15, 2309.
43. Richrdit, H. NACA TM, 1047, 1943
44. Rosen C. and C. Tragradh, (1995), "Prediction of turbulent high Schmidt number mass transfer using a low Reynolds number k–e turbulence model, Chem Eng. J., 59, 153-159.
45. Sherwood T. K., K. A. Smith, and P.E. Fowles, (1968) The Velocity and Eddy Viscosity Distribution in the wall Region of Turbulent Pipe Flow"; Chem. Eng. Sci., Vol.23, P. 1225-1236,.
46. Shlanchyauskas, A. A., P. P.Vaitekunas, and A. A. Zhukauskas, (1974), Calculations of heat Transfer in Turbulent Boundary Layer of Viscous liquids and an Evaluation pf the Effect of Prt", Int. Chem. Eng., 14, (1), 33.
47. Simpson, R. L, D.G. Whitten, R. J. Moffat, (1970), "An experimental study of the turbulent Prandtl number of air with injuction and suction", Int. J. of Heat and Mass Transfer, Vol.13, pp. 125.
48. Sleicher C. A, A. S. Awad, R. H. Notter, (1973), " Temperature and Eddy Diffusivity Profile in NaK", Int. J. Heat Mass transfer, 16, 1565.
49. Sleicher C.A., J.R. Emeryville, and Calif, (1958) Experimental Velocity and Temperature Profile for Air in Turbulent Pipe Flow, Trans. ASME, pp. 693, April.
50. Thomas L. C., and R. Rajagopal, (1974) Adaptation of Stochastic Formulation of the Surface Rejuuvenation Model to Turbulent Convection Heat Transfer, Chem. Eng. Sci. 29, 1639.
51. Toorman, E.A., (2003), "Validation of macroscopic modeling of particle-laden turbulent flows", Proceedings 6th Belgian National Congress on Theoretical and Applied Mechanics,Gent, 26-27 May.
52. Townsend A.A ,(1961) J. Fluid Mech., Vol. 11, P. 97.
53. Tyldesley J.R. and R. S. Silver, (1968) The Prediction of Transport Properties of a Turbulent Flow, Int. J. Heat Mass Trans., 11,1325..
54. Von Karman T., (1939) "Mechanishe Ahnlichkeit und Turbulenz" Proc. 3rd. Int. Congress Apll. Mech. Stockholm 1, 85.
55. Wang S. and S. Nesic (2003) " On Coupling Co2 Corrosion and Multiphse Flow Model", Corrosion J., Paper No. 03631, P.1.
56. Wasan, D.T and C. R. Wike, (1964), "Turbulent exchange of momentum, mass and heat between fluid streams and pipe wall", Int. J. Heat Mass Trans.,Vol. 7, P. 87.

NOMENCLATURE

A	Area, m2
Cp	Specific heat, kJ/kg.K
d	Diameter
f	Fanning friction factor
h	Heat transfer coefficient, W/m2. C
Nu	Nusselt number, hd/k
Pr	Prandtl number
Prt	Turbulent Prandtl Number
qw	Heat Flux at the wall (W/m2)
qt	Turbulent heat Flux at the wall (W/m2)
R+	Dimensionless radius
Re	Reynolds number, ρdu/μ
T	Temperature
T+	Dimensionless temperature
T'	Fluctuating temperature
u	Local velocity
u*	Friction velocity, m/s
y+	Dimensionless distance from the wall
R	Radius
y	Distance from the wall
u'	Fluctuating velocity in axial direction
v'	Fluctuating velocity in redial direction

GREEK LETTERS

α	Molecular thermal diffusivity, (m2/s).
δT	Thermal layer thickness, μm.
δb	Viscous sublayer thickness, μm.
εm	Eddy diffusivity for momentum transfer, m2/s.
εh	Eddy diffusivity for heat transfer, m2/s.
ν	Kinematic Viscosity m2/s
τ	Shear Stress (N/m2)
ρ	Density (kg/m3)
η	Ratio of the distance from the wall to pipe radious

SUBSCRIPTS

b	Bulk
m	Momentum
h	Heat
w	Wall
t	Turbulent

استخدام عدد براندتل الاضطرابي لحساب معامل انتقال الحرارة للسوائل

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الخلاصة

تم إجراء دراسة نظرية لتحديد عدد براند تل الاضطرابي(Pr_t) للسوائل تحت ظروف جريان مضطرب لعدد رينولد من (100000- 10000) وعدد براندتل ألجزيئي من ١ إلى ٦٠٠ وذلك بتحليل النتائج العملية المستحصلة من دراسات انتقـــال الزخم وانتقال الحرارة التي اجريت من قبل باحثين آخرين. تم الحصول على علاقة لوصف(Pr_t) كدالة لعددي رينولد (Re) و برانتل ألجزيئي (Pr) . تم استخدام هذه العلاقة لحساب معامل انتقال الحرارة كما تم الحصول على علاقة تصـــف الانتشـــارية الدوامية للزخم . دلت النتائج على إن (Pr_t) اقل من 0.7 ويتغير مع Re و Pr حسب العلاقة التالية :

$$Pr_t = 6.374\,Re^{-0.238}\,Pr^{-0.161}$$

كذلك تم اختبار إمكانية بعض العلاقات ال (Pr_t) الموضوعة سابقا من قبل باحثين آخرين لحساب معامل انتقال الحرارة . وجد إن علاقة (Cebeci 1973) تعطي نتائج جيدة إذا استخدمت مع علاقة الانتشارية الدوامية للزخم التي تم الحصول عليهـــا فـــي هذا البحث.