Modeling and Removing Noise under *Statistical Distribution*

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Abstract

Noise is undesirable information in images. This noise occurs due to transmission errors, malfunctioning pixel elements in the camera sensors, faulty memory locations, and timing errors in analog-to-digital conversion.

The researchers were developing several methods to remove these noises such as mean filter, median filter and etc…. in this research we propose a new method to remove the noise from images. In this research we will remove noise using traditional filters and then modeling the total signal image thus modeling the noise generated from the image under the statistical distribution (Multivariate Normal Distribution and Multivariate Poisson Distribution) in order to remove the noise by using the proposed algorithm. Finally, the performance of our proposal is measured by evaluating the MSE and PSNR values.

Keyword: ***Multivariate Poisson distribution, Multivariate Normal Distribution, PSNR, MSE.***

المستخلص

الضوضاء هي اي معلومات غير مرغوب بها في الصورة. الضوضاء تحصل بسبب اخطاء اثناء ارسال الصور صورة بكسل تضررت بسبب هذه الضوضاء ايضاً يحدث نتيجة خلل في اجهزة الاستشعار(الكاميرا)، مواقع الذاكرة الخاطئة واخطاء في توقيت التحويل من تناظرية الى رقمية. الباحثين قاموا بتطوير عدة طرق لازالة الضوضاء مثل Mean filter, median filter الى اخره في هذا البحث اقترحنا طريقة جديدة للازالة الضوضاء من الصور. حيث سنقوم بازالة الضوضاء باستخدام الفلاتر التقليدية ثم نمذجة الاشارة الكلية للصورة، بالتالي نمذجة الضوضاء الناتجة من الصورة تحت التوزيعات الاحصائية (توزيع متعدد المتغيرات عادي و توزيع متعدد المتغيرات بواسون) لكي يتم ازالة الضوضاء

باستخدام الخوارزمية المقترحة. واخيراً اداء الطريقة المقترحة يقاس بواسطة تقييم قيم MSE, PSNR.

1. Introduction

Noise reduction can be represented a pre- processing for high level of image processing, such as edge detection, enhancement, compression …etc [2].

There are many resource for noise in digital image such as (transmission errors, malfunctioning pixel elements in the camera sensors, faulty memory locations, and timing errors in analog-to-digital conversion) that are corrupted an image. In most applications, denoising the image is fundamental to subsequent image processing operations. The goal of noise removal is to suppress the noise while preserving image details.[3]. There are many works on the restoration of images corrupted by noise[1].

Image filters produce a new image from an original by operating on the pixel values. The image signal usually has noise which is not easily eliminated in image processing. According to actual image characteristic noise statistical property.[2]

Filters are used to suppress noise, enhance contrast, find edges, and locate features. If we want to enhance the quality of images, we can use various filtering techniques which are available in image processing. There are various filters which can remove the noise from images and preserve image details and enhance the quality of image.[3]

The success of image demising algorithms is usually evaluated by PSNR values, which essentially measure the mean squared error (MSE) to minimize MSE, maximize PSNR[4].

2. NOISE

Noise is any undesired information that contains an image. Noise appears in images from a variety of sources. The digital image acquisition process, which converts an optical image into a

continuous electrical signal that is then sampled, is the primary process by which noise appears in digital images. At every step in the process there are fluctuations caused by natural phenomena that add a random value to the exact brightness value for a given pixel. In typical images the noise can be modeled with either a Gaussian (normal), or Poisson.[2]

2.1 Noise Categories

The classification of noise relies mainly on the characterizing probabilistic specifications. There are two types of noise Categories in image processing

1. **Gaussian noise**
2. **Poison noise[5]**

3. Traditional Filters

We used number of traditional method to remove noise from digital image which

Are:

3.1 Mean Filter

One of the simplest linear filters, Also cawlled Neighborhood Averaging and it is simple method for remove noise from image the concept of this method where the value of each pixel is replaced by the average of all the values in the local and capture neighborhood for it within window determine size N*N and process the whole image used concept shifting window and the equation explained Mathematically representation of this method:

$$\boldsymbol{Arithmetic\ Mean} = \frac{\mathbf{1}}{\boldsymbol{N^2}} \sum_{(\boldsymbol{r},\boldsymbol{c})\in} \boldsymbol{d}(\boldsymbol{r},\boldsymbol{c}) \qquad (5)$$

Where f(r,c) represent the value of normal gray on location r,c and N^2 represent number of pixel in the N*N window

3.2 Median Filter

It is an order filter. There are several common filter's sizes are used such as (3*3), (5*5), (7*7). The process of this type of filter based on ordering the pixel values in the filter in ascending order, then selects the middle value of the order list and can be explained in the figure

3*3 Sub Image

110	110	114
100	108	104
95	88	85

Median Filter $\Leftrightarrow$ {85, 88, 95,100,104,108,110,110,114}

Median Value: 104

Figure (1): Median Filter

The median filter selects the middle point value from the ordered set, the median filter select the value 104 because there are 4 values above it and 4 values below it. The median filtering operation is performed on an image by applying the sliding window concept, similar to what is done with convolution and the properties of this method not display any unexpected value because result process is value of one of image point.[2]

4. Estimation Of Quality Of Reconstruct Images

4.1 Mean Square Error (MSE)

In statistics and signal processing, a Mean square error (MSE) estimator describes the approach which minimize the mean square error (MSE), which is a common measure of estimator quality.
Let X is an unknown random variable, and let Y be known random variable (the measurement). An estimator $\breve{X}(y)$ is any function of the measurement Y, and its MSE is given by

$$MSE = E\{(\breve{X} - X)^2\} \qquad (6)$$

Where the expectation is taken over both X and Y. the MSE estimator is then defined as the estimator achieving minimal MSE. In many cases, it a is not possible to determine a closed form for the MMSE estimator. In these cases, one possibility is to seek the technique minimize the MES within a particular class, such as the class of linear estimator.[2]

4.2 Peak to Signal Noise Ratio (PSNR)

The phrase peak signal-to-noise ratio, often abbreviated PSNR, is an engineering term for the ratio between the maximum = possible power of a signal and the same content. It is most easily defined via the mean square error (MSE) which for two m*n monochrome images I and K where one of the images is considered a noisy approximation of the other is defined as:

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2 \qquad (7)$$

The PSNR is defined as: $PSNR = 10.\log_{10}\left(\frac{MAX_I^2}{MSE}\right)$

$$= 20.\log_{10}\left(\frac{MAX_I^2}{\sqrt{MSE}}\right) \qquad (8)$$

Here, MAX_I is the maximum possible pixel value of the image. The definition of PSNR is the same except the MSE is the sum over all squared value differences divided by image size and by three.[2]

4.3 Mean

Is the average value of the image brightness points, when owned greater value refer to better image, by applying the following equation:

$$Mean = \frac{1}{m*n}\sum i(m,n) \qquad (9)$$

m,n: Number of rows and columns of image.

4.4 Variance

Is the value of image brightness areas derivation from edges, notice decrease value refer to better image, by applying the following equation:

$$Var = \frac{\sum_{i=1}^{N} Xi^2}{N} - Mean^2 \qquad (10)$$

4.5 Median

Is the center point for image brightness from edges, when the value is increase refer to better image, by applying the following equation:

5. Proposed Algorithm

5.1 Noise model

In this section will modeling total image signal that involving any type from noise as

$$\upsilon(\varsigma) = \chi(\varsigma) + \eta(\varsigma) \qquad (11)$$

Where $\upsilon(\varsigma)$ is image gradient

$\chi(\varsigma)$ is original image

$\eta(\varsigma)$ is noise value

Also can be represented alone type of noise in model , that is we have two models for gaussian and poisson , respectively as follows

$\psi(\varsigma) = \omega(\varsigma) + \epsilon(\varsigma) \qquad (12)$ **where $\epsilon(\varsigma)$ is gaussian noise**

$\varphi(\varsigma) = \phi(\varsigma) + \varepsilon(\varsigma)$ **(13)** **Where** $\varepsilon(\varsigma)$ **is poisson noise**

5. 2 Modeling Noise Under Statistical Distribution

The noise can be modeled under statistical distribution as a function of probability density function , given by :

5.2.1 Multivariate Normal Distribution (MVND)

Derivation of the joint density function for the multivariate normal is complex since it involves calculus and moment-generating functions or a knowledge of characteristic functions which are beyond the scope of this text. To motivate its derivation, recall that a random variable $\tau_i$has a normal distribution with mean $\mu_i$and variance σ^2,written $\tau_i \sim N(\mu_i, \sigma^2)$
if the density function of $\tau_i$has the form

$$f_{\tau_i}(\tau_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(\tau_i - \mu_i)^2}{2\sigma^2}\right] \quad (14)$$

Letting $\tau' = [\tau_1, \tau_2, \ldots, \tau_p]$ where each τ_i is independent normal with mean μ_i and variance σ^2,then the joint density of $\tau'$is

$$f_\tau(\tau) = \prod_{i=1}^{P} f_{\tau_i}(\tau_i) \quad (15)$$

$$f_\tau(\tau) = (2\pi)^{-P/2} |\sigma^2 I_P|^{-1/2} \exp\left[-\frac{(\tau-\mu)'(\sigma^2 I_P)^{-1}(\tau-\mu)}{2}\right] \quad (16)$$

This is the joint density function of an independent multivariate normal distribution, written as $\tau' \sim Np(\mu, \sigma^2 I)$, where the mean vector μ and covariance matrix $\sigma^2 I$ are :

$$E(\tau) = \mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_P \end{bmatrix} \quad , \text{ and } \quad cov(\tau) = \begin{bmatrix} \sigma^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma^2 \end{bmatrix} = \sigma^2 I_P \quad (17)$$

Respectively.
More generally, replacing $\sigma^2 I_P$with a positive definite covariance ME.
a generalization o the independent multivariate normal density to the multivariate normal (MVN) distribution is established

$$f_\tau(\tau) = (2\pi)^{-P/2} |\Sigma|^{-1/2} \exp\left[-\frac{(\tau-\mu)'(\Sigma)^{-1}(\tau-\mu)}{2}\right] - \infty \le \tau_i \ge \infty (18)$$

$$\Sigma = \begin{bmatrix} \sigma^2{}_{11} & \cdots & \sigma^2{}_{1P} \\ \vdots & \ddots & \vdots \\ \sigma^2{}_{P1} & \cdots & \sigma^2{}_{PP} \end{bmatrix} \quad , \text{ and } \quad \Sigma^{-1} = adj(\Sigma)/\mathrm{dat}(\Sigma)(19)$$

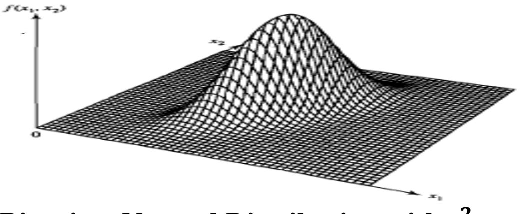

Figure (2): Two Bivariate Normal Distribution with $\sigma^2{}_{11} = \sigma^2{}_{22}, P = 0.75$

5.2.2 Multivariate Poisson Distribution (MVPD)

The joint density function for the multivariate Poisson is complex since it involves calculus and moment-generating functions or a knowledge of characteristic functions which are beyond the scope of this text, Let $\varsigma_i \sim$ Poisson(θ_i) , $i = 0, 1, 2, 3, \cdots m$, then.
Contain from the structure variables as Consider the random variables ζ_i
As follows:

$$\begin{aligned} \zeta_1 &= \varsigma_1 + \varsigma_0 \\ \zeta_2 &= \varsigma_2 + \varsigma_0 \\ \zeta_3 &= \varsigma_3 + \varsigma_0 \\ &\vdots \\ \zeta_m &= \varsigma_m + \varsigma_0 \ (20) \end{aligned}$$

Then $\zeta' = (\zeta_1, \zeta_2, \zeta_3 \dots \dots \zeta_m)$ jointly follow a multivariate Poisson distribution
Given by:

$$p(\zeta) = p(\zeta_1 = \zeta_1, \zeta_2 = \zeta_2 \cdots \cdots \zeta_m = \zeta_m) \qquad (21)$$

$$= exp\left(-\sum_{i=1}^{m} \theta_i\right) \prod_{i=1}^{m} \frac{\theta_i{}^{\zeta_i}}{\zeta_i!} \left(\sum_{i=0}^{s} \prod_{j=1}^{m} \binom{\zeta_i}{i} i! \left(\frac{\theta_0}{\prod_i^m \theta_i}\right)^i\right) (22)$$

Where $s = min(\zeta_1, \zeta_2, \zeta_3 \dots \dots \zeta_m)$
This is the joint density function of unindependent multivariate Poisson distribution, written as $\zeta' \sim$Pm(Θ), where the mean and covariance vector as :

$$E(\zeta) = \Theta = \begin{bmatrix} \theta_1 + \theta_0 \\ \vdots \\ \theta_m + \theta_0 \end{bmatrix} = cov(\zeta) \qquad (23)$$

We must having the following notes

- **Marginally each $\zeta_i$follows a poisson distribution with parameter**
  $\theta_i + \theta_0$
- **Parameter θ_0 is the covariance between all the pairs of random**
  Variables
- **If $\theta_0 = 0$, then the variables are independent**

5.3 Denoise

A frequently asked question in studies involving multivariate data as for signal image is whether there is a image under noise difference in mean of (m or p) variables. A special case of this general problem is whether two noise contained two groups are different on p and m variables of gaussion and Poisson noise, respectively. In practice, it is most often the case that the sample sizes of the groups are not equivalent possibly due to several factors including study dropout. in this method we maximize image mean and minimize variance for a types noise by using our method, given by

- **The total image signal can be represented under the assume model**
- **[$\upsilon(\varsigma) = \chi(\varsigma) + \eta(\varsigma)$], as figure**

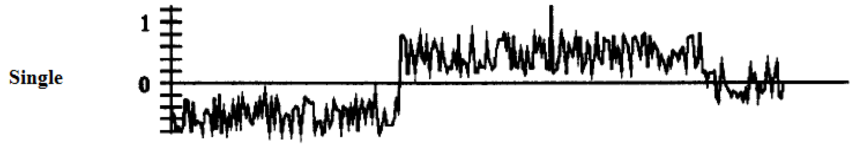

Figure (3):Total Image Signal

- **The total image signal can be represented under the assume model after one denoise [$\upsilon^*(\varsigma) = \chi(\varsigma) + \eta^*(\varsigma)$], as figure**

Figure (4): Image Signal with One Denoise

- **The total image signal can be represented under our method for the model [$\upsilon^*(\varsigma) = \frac{1}{2}\chi(\varsigma) + \eta^*(\varsigma)$], as figure**

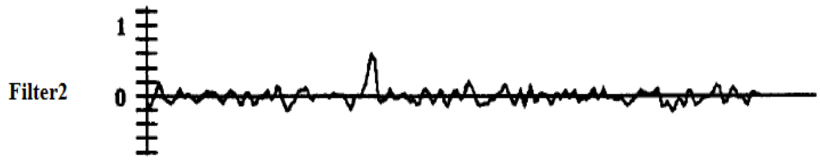

Figure (5): Image Signal with Two Denoise

Our method working on derivation for image signal a function , that is maximize mean of a noise to clean the image from any noise as follows

$$\frac{d}{d\tau}f_\tau(\tau) = \frac{d}{d\tau}\left((2\pi)^{-P/2}|\Sigma|^{-1/2}\, exp\left[-\frac{(\tau-\mu)'(\Sigma)^{-1}(\tau-\mu)}{2}\right]\right) \qquad (24)$$

We know that a parts $\left[\frac{d}{d\tau}((2\pi)^{-P/2})\right]$ *and* $\left[\frac{d}{d\tau}\left(|\Sigma|^{-1/2}\right)\right]$ (23)equal to zero and written above equation by trace for quadric form, leads:

$$\frac{d}{d\tau}f_\tau(\tau) = -(\Sigma)^{-1}(\tau-\mu)\, exp\left[-\frac{(\tau-\mu)'(\Sigma)^{-1}(\tau-\mu)}{2}\right] \qquad (25)$$

Now the [figure 5] draw image signal first derivation a function such as satisfied the model [$\psi^*(\varsigma) = \frac{1}{2}\omega(\varsigma) + \epsilon^*(\varsigma)$] and due clean signal

Under model that is denoise gaussian as:

$$Z(\varsigma) = \frac{1}{2}\omega(\varsigma) + \frac{1}{2}\omega(\varsigma) \qquad (26)$$

$$Z(\varsigma) = \omega(\varsigma) \qquad (27)$$

Where $Z(\varsigma)$ clean image singel by using our method.

$$\frac{d}{d\zeta}p(\zeta)$$
$$=\frac{d}{d\zeta}exp\left(-\sum_{i=1}^{m}\theta_i\right)\prod_{i=1}^{m}\frac{\theta_i^{\zeta_i}}{\zeta_i!}\left(\sum_{i=0}^{s}\prod_{j=1}^{m}\binom{\zeta_i}{i}i!\left(\frac{\theta_0}{\prod_i^m\theta_i}\right)^i\right) \tag{28}$$

$$= exp\left(-\sum_{i=1}^{m}\theta_i\right)\left[\prod_{i=1}^{m}\frac{\theta_i^{\zeta_i}\log\theta_i\zeta_i! - \theta_i^{\zeta_i}\frac{d}{d\zeta}\zeta_i!}{(\zeta_i!)^2}\left(\sum_{i=0}^{s}\prod_{j=1}^{m}\binom{\zeta_i}{i}i!\left(\frac{\theta_0}{\prod_i^m\theta_i}\right)^i\right)\right.$$
$$\left.+\prod_{i=1}^{m}\frac{\theta_i^{\zeta_i}}{\zeta_i!}\left(\sum_{i=0}^{s}i!\left(\frac{\theta_0}{\prod_i^m\theta_i}\right)^i\right)\prod_{j=1}^{m}\frac{d}{d\zeta}\binom{\zeta_i}{i}\right] \tag{29}$$

Now the [figure 5] draw image signal first derivation a function such as satisfied the model [$\varphi^*(\varsigma) = \frac{1}{2}\phi(\varsigma) + \varepsilon^*(\varsigma)$] and due clean signal
under model that is denoise poisson as:

$$K(\varsigma) = \frac{1}{2}\phi(\varsigma) + \frac{1}{2}\phi(\varsigma) \tag{30}$$
$$K(\varsigma) = \phi(\varsigma) \tag{31}$$

Then can be fined MSE of a Gaussian and poisson noise ,respectively as:

$$mse(\tau) = var(\tau) + [bais(t)]^2 \quad and \quad mse(\zeta) = var(\zeta) + [bais(\zeta)]^2$$

Where $K(\varsigma)$ clean image signal by using our method, we will interview the image called koala under all noise

6. Experimental Results:

Figure (6): Original Image

Figure (7): Image with Gaussian Noise

Figure (8): Image with Poisson Noise

GAUSSIAN DENOISE

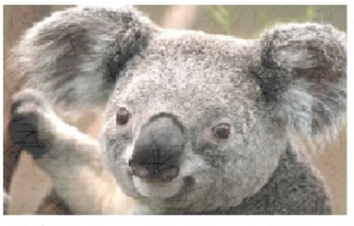

Figure (9): Filter Mean & PSNR =15.60dB

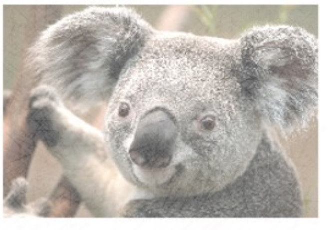

Figure (10): Filter Median & PSNR=17.08dB

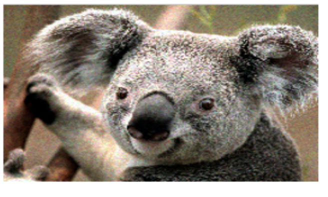

Figure (11): Proposed Method & PSNR=18.01dB

POISSON DENOISE

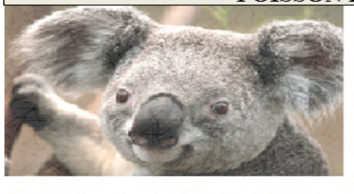

Figure (12): Filter Mean & PSNR =18.70dB

Figure (13): Filter Median & PSNR=18.99dB

Figure (14): Proposed Method & PSNR=20.01dB

TABLE (1): KOALA WITE PSNR&MSE AND OTHER STATISTICAL

METHOD	MEAN	MEDIAN	VARIANCE	MSE	PSNR
ORGENAL	114	112	61.79	81.52	12.33
GAUSSIAN	99.87	97	70.02	82.50	13.03
POISSON	111	109	69.76	81.99	12.87

TABLE2: DENOISE FOR GAUSSIAN NOISE WITH PSNR&MSE

METHOD	MEAN	MEDIAN	VARIANCE	MSE	PSNR
FILTER1	101	99	68.34	81.90	15.60
FILTER2	106	100	65.20	80.87	17.08
OURMETHOD	112	108	62.01	79.77	18.01

TABLE3: DENOISE FOR POISSON NOISE WITH PSNR&MSE

METHOD	MEAN	MEDIAN	VARIANCE	MSE	PSNR
FILTER1	102	99.800	66.43	80.87	18.70
FILTER2	108	103.60	64.23	79.77	18.99
OURMETHOD	113	109.87	61.09	77.09	20.01

We apply the proposed algorithm to many jpg color images, we notice from our experimental results that PSNR value for proposed method to remove Gaussian noise from an image1, image2, and image3 are (18.01 dB, dB, dB) respectively that is better than use mean filter (PSNR=15.60 dB) and median filter (PSNR=17.08 dB).

Also, when remove Poisson noise from an image we see that PSNR value is equal to (20.01dB) which is better than mean filter (PSNR=18.70dB) and median filter (PSNR=18.99 dB).Eventually, we get best method to remove noise from captured image than a traditional method such as mean filter and median filter.

7. Conclusions

The aim of this paper is to remove Gaussian and Poisson noise by using modeling noise under statistical distribution ((MVND) and (MVPD) this method based on the concept of statistical distribution which leads to best results comparing with traditional method. Through by increase (Mean, PSNR value), decrease (variance, MSE).

Comparing the deduced statistical parameters that produced from denoised image are better than traditional methods.

8. Future Work:

1. **For future work, we applied proposed method to both type of noise (Gaussian and Poisson).**

9. References:

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