



Tikrit Journal of Administration and Economics Sciences

مجلة تكريت للعلوم الإدارية والاقتصادية

ISSN: 1813-1719 (Print)



Using Proposed Hybrid method for neural networks and wavelet to estimate time series model

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Abstract:

The research presents a new hybrid model that proposes its use for accurate time series prediction, which combines wavelet transformations to remove de-noise of the data before using it in artificial neural network and applied for time series. To find out the effectiveness and efficiency of the proposed method on artificial neural network models in prediction, the proposed method was firstly applied to the generation time series data (first-order auto-regression) through several simulation examples by changing the value of the parameters and sample size with the generation data being repeated 25 times, secondly the application on the real data represents the monthly average of the price of an ounce of gold in the Kurdistan Region, To compare the simulation results and the real data of the proposed and traditional method, then design a program in Matlab language for this purpose and based on the criteria (MSE, MAD, R2). The results of the research concluded that the proposed method is more accurate than the traditional method in estimating the parameters of the time series model.

Keywords: neural networks, Wavelet transform, Time series, Gold price.

استخدام طريقة مقترحة هجينة للشبكات العصبية والموجة لتقدير أنموذج
السلسلة الزمنية

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المستخلص:

يقدم البحث نموذجاً هجيناً جديداً يقترح استخدامه للتنبؤ الدقيق للسلاسل الزمنية، والذي يجمع بين تحويلات الموجات لتقليل الضوضاء من البيانات قبل استخدامها في الشبكة العصبية

الاصطناعية وتطبيقها على السلاسل الزمنية. لمعرفة مدى فعالية وكفاءة الطريقة المقترحة على نماذج الشبكات العصبية الاصطناعية في التنبؤ، تم تطبيق الطريقة المقترحة أولاً على بيانات السلاسل الزمنية للتوليد (الانحدار الذاتي من الدرجة الأولى) من خلال العديد من أمثلة المحاكاة عن طريق تغيير قيمة المعلمات وحجم العينة مع تكرار بيانات التوليد 25 مرة، وثانياً التطبيق على بيانات حقيقية تمثل المتوسط الشهري لسعر أونصة الذهب في إقليم كردستان. لمقارنة نتائج المحاكاة والبيانات الحقيقية للطريقة المقترحة والتقليدية، ثم تصميم برنامج بلغة ماتلاب لهذه الغرض واعتماداً على المعايير (MSE, MAD, R²). وتوصلت نتائج البحث الى أن الطريقة المقترحة أكثر دقة من الطريقة التقليدية في تقدير معلمات نموذج السلسلة الزمنية.

الكلمات المفتاحية: الشبكات العصبية، التحويل المويجي، السلاسل الزمنية، سعر الذهب.

1. Introduction

Researchers have been cumulatively interested in the development of Artificial Neural Networks (ANN) in the last few years and their application in many research areas such as time series (Ouyang et al, 2021). Despite of acceptable results obtained from neural networks but it requires searching of accurate and modern methods techniques for development, the use of wavelet analysis is one of the most important modern development methods and powerful tool with time series. The idea of combine (ANN) and wavelet transformation is based on the proposal of the hybrid method (proposed model).

Wavelet Transform (WT) analyzes in the time series reduces noise of the data (de-noise), and to generalize the results of wavelet transformation. We will have some applications of time series for non-stationary data that are completely different among themselves, which were obtained from simulation and a real data; after using wavelet transform data are prepare to use in neural networks and get results of hybrid method. The importance of studying the hybrid method with many different models allows us to consider the properties and understanding the behavior of this method when it will be used in the future. The aim of this paper is to analyze the efficiency and capability of (WT) in modeling (ANN), and to select the best suitable model according to the performance criteria (MSE, MAD, and R²).

2. The Methodology:

2-1. Introduction to Time Series Analysis: A time series is a sequential set of data points collected based on time, measured typically over successive times. It is mathematically defined as a set of vectors y_t , $t = 0, 1, 2, \dots, n$ where t represents the time elapsed. The variable y_t is treated as

a random variable. The measurements were taken during an event in a time series are arranged in a proper chronological order. (Adhikari,2013). The importance of time series analysis and forecasting is shown for the future data values are predicted financial, business, and industry production process. Financial time series analysis contains an element of uncertainty, as a result of the added uncertainty in model, statistical theory and methods play an important role in financial time series analysis. (Tsay,2005) (Palit, 2005)

Time series features can be briefly described using time series models. The most popular time series models:

- Auto-regression model (AR).
- moving-average model (MA).
- ARMA model.
- ARIMA model.
- SARIMA models.

2-2. Autoregressive Model: Autoregressive processes are as their name suggests-regressions on themselves. Specifically, a Pth-order autoregressive process $\{y_t\}$ satisfies the equation:

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + e_t \dots (1)$$

The current value of the series Y_t is a linear combination of the p most recent past values of itself plus an “innovation” term e_t that incorporates everything new in the series at time t that is not explained by the past values. Thus, for every t, we assume that e_t is independent of y_{t-1} , y_{t-2} , ..., (Yule 1926) carried out the original work on autoregressive processes. It is instructive to consider the first-order model, abbreviated AR (1), in detail. Assume the series is stationary and satisfies. (Cryer and Chan, 2008).

$$y_t = \delta + \phi_1 y_{t-1} + e_t \dots (2)$$

A commonly used statistic to measure goodness of fit of a stationary model is the R-square (R^2) defined as (Tsay, 2005)

$$R^2 = 1 - \frac{\text{Residual sum of squares}}{\text{Total sum of squares}}$$

For a stationary AR(p) time series model with T observations $\{r_t | t = 1, \dots, T\}$, It is easy to show that $0 \leq R^2 \leq 1$. Typically, a larger R^2 indicates that the model provides a closer fit to the data. However, this is only true for a stationary time series. (Tsay, 2005)

2-3. Moving average model: Constructing a moving average is a simple method for smoothing time series with random fluctuations:

$$y_t = \delta + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \dots \dots (3)$$

We call such a series a moving average of order q and abbreviate the name to MA(q).

2-4. The Mixed Autoregressive Moving Average Model: In an autoregressive integrated moving average model, the variable is assumed to be a linear function and stationary of several past observations and random errors. The type of model is determined by using the autocorrelation function and the partial autocorrelation function when estimating the model (Kitagawa, 2010). If we take a look at the two different functions that can be used to identify autoregressive and moving average processes. The partial autocorrelation function (PACF) is a useful tool to help identify AR(p) models, breaks off after a finite number of lags (p), but the ACF is continuous for the length of the number of lags (Exponential decay to zero). The autocorrelation function (ACF) used to identify MA(q) model breaks off after a finite number of lags (q) and the PACF is continuous for the length of the number of lags.

The model which contain both an autoregressive (AR) term of order p and a moving average (MA) term of order q . Hence, these mixed processes are denoted as ARMA(p,q) model. They enable us to describe processes in which neither the autocorrelation nor the partial autocorrelation function breaks off after a finite number of lags.

$$y_t = \delta + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \dots + \varphi_p y_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \dots (4)$$

Sometimes the model needs differences and the model becomes ARIMA (p,d,q), or there are seasonal fluctuations also the model changes to seasonal SARIMA (p,d,q) \times (P, D,Q). (Kirchgässner and Wolters, 2007) and (Zivot and Wang, 2005)

2-5. Artificial neural networks: Artificial neural networks (ANN) are computer programs or systems based in principle on simulating the work of brain neurons in order to process data and accomplish tasks in a variety of fields, and they are the most famous patterns and methods of machine learning aimed at providing algorithms and software capable of learning with experience, are nowadays (ANN) used in a large variety of modeling and forecasting problems. Neural networks have found their way in recent years into financial analysis. (Arbib, 2003) and (Franses and Dijk, 2000).

The main reason for the increased popularity of artificial neural networks is that these models have been shown to be able to arbitrarily approximate almost any nonlinear function. Hence, when applied to a time series characterized by truly nonlinear dynamic relationships, without the need to construct a specific nonlinear time series model. An often-cited disadvantage of artificial neural networks is that the parameters in the model are difficult to interpret. The estimated ANN does not (necessarily) provide information about the type of parametric time-series models that might be suitable for describing the detected nonlinear patterns. For this reason, and because it is usually difficult to assign meaning to parameter values, ANNs are often considered "black box" models and are built primarily for the purpose of pattern recognition and prediction. (Franses and Dijk, 2000)

2-6. Artificial Neural Networks Architecture: The work of artificial neural networks as a parallel group of simple processing elements (nodes), and the interconnections between these nodes are of special importance when constructing the network, as the arrangement of nodes in layers, and the form of interconnections between layers input and the output layers and hidden layers is called the architecture of the neural network (Figure 1).

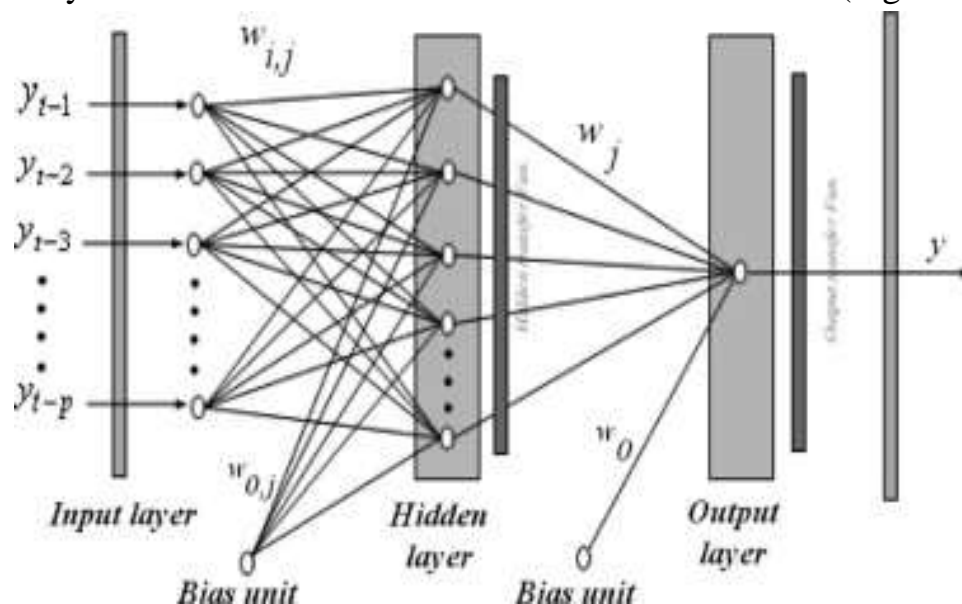


Figure (1): Artificial Neural Networks

The number of layers in the network is calculated without calculating the input layer because no arithmetic operation is performed in it, or it can be known from the number of connections between neurons (Fausett, 1994), (Hagan et al, 1996).

2-7. Artificial Neural Networks in Time Series: Several techniques have been proposed in traditional statistical forecasting methods forecasting time series, for example (ARIMA) have relied on linear models (Box & Jenkins, 1976). However, ARIMA is a general univariate model and it is developed based on the assumption that the time series being forecasted are linear and stationary. However, alternative methods have been researched and developed to overcome the shortcomings of the linear model, among which ANNs emerged as a promising prediction tool. Over the last decade, researchers and practitioners alike have shown growing interest in applying ANNs in time series analysis and forecasting. ANNs are an effective tool to realize any nonlinear input-output mapping. It has been demonstrated that, with sufficient number of hidden layer units, an ANN is capable of approximating any continuous function to any desired degree of accuracy. Due to the nature of their learning process, ANNs can be regarded as nonlinear autoregressive models. (Kamruzzaman, et al, 2006).

2-8. The Basic Steps in Neural Networks and Time Series: In order to implement ANN networks, we need to define main steps in general, and these steps are: (Zhang, 2003) and (Hamid, 2011)

- The first step in designing a time-series neural network is to determine which variables the network takes, whether it is univariate or multivariate.
- Most of the researchers use a univariate model, which the only variable taken into account by the ANN is the time series of intervals (daily, monthly,...), and an ANN with an input layer of N neurons and output layer with only one neuron.

The ANN model of performs a nonlinear functional mapping from the past observations (y_{t-1} , y_{t-2} , y_{t-p}) to the future value y_t , i.e., $y_t = f(y_{t-1}; y_{t-2}; \dots, y_{t-p}; W) + t$

Relationship between the output(y_t) and the inputs (y_{t-1} , y_{t-2} , ..., y_{t-p}) has the following mathematical representation (5)

$$y_t = w_0 + \sum_{j=1}^q w_j \cdot g \left(w_{0j} + \sum_{i=1}^p w_{ij} \cdot y_{t-i} \right) + \varepsilon_t \dots \quad (5)$$

Where, W_{ij} and W_j are model parameters often called connection weights and ε_t represent the difference between target and output. (Zhang, 2003)

- Determining the architecture of the neural network, i.e. determining the number of layers required and the number of neurons within each layer.

- The number of neurons in the hidden layer determines the network's ability to approximate the nonlinear relationships between time series delays and the resulting predictions.
- Initializing the data entered into the neural network, this may help in improving the performance of the network. The data is initialized with some arithmetic transformation it.
- Choosing the appropriate training algorithm is one of the most important factors in the applications of artificial neural networks

2-9. Wavelet: Wavelet are small waves that can be grouped together to form larger waves or different waves. A few fundamental waves were used, stretched in infinitely many ways, and moved in infinitely many ways to produce a wavelet system that could make an accurate model of any wave.

Consider generating an orthogonal wavelet basis for functions $f \in L^2(\mathbb{R})$ (the space of square integrable real functions), starting with two parent wavelet: the scaling function ϕ (also called father wavelet) and the mother wavelet ψ . Other wavelets are then generated by dilations and translations of ϕ and ψ (Donald et al., 2004). The dilation and translated of the functions are defined by formulas (6) and (7).

$$\phi_{k,q}(y) = 2^{k/2} \phi(2^k y - q) \quad k, q \in \mathbb{Z} \quad (6)$$

$$\psi_{k,q}(y) = 2^{k/2} \psi(2^k y - q) \quad k, q \in \mathbb{Z} \quad (7)$$

The discrete wavelet transforms (DWT) is a widely applicable observation processing algorithm that is used in various applications, for instance, science, engineering, mathematics and computer science. DWT decomposes an observation by using scaled and shifted versions of a compact supported basis function (mother wavelet), and provides multiresolution representation of the observation (Iolanda, 2007). Given a vector of an observation \mathbf{y} consisting of 2^k observations, where k is an integer and the DWT of \mathbf{y} due to formula (8).

$$W = w\mathbf{y} \quad (8)$$

Where w is wavelet matrix with $(n \times n)$ dimensions, W is a vector with $(n \times 1)$ dimensions including both scaling and wavelet coefficients. The vector of wavelet coefficients can be organized into $(k+1)$ vectors. $W = [W_1, W_2, \dots, W_k, V_{k0}]^T$. At each DWT, the approximation coefficients are divided into bands using the same wavelet as before, with the result that the

details are appended with the details of the latest decomposition, as in the following formula:

$$y = WW^T = \sum_{k=1}^{k_0} W_k^T W_k + V_{k_0}^T V_{k_0} \quad (9)$$

At each level (k), the observations can be reconstructed of the de-noise data (reduce of the contamination) by the inverse DWT (Gengay, 2002).

2-10. Proposed Method: The proposed model represents Wavelet Transformation (WT) and neural networks called hybrid model (WNN method), the idea of a wavelet network is to adapt the wavelet basis to the training data. Usually the wavelet network has the form of a three-layer network; the first layer is the input layer, use the approximation and details of the original time series components that as the input of the neural network are determined by one of the wavelet method. Each wavelet transform with associated wavelet can be applied to the given signal and produces different level of information. The outcome may differ by the choices of wavelets and approximation levels.

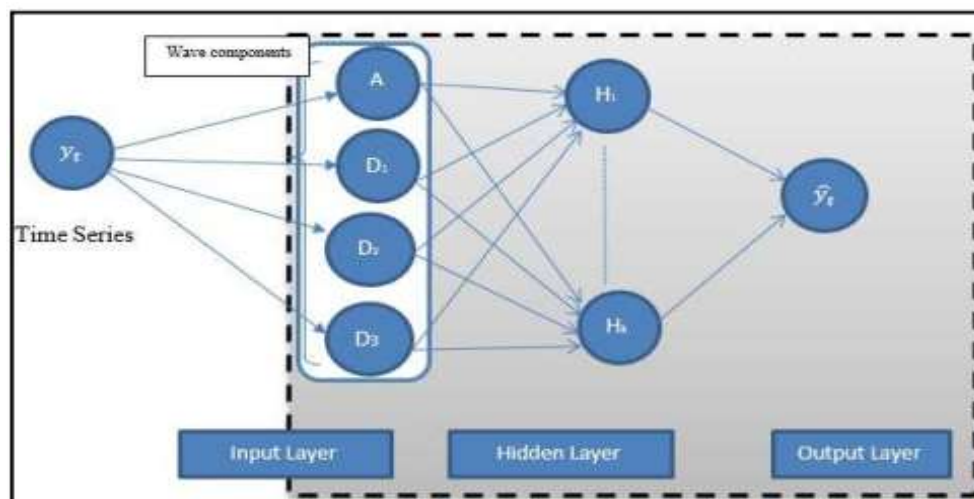


Figure (2): The proposed Hybrid method

Where; y_t is input to wavelet transformation; A, D_1, D_2, D_3 is to ANN; H_1, H_2, \dots, H_K is number of nodes in hidden layer; \hat{y}_t is output of ANN.

The equation of the proposed Hybrid method (WNN) for the input layer of the neural network, can be defined as follows formula 3. It is obtained by replacing the term of wavelet components instead of the time series lag term.

$$y_t = w_0 + \sum_{j=1}^q w_j \cdot g \left(w_{0j} + \sum_{i=1}^p w_{ij} \cdot A, D1, D2, D3 \right) + \varepsilon_t \dots \quad (10)$$

3. The Application: In this section, we will apply the proposed hybrid method of neural networks and wavelets to estimate the time series model through the first application using simulation to generate data and comparing different training algorithms and second application using real data, to know the effectiveness of the waves in the neural network and comparing the results with the neural networks.

3-1. Simulation Study: In this section, simulation studies have been performed that include creating data to understand and evaluate the performance of the proposed method (using wavelets to reduce errors based on the diagram 1) and this allows us to consider the properties and understanding of the behavior of this method, which is indicated by the proposal, and to compare its performance with the classic method.

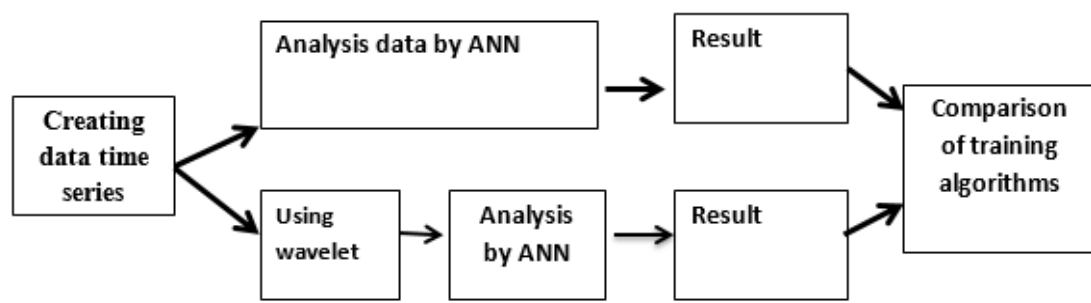


Diagram (1): Comparison between proposed hybrid method and ANN

We started the experiments by writing program designed for this purpose (in Appendix) to generate time series data (first-order autoregression) by MATLAB language and to obtain a set of data we changed the values ($\theta_0 = 0.5$ and 0.6 , $\theta_1 = 0.5$, 0.7 and 0.8) with changing sample size ($n = 100$ and 150) are generated $x_t = \theta_0 + \theta_1 x_{t-1} + e_t$. The first simulation experiment generated data by using ($\theta_0 = 0.5$, $\theta_1 = 0.5$ and $n = 100$), first step in analyzing the original data (the first generated data) using neural networks time series model and calculate (θ_0 , θ_1 , MSE, MAD and R^2), and second step we use wavelets to reduce de-noise from original data and then calculate the previous indicators, and then perform the comparison process between them.

3-2. Comparison between training algorithms (ANN and WNN): As we mentioned in the previous paragraph, the first step is to use the nonlinear autoregressive network with feedback connections enclosing several layers of the network, the following figure (3) shows this model. The NARX model is based on the linear ARX model, which is commonly used in time-

series modeling, based on the MATLAB language and defining the non-linear dynamic model of the neural networks appropriate for the distribution of the series. The future values of a predicted time series are only from the preceding values for that series. Through the program

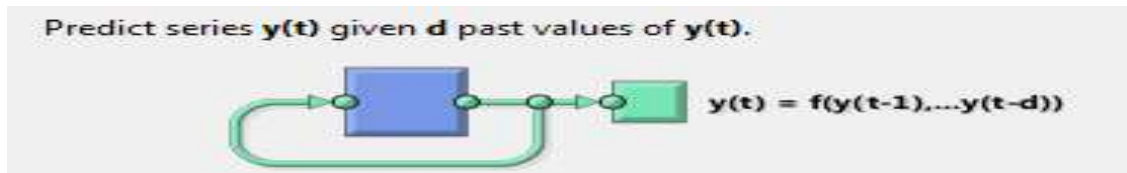


Figure (3): nonlinear autoregressive (NAR) Diagram

The input and predicted vectors will be randomly divided into three groups as follows:

- The first group: 70% will be used for training set.
- The second group: 15% will be used to validation set the network generalization and stop the training before installing it.
- The third group: The last 15% will be used as a completely independent test set to generalize the network.

We compared the proposed method (WNN) and classical method (ANN) to testing performance and validation using statistical indicators: mean squared error (MSE), Mean absolute error (MAD) and R^2 values to show which approach was most effective, table (1) shows the results obtained of the simulation experiment by the following:

Each of the generate data is repeated (500) times for each training experience using different initial random seeds for the error terms, by change the values of parameters(θ_0 and θ_1) and changing sample size. Calculating the average (MSE, MAD, and R^2) for each experiment.

Table (1): The average of main results for the training algorithms (ANN and WNN) with simulated experiment

Sample size $n = 100$							
θ_0	θ_1	MSE (ANN)	MSE (WNN)	R^2 (ANN)	R^2 (WNN)	MAE (ANN)	MAE (WNN)
0.5	0.5	1.138	0.003	23.218	97.608	0.436	0.033
0.6	0.7	1.167	0.051	56.156	96.263	0.782	0.027
0.5	0.8	0.995	0.016	56.899	97.157	0.438	0.032
Sample size $n = 150$							
0.5	0.5	1.008	0.002	25.508	97.751	0.669	0.019
0.6	0.7	1.000	0.013	43.551	97.464	0.791	0.031
0.5	0.8	1.089	0.025	61.113	97.224	0.623	0.033

Table (1) shows the simulation results and that the proposed hybrid method is better than the classic method, where we note that the values of MSE and MAD are less and that the values of R^2 are greater than the value of classical method for all cases of parameter values and all sample.

We used figures (4 and 5) in the first simulation experiment to illustrate the effect of wavelet on error reduction when estimating series data values and comparing them with the original values:

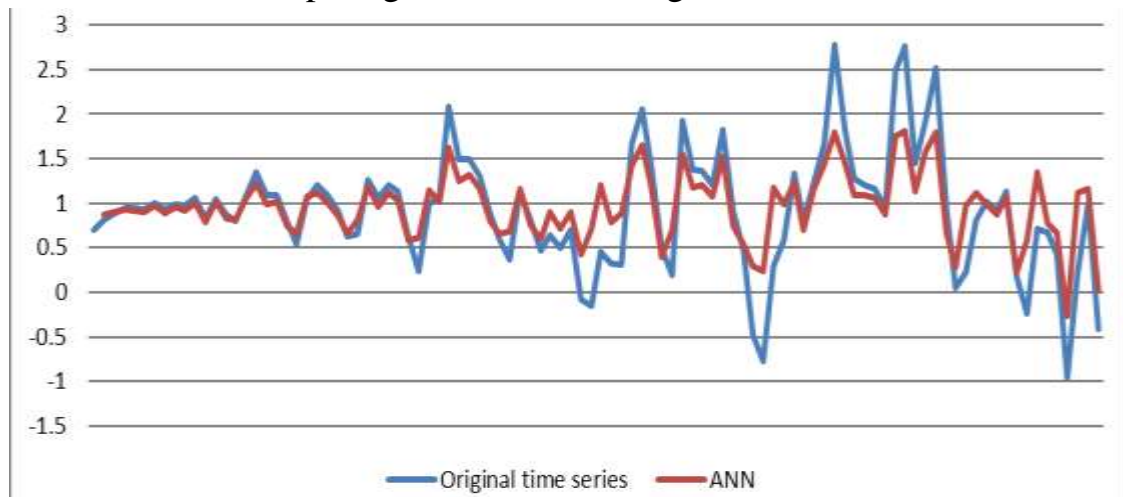


Figure (4): The original (red line) and ANN time series (blue line)

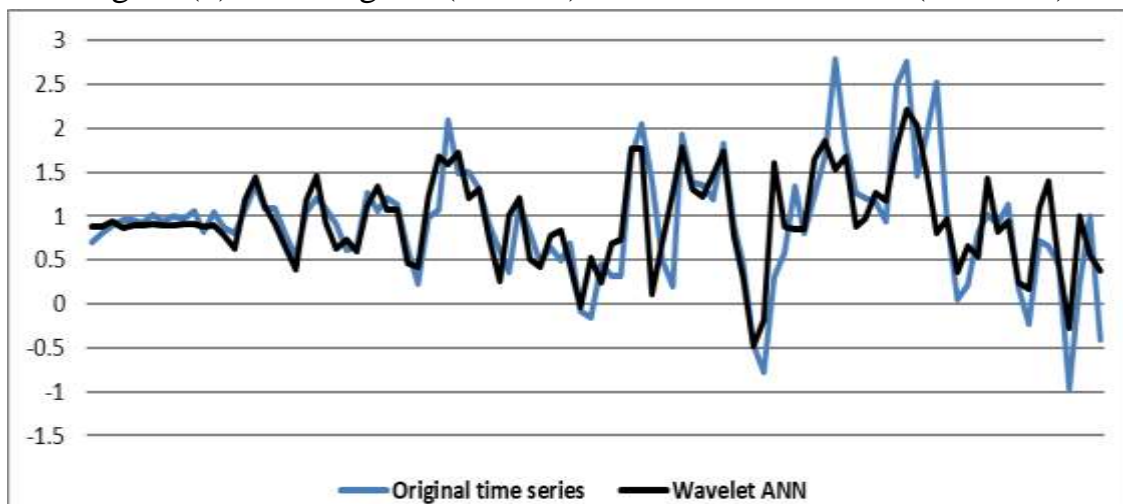


Figure (5): The original (red line) and Wavelet ANN time series (black line)

3-3. Application of real data: To generalize the results obtained through simulation models to use the proposed hybrid method for neural networks and wavelets to estimate the time series model will be used based on the proposed diagram (1) and the comparison of their efficiency by applying on sample of time series data represents the average monthly gold price per ounce measured by Iraqi dinars in the Kurdistan Region for a period of ten years (120 observations) from January 1, 2011 until 31 December 2020, as

in Table (2). To describe and analyze a time series (gold price) data we draw to assess patterns and behavior in the data over time, where we note the high price of gold from mid-2011 to mid-2013 and then it begins to decrease and moderation to mid-2019, then begins to increase and rise more at the end of 2020 as a result of the change in the price of the dollar in Iraq. We cannot see a general trend or seasonal component during this period, as it is shown in figure (4). The data have been taken from the Central Statistical Office of the Kurdistan Region.

Table (2): average monthly gold price in the Kurdistan Region
for the period 1/1/2011 - 31/12/2020

Months	Years									
	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
1	227000	285000	288000	217000	220000	194000	221000	232000	219000	265000
2	231000	303000	280000	228000	218000	206000	222000	226000	223000	269000
3	233000	293000	285000	227000	215000	216000	222000	230000	223000	269000
4	242000	297000	261000	222000	216000	220000	224000	229000	220000	280000
5	256000	280000	248000	221000	216000	224000	221000	227000	220000	293000
6	257000	280000	239000	224000	221000	229000	218000	223000	232000	298000
7	268000	280000	228000	221000	204000	231000	221000	220000	237000	305000
8	306000	288000	234000	218000	194000	237000	223000	215000	247000	335000
9	299000	295000	229000	210000	194000	240000	232000	212000	255000	335000
10	279000	293000	232000	215000	196000	230000	232000	209000	253000	330000
11	289000	294000	216000	206000	189000	222000	231000	208000	252000	345000
12	282000	291000	210000	209000	186000	213000	227000	212000	250000	330000

In the study of the performance of the proposed hybrid method of neural networks and wavelets to estimate a model using historical data for gold prices, the gold price data was divided into three groups with the same ratio mentioned previously, and then it was calculated (MSE, MAD and R^2) to evaluate the performance of the model and Table (2) shows the results that were Obtaining it as we note that the value (MSE and MAD) is less and the value (R^2) is greater in the hybrid method compared to the artificial neural network, and therefore it can be said that the proposed model using the hybrid method for neural networks and wavelets is better than the artificial neural network.

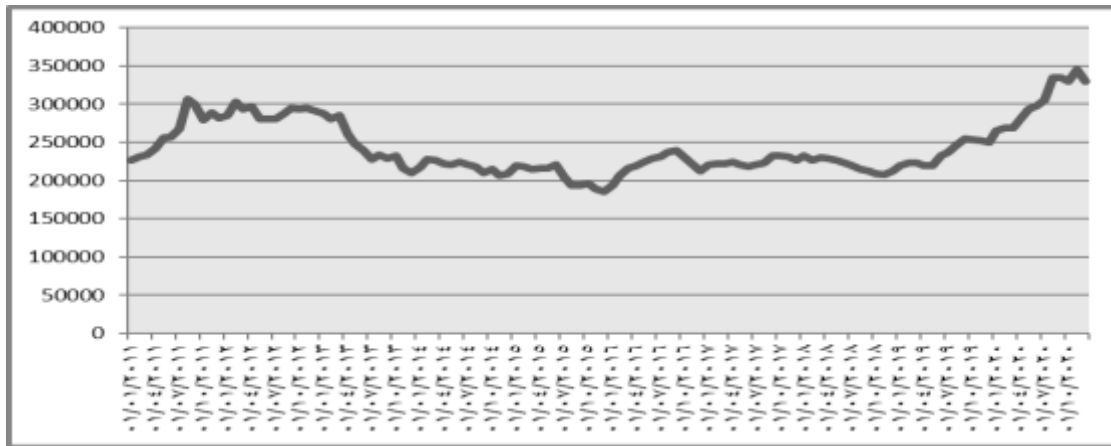


Figure (6): Gold price 1/1/2011 to 1/12/2020

Table (2): The main results of the training algorithms (ANN and WNN) for Gold Price

Method	MSE	MAD	R^2
ANN	89.83	4.85	93.56
WNN	36.34	3.22	97.04

We compared the two models through statistical indicators, and we illustrated the comparison further through a graphic, where figure (7) shows the original gold data and the predicted value by ANN method, as for the figure (8) shows the original data and hybrid method (WNN).

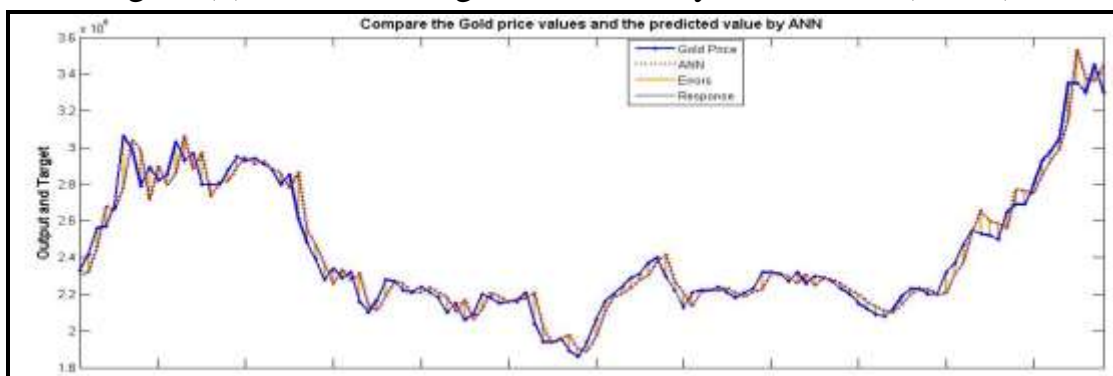


Figure (7): Compare the Gold price values and the predicted values by ANN



Figure (8): Compare the actual values and the predicted values by WNN

4. Conclusion: The study reached a set of conclusions based on the results of the simulation study and the real data as follows:

- A. In this paper, neural network models and wavelet transforms have been combined to form the proposed method (WNN), where the wavelet neural network (proposed method) has reduce the noise of data (de-noise) better than removing by the classical method.
- B. For the simulation study, a set of simulation examples were given by changing the value of the parameters and sample size with the generation data being repeated 25 times. The results showed that the proposed model (hybrid wavelet neural network WNN) enjoys the efficiency, quality and high accuracy (through Statistical measures) better than the traditional methods (ANN). This demonstrates the performance and effectiveness of the proposed model.
- C. To compare the models, real data representing the average monthly price of gold from January 2011 to December 2020 was used. Statistical measures (MSE, MAD and R^2) were used to evaluate the performance of the hybrid wave neural network (WNN) and the classical method (ANN). The results showed that the WNN method is better for modeling gold prices and can be used for forecasting.
- D. The study of simulation and real data concludes that the results of the proposed method (WNN) are more accurate than the classical method (ANN) because the wavelet transforms contribute to improving the data of the (ANN) model and provide useful analyzes of the original time series.

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Appendix (1) Simulation by Matlab

```

clc
clear
t=100;theta0=0.5;theta1=0.5;e=randn(t,1);x(1)=rand(1,1)
for i=2:t+1
    x(i)=x(i-1)*theta1+theta0+randn*i*.01
end
x=x(2:t+1)'
plot(x)

```

Appendix (2) Code Matlab

```

% Program code of ANN
targetSeries = tonndata(q,true,false); feedbackDelays = 1:2; hidden Layer
Size = 10;
net = narnet (feedback Delays, hidden Layer Size); net.
inputs{1}.processFcns
={ 'removeconstantrows','mapminmax'};[inputs,inputStates,layerStates,targ
ets] = preparets (net,{}, {}, targetSeries); net.divideFcn = 'dividerand'; %
Divide data randomly
net.divide Mode = 'time'; % Divide up every value
net. Divide Param.trainRatio = 70/100; net.divideParam.valRatio = 15/100;
net.divideParam.testRatio = 15/100; net.trainFcn = 'trainlm'; % Levenberg-
Marquardt
net.performFcn = 'mse' % Mean squared error
net. plotFcns = {'plotregression', 'plotperform', 'plottrainstate',
'plotresponse',...
'ploterrcorr', 'plotinerrcorr'};
[net,tr] = train (net, inputs, targets, input States, layer States); % Train the
Network
outputs = net (inputs, input States, layer States); % Test the Network
errors = gsubtract (targets, outputs);
performance = perform(net,targets,outputs)
% Recalculate Training, Validation and Test Performance
trainTargets = gmultiply (targets, tr.train Mask); valTargets = gmultiply
(targets, tr. valMask);
testTargets = gmultiply (targets, tr.test Mask); train Performance =
perform(net, train Targets, outputs); val Performance = perform(net, val
Targets, outputs);

```

```
test Performance = perform(net,testTargets,outputs);
netc = closeloop (net); [xc, xic, aic, tc] = preparets (netc,{}, {}, target
Series);
yc = netc (xc,xic,aic); perfc = perform(net,tc,yc);MAE=mae(errors)
nets = removedelay(net); [xs,xis,ais,ts] = preparets(nets, {}, {},targetSeries);
ys = nets (xs,xis,ais); closed Loop Performance = perform(net,tc,yc);
.....
.....
% Program code of Wavelet
Denoisewavelet=wdenoise(d,'Wavelet','sym2','DenoisingMethod','SURE','T
hresholdRule','soft');
```