An iterative learning controller for robot manipulators with flexible joints

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ABSTRACT

In this paper a higher order iterative learning control scheme is proposed for flexible-joint manipulators to improve their tracking accuracy. Tracking control can be carried out by using the proposed learning law as a repeatation of the manipulation task without perfect knowledge of the robot dynamics flexible joint. The proposed learning control scheme utilize more than one past error history in the trajectories that are generated at prior iterations, the convergence proof is given and examples are provided to show the effectiveness of the algorithm . simulation results indicate that the proposed learning control method has better convergence speed and robustness of the algorithm against error in initial setting and disturbances is studied through computer simulations .

التحكم المتعلم بمعالجات الروبوت ذات المفصل المرن

الخلاصة

في هذا البحث تم اقتراح طريقة تحكم متعلم ذات درجة أعلى للتحكم بمعالجات الروبوت ذات المفصل المرن وتحسين أداءها . أن طريقة التحكم المتعلم هي طريقة تحكم تتابعية لا تحتاج إلى حسـابات كبيـرة ويكفيهـا معرفه مسبقة بالنظام المراد التحكم بهِ. أن الطريقة ألمقترحه تقوم بتحسين الأداء في المحاولة الجديدة اعتماداً على بيانات الاشتغال الناتجة في أكثر من محاولة سابقة يتضمن البحث وضع أثبات رياضي لصحة الطريقـة المقترحة بالإضافة إلى وجود مجموعة أمثلة لتوضيح فعالية هذهِ الطريقة. لقد تم دراسة الحساسية لكـل مـن تأثير الضوضاء والخطاء في العودة إلى الوضع الابتدائي من خلال المحاكاة بالحاسبة. أن النتائج تبين الأداء الجيد للطريقة المقترحة.

1.Introduction

A control system with the ability to learn is called learning control system . Although a mechanical device (controller) is not as intelligent as a human operator ,one can still incorporate a certain degree of learning capability in control algorithms, particularly in situations where the system is performing repetitive operations.

Many applications of industrial robots are of this nature ,and hence we can introduce a learning control law to improve the tracking performance of the robot. In learning control, the controller adjusts the system input as the trials repeated such that the output tracks a desired trajectory ,as the number of trials increases . The system input will approach the ideal one. Hence ,the system learns from previous trial to adjust the system input of the next trial. Recently much effort has been directed to the learning control design for robot repetitive operations [1-4].

Many papers apply the learning control method for the flexible joints robot for example in [5] ,a two –stage control scheme has been proposed ,first the motor reference trajectory is learned iteratively and then the required torque input is learned iteratively . In [6,7] a linearized robot dynamics have been used , further, Wang [8] has proposed a simple iterative learning control law for improving the tracking performance for robot manipulator with flexible joint. It was shown that the learning control algorithm may use position, velocity and acceleration error in updating the command input . In [9] a simple iterative learning control scheme using internal model is used for the control of robot manipulator with flexible joints. In this paper, an iterative learning control scheme is used for the control of robot manipulator with flexible joints , the control signal is synthesized from three past-history data. A convergence proof is given and shows that the proposed third order learning control method does not need exact mathematical description of the system being controlled. Many examples are considered and simulation results for a two-link flexible joint robot manipulator are given and show the effectiveness of the proposed algorithm in reducing the error in motion of the flexible joint robot .Robustness of the algorithm against error in initial settings and disturbances is studied through computer simulations.

2. Learning control design

Consider a robot manipulator with flexible joints .The flexibility is assumed only on the rotary direction of the rotor and link angles ,which are referenced to the same axis .The control input is applied to the motor ,and the motor torque is passed through the flexible transmission to turn the link on the other side. The dynamics of robot with flexible joints can be modeled by the following equations [7]

$$\begin{bmatrix} M(q) & R(q) \\ R^T(q) & J \end{bmatrix} \cdot \begin{bmatrix} \ddot{q} \\ \ddot{\theta} \end{bmatrix} +$$

$$\begin{bmatrix} (\beta(q,\dot{q}) + N_1(q,\dot{\theta})\dot{q} + N_2(q,\dot{q})\dot{\theta} + g(q) + k(q-\theta) + f_1\dot{q}) \\ N_3(q,\dot{q})\dot{q} - k(q-\theta) + f_m\dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \text{.......(1)}$$

In eq.1, q is *nx1* vector comprising the n link angles, $\dot{q}$ is the link angular velocity vector , θ is nx1 vector comprising the rotor angles . $\dot{\theta}$ is the rotor angular velocity vector and u is nx1 comprises the control inputs applied at the n joint motors. $M(q)$ and J are the inertia matrices of the manipulator and the joint motors respectively, while

$$\beta(q,\dot{q}) = \left[\frac{d}{dt}M(q)\right]\dot{q} - \frac{\partial}{\partial q}\left[\frac{1}{2}\dot{q}^T M(q)\dot{q}\right]$$

combines the centrifugal and coriolis terms ,and $g(q)$ is the gravity term . The stiffness of the elastic joints are characterized by the nxn matrix k ,whose diagonal elements are the stiffness of the flexible joint transmissions.

The gyroscopic coupling matrix $R(q)$ is strictly upper triangular and the matrices N_i , $i=1,2,3$,..... have linear dependent on velocity and are zero when $R(q)$ is a constant matrix . The matrices f_1 and f_m are diagonal positive - semi definite, representing friction at the link and at the motor side of the transmissions. To use a learning controller to improve the tracking performance of such robots with joint flexibility , a feed back controller is assumed available to guarantee the closed –loop stability. A simple PD (proportional plus derivative) feedback type with gravity compensation is used to guarantee global stability of flexibility of flexible joint robots[7].

The objective of this paper is to design a learning controller for flexible joint robots. In particular, $q_d(t), \dot{q}_d(t),$ and $\ddot{q}_d(t)$ are given as link angle trajectories to be followed repeatedly in the time interval [$0,T$] .We assume that the rotor angles are available for on-line feedback control purposes . We also assume that the link angles are available for off-line computation after an operation task is completed.

The controller is made of two parts in the k^{th} operation. These are

$$u = u_1 + m_k \qquad \text{...... (2)}$$

Where u_1 and m_k are designed to guarantee stability and to improve tracking performance respectively . The first part is a stabilization feedback controller that is the same at every operation cycle, as PD controller

$$u_1 = C\left(q_d - \theta, \dot{q}_d - \dot{\theta}\right) \qquad \text{......(3)}$$

The controller uses q_d and $\dot{q}_d$ as reference trajectory for θ and $\dot{\theta}$ because computation of $\dot{\theta}_d$ and θ_d requires exact knowledge of robot dynamics and parameters , which are not available. But we know that q_d and θ_d are close if the joint transmissions are stiff [9] .Design of such stabilization controllers has been successfully carried out in many papers, for example in [5-9]. Since such feedback control can guarantee closed-loop stability [8].
It can be assumed the velocity tracking error is uniformly bounded ,i.e.
$\left\|\dot{q}_d(t) - \dot{q}(t)\right\| \le \varepsilon$ where ε is a constant .

3.Learning control of motion for robot manipulators :

The closed-loop robot dynamics can be written as

$$\begin{bmatrix} M(q) & R(q) \\ R^T(q) & J \end{bmatrix} \cdot \begin{bmatrix} \ddot{q} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} (\beta(q,\dot{q}) + N_1(q,\dot{\theta})\dot{q} + N_2(q,\dot{q})\dot{\theta} + g(q) + k(q-\theta) + f_1\dot{q}) \\ N_3(q,\dot{q})\dot{q} - k(q-\theta) + f_m\dot{\theta} - C(\theta,\dot{\theta},q_d,\dot{q}_d) \end{bmatrix} = \begin{bmatrix} 0 \\ m \end{bmatrix} \qquad \text{......(4)}$$

Define

$$y = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \quad , \quad z = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \quad , \quad x = \begin{bmatrix} y \\ z \end{bmatrix}$$

The system eq. 4 can be rewritten in state space form

Where $\dot{x} = D^{-1}(q)F(x) + D^{-1}(q)E \cdot m$ ……..(5)

$$D(q) = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & M(q) & 0 & R(q) \\ 0 & 0 & I & 0 \\ 0 & R^T(q) & 0 & J \end{bmatrix}$$

$$F(x) = -\begin{bmatrix} \dot{q} \\ \beta(q,\dot{q}) + N_1(q,\dot{\theta})\dot{q} + N_2(q,\dot{q})\dot{\theta} + g(q) + k(q-\theta) + f_1\dot{q} \\ \dot{\theta} \\ N_3(q,\dot{q})\dot{q} - k(q-\theta) - C(\theta,\dot{\theta},q_d,\dot{q}_d) + f_m\dot{\theta} \end{bmatrix}$$

$$E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Without loss of generality of robot systems ,we assume that eq.5 has the following properties :
1. Every operation ends in a finite time interval T; i.e. $t \in [0,T]$.
2. The functions $D(q(t))$ and $F(x(t))$ are bounded on the interval $[0,T]$

3. The functions $D(q(t))$ and $F(x(t))$ are Lipschitzain functions of their arguments in the interval $[0,T]$.
4. Repeatability of the initial setting is satisfied , i.e , the initial state $x_k(0)$ of the system is set to the same value at the beginning of each operation, as $x_k(0) = x_0$ for $k=1,2,\ldots$.. where k denotes the trial number of operation.
5. Each output trajectory $y_k(t)$ can be measured without noise and hence the error signal $e_k = y_d(t) - y_k(t)$ can be used in the construction of the next input.
6. The desired operations trajectory is given as $x_d(t)$ prior over the time duration $t \in [0,T]$. The closed loop equation for the k^{th} operation is given by

$$\dot{x}_k = D^{-1}(q_k)F(x_k) + D^{-1}(q_k)E \cdot m_k \qquad \text{……..(6)}$$

The second part of eq. 2 is a biased function. It is updated between two consecutive cycles according to the following learning law :

$$m_{k+1}(t) = A_1 \cdot m_k(t) + A_2 \cdot m_{k-1}(t) + A_3 \cdot m_{k-2}(t) + B_1(\dot{x}_d(t) - \dot{x}_k(t)) + B_2(\dot{x}_d(t) - \dot{x}_{k-1}(t)) + B_3(\dot{x}_d(t) - \dot{x}_{k-2}(t)) \qquad \text{………(7)}$$

Where $A_1, A_2, A_3, B_1, B_2, B_3$ are the bounded learning gains which are *nxn* symmetric and positive-definite and satisfies the conditions

$$\left\|A_1 - B_1 D^{-1}(q_k)E\right\|_\infty + \left\|A_2 - B_2 D^{-1}(q_{k-1})E\right\|_\infty + \left\|A_3 - B_3 D^{-1}(q_{k-2})E\right\|_\infty \langle 1 \qquad \text{.....(8)}$$

and $$A_1 + A_2 + A_3 = I \qquad \text{...…….(9)}$$

Assume that at every operation the initial state is set the same that is , $x_k(0) = x_d(0)$ for *k=0,1,2,3,…*Then a sequence of input $m_k(t)$ will be generated such that $m_k(t) \to m_d(t)$ uniformly for $t \in [0,T]$ and the state variables $x_k(t)$, generated by this control are such that $x_k(t) \to x_d(t)$ uniformly for $t \in [0,T]$ as $k \to \infty$.

In proving the convergence of the proposed learning control algorithm, we use the following norm definitions : the norm of an n – vector f is $\|f\| = \sqrt{\sum_{i=1}^{n} x_i^2}$

,and the induced norm of a matrix A is $\|A\| = \sqrt{\max_{eignvalue} \cdot A^T \cdot A}$; the α norm for a function h, is $\|h\|_\alpha = \sup \cdot e^{-\alpha t}\|h\|_\infty \quad t \in [0,T]$

Define $w_1 > 0$ for all q so that $w_1 I \le \begin{bmatrix} M(q) & R(q) \\ R^T(q) & J \end{bmatrix}$.

4.Convergence proof :

Consider the feedback control robot system with joint flexibility as given by eq.1- 7. It was shown in [5] that when the desired link angle trajectories q_d are

given, the desired motor angle variables $\theta_d(t)$, $\dot{\theta}_d(t)$, $\ddot{\theta}_d(t)$ and the desired biased term $m_d(t)$ exit. The dynamics at the desired state trajectory is given by

$$\dot{x}_d = D^{-1}(q_d)\cdot F(x_d) + D^{-1}(q_d)E\cdot m_d \qquad \text{............(10)}$$

taking the difference of eq. 6 and eq.10.

$$\dot{x}_d - \dot{x}_k = D^{-1}(q_k)\cdot[F(x_d) - F(x_k) + E(m_d - m_k)] + \lfloor D^{-1}(q_d) - D^{-1}(q_k)\rfloor$$
$$\cdot[F(x_d) + E\cdot m_d]$$

............(11)

with the assurance of tracking stability ,Local Lipschitz conditions [8] are assumed as follows

$$\|D^{-1}(q_d) - D^{-1}(q_k)\| \le a_1\|q_d - q_k\| \quad , \quad \|F(x_d) - F(x_k)\| \le a_2\|x_d - x_k\|$$

Where a_1 and a_2 are the corresponding local Lipschitz constants . From eq. 7 and 11 we have

$$m_d - m_{k+1} = A_1(m_d - m_k) + A_2(m_d - m_{k-1}) + A_3(m_d - m_{k-2}) - B_1(\dot{x}_d - \dot{x}_k)$$
$$- B_2(\dot{x}_d - \dot{x}_{k-1}) - B_3(\dot{x}_d - \dot{x}_{k-2})$$

With

$$\delta m_k(t) = m_d(t) - m_k(t) \qquad \text{and} \qquad \delta F(x_k) = F(x_d) - F(x_k)$$

$$\delta m_{k+1} = A_1\delta m_k + A_2\delta m_{k-1} + A_3\delta_{k-2}\,\mathrm{m}$$
$$- B_1\lfloor D^{-1}(q_k)[\delta F(x_k) + E\cdot\delta m_k] + \lfloor D^{-1}(q_d) - D^{-1}(q_k)\rfloor\cdot[F(x_d) + E\cdot m_d]\rfloor$$
$$- B_2\lfloor D^{-1}(q_{k-1})[\delta F(x_{k-1}) + E\cdot\delta m_{k-1}] + \lfloor D^{-1}(q_d) - D^{-1}(q_{k-1})\rfloor\cdot[F(x_d) + E\cdot m_d]\rfloor$$
$$- B_3\lfloor D^{-1}(q_{k-2})[\delta F(x_{k-2}) + E\cdot\delta m_{k-2}] + \lfloor D^{-1}(q_d) - D^{-1}(q_{k-2})\rfloor\cdot[F(x_d) + E\cdot m_d]\rfloor$$

..............(12)

Taking norm and using the bounds and Lipschitz conditions, and applying Gronwall-Bellman Lemma [11],we obtain

$$\|\delta m_{k+1}\| \le \|A_1 - B_1D^{-1}(q_k)E\|\cdot\|\delta m_k\| + \|B_1\|\cdot\|D^{-1}(q_k)\|\cdot\|\delta F(x_k)\|$$
$$+\|B_1\|\cdot\|D^{-1}(q_d) - D^{-1}(q_k)\|\cdot\|F(x_d) + E\cdot m_d\| + \|A_2 - B_2D^{-1}(q_{k-1})E\|\cdot\|\delta m_{k-1}\|$$
$$+\|B_2\|\cdot\|D^{-1}(q_{k-1})\|\cdot\|\delta F(x_{k-1})\| + \|B_2\|\cdot\|D^{-1}(q_d) - D^{-1}(q_{k-1})\|\cdot\|F(x_d) + E\cdot m_d\|$$
$$+\|A_3 - B_3D^{-1}(q_{k-2})E\|\cdot\|\delta m_{k-2}\| + \|B_3\|\cdot\|D^{-1}(q_{k-2})\|\cdot\|\delta F(x_{k-2})\|$$

$$+\|B_3\|\cdot\left\|D^{-1}(q_d)-D^{-1}(q_{k-2})\right\|\cdot\|F(x_d)+E\cdot m_d\|$$
$$\le S_1\|\delta m_k\|+S_2\|\delta m_{k-1}\|+S_3\|\delta m_{k-2}\|+b_{B_1}\cdot c\cdot\|\delta x_k\|+b_{B_2}\cdot c\cdot\|\delta x_{k-1}\|$$
$$+b_{B_3}\cdot c\cdot\|\delta x_{k-2}\| \qquad \text{................(13)}$$

Where

$$S_1=\left\|A_1-B_1D^{-1}(q_k)E\right\| \quad , \quad S_2=\left\|A_2-B_2D^{-1}(q_{k-1})E\right\| \quad ,$$
$$S_3=\left\|A_3-B_3D^{-1}(q_{k-2})E\right\|$$
$$c=d\cdot a_2+a_1\cdot a_3 \quad , \quad d=\left\|w_1^{-1}\right\|.$$

b_{B_1}, b_{B_2} and b_{B_3} are the norm bound for B_1 , B_2 and B_3, espectively[11]

$$a_3=\sup_{t\in[0,T]}\|F(x_d)+Em_d\|$$

Because $x_k(0)=x_d(0)$,for all k ,we have from eq. 11

$$\|x_d-x_k\|\le\left\|\int_0^t\left\{D^{-1}(q_k)[F(x_d)-F(x_k)+E(m_d-m_k)]\right.\right.$$
$$\left.\left.+\left[D^{-1}(q_d)-D^{-1}(q_k)\right]\cdot[F(x_d)+E\cdot m_d]\right\}d\tau\right\| \qquad \text{……..(14)}$$

$$\|x_d-x_k\|\le\left\|\int_0^t\left\{\left\|D^{-1}(q_k)\right\|\cdot[\|F(x_d)-F(x_k)\|+\|m_d-m_k\|]\right.\right.$$
$$\left.\left.+\left\|D^{-1}(q_d)-D^{-1}(q_k)\right\|\cdot\|F(x_d)+E\cdot m_d\|\right\}d\tau\right\| \qquad \text{……..(15)}$$

$$\le\int_0^t c\|x_d-x_k\|+d\|m_d-m_k\|d\tau \qquad \text{……..(16)}$$

$$\|x_d-x_k\|\le d\int_0^t e^{c(t-\tau)}\|\delta m_k\|d\tau \qquad \text{..……..(17)}$$

Similarly

$$\|x_d-x_{k-1}\|\le d\int_0^t e^{c(t-\tau)}\|\delta m_{k-1}\|d\tau \qquad \text{..……...(18)}$$

$$\left\| x_d - x_{k-2} \right\| \leq d \int_0^t e^{c(t-\tau)} \left\| \delta m_{k-2} \right\| d\tau \qquad \text{.........(19)}$$

Substituting eqs. 17 ,18 and 19 into 13 gives :

$$\left\| \delta m_{k+1} \right\| \leq S_1 \cdot \left\| \delta m_k \right\| + b_{B_1} \cdot c \cdot d \int_0^t e^{c(t-\tau)} \left\| \delta m_k \right\| d\tau + S_2 \cdot \left\| \delta m_{k-1} \right\| + b_{B_2} \cdot c \cdot d \int_0^t e^{c(t-\tau)}$$

$$\left\| \delta m_{k-1} \right\| d\tau + S_3 \cdot \left\| \delta m_{k-2} \right\| + b_{B_3} \cdot c \cdot d \int_0^t e^{c(t-\tau)} \left\| \delta m_{k-2} \right\| d\tau$$

$$\text{.............(20)}$$

Multiplying both sides by $e^{-\alpha t}$ we have the following:

$$e^{-\alpha t} \left\| \delta m_{k+1} \right\| \leq e^{-\alpha t} \Big[S_1 \left\| \delta m_k \right\| + b_{B_1} \cdot c \cdot d \int_0^t e^{c(t-\tau)} \left\| \delta m_k \right\| d\tau + S_2 \cdot \left\| \delta m_{k-1} \right\|$$

$$+ b_{B_2} \cdot c \cdot d \int_0^t e^{c(t-\tau)} \left\| \delta m_{k-1} \right\| d\tau + S_3 \left\| \delta m_{k-2} \right\| + b_{B_3} \cdot c \cdot d \int_0^t e^{c(t-\tau)} \left\| \delta m_{k-2} \right\| d\tau \Big]$$

$$\text{...............(21)}$$

with $R_1 = \lfloor b_{B_1} \cdot c \cdot d \rfloor$, $R_2 = \lfloor b_{B_2} \cdot c \cdot d \rfloor$, $R_3 = \lfloor b_{B_3} \cdot c \cdot d \rfloor$ we have

$$e^{-\alpha t} \left\| \delta m_{K+1} \right\| \leq S_1 e^{-\alpha t} \left\| \delta m_k \right\| + R_1 \int_0^t e^{-\alpha t} e^{(c-\alpha)(t-\tau)} \left\| \delta m_k \right\| d\tau + S_2 e^{-\alpha t} \left\| \delta m_{k-1} \right\|$$

$$+ R_2 \int_0^t e^{-\alpha t} e^{(c-\alpha)(t-\tau)} \left\| \delta m_{k-1} \right\| d\tau + S_3 e^{-\alpha t} \left\| \delta m_{k-2} \right\| + R_3 \int_0^t e^{-\alpha t} e^{(c-\alpha)(t-\tau)} \left\| \delta m_{k-2} \right\| d\tau$$

$$\text{.............. (22)}$$

Hence ,we have

$$\left\| \delta m_{k+1} \right\|_\alpha \langle \hat{S}_1 \left\| \delta m_k \right\|_\alpha + \hat{S}_2 \left\| \delta m_{k-1} \right\|_\alpha + \hat{S}_3 \left\| \delta m_{k-2} \right\|_\alpha \qquad \text{...............(23)}$$

Where $\hat{S}_1 = S_1 + \dfrac{R_1}{\alpha - c}\left(1 - e^{(c-\alpha)T}\right) \qquad \hat{S}_2 = S_2 + \dfrac{R_2}{\alpha - c}\left(1 - e^{(c-\alpha)T}\right)$

$$\hat{S}_3 = S_3 + \frac{R_3}{\alpha - c}\left(1 - e^{(c-\alpha)T}\right)$$

If we choose $S_1 + S_2 + S_3 \langle\ 1$, and $\alpha > c$ and large enough so that $\hat{S}_1 + \hat{S}_2 + \hat{S}_3 \langle 1$, then eq. 23 converges such that $m_k \to m_d$ uniformly for $t \in [0,T]$ as $k \to \infty$ .Applying the same argument to eq. 17 , we have

$$\left\| x_d - x_k \right\|_\alpha < \frac{d}{\alpha - c}\left(1 - e^{(c-\alpha)T}\right)\left\| m_d - m_k \right\| \qquad \text{……………(24)}$$

Hence the state variables $x_k(t)$ also converge such that $x_d \to x_k$ as $k \to \infty$ .We can notice the following :

1.In the case of perfect repeatability of initialization i.e. $\delta.x_k(0) = 0$ for all k , an exact knowledge of robot dynamics, which makes $\delta m_o = 0$,the tracking error bonds will be zero, and this implies the convergence of the algorithm to the desired trajectories .

2. Although present industrial robots are quite good at repeatability precision, they do not satisfy the repeatability condition thoroughly [9,11]. Therefore it is reasonable to relax some of the conditions mentioned in section (3) ,the following properties are used :

a. The system is reinitialized at the beginning of each operation , namely the initial state x_k at *t=0* .can be set as possible to the specified state x_o .Thereby there exists a sufficiently small constant $\varepsilon_1 > 0$,such that $\left\| x_k(0) - x_o \right\| < \varepsilon_1$ for every *k*, where *k* denotes the trial number of operation . when the knowledge of the robot parameters is limited, $\left\| \delta m_o \right\|_\lambda$ could be dominant in the tracking error bound in eq.(22) and (23) .To reduce the effect due to this factor , we can reduce $\left\| \delta m_o \right\|_\lambda$ from cycle to cycle of operation .The first way is to reduced the weighting of the inaccuracy of the initial guess m_o in the bounds . The second way can be implement by refreshing the memory content of m_o by m_k after the k^{th} operation, where $\left\| \delta x_k(0) \right\|$ is observed to be less than $\left\| \delta x_o \right\|_\alpha$,the bias term m_k is considered a better guess than the initial guess m_o.

b. Each time ,the output $y_k(t)$ is measured within a small specified noise, i.e. $e_k(t) = y_d(t) - [y_k(t) + n_k(t)]$,the noise $n_k(t)$ must satisfy $\left\| n_k(t) \right\|_\infty \le \varepsilon_2$ for some small constant $\varepsilon_2 > 0$.

3.The tracking error bounds remain the same if additional disturbances exist ,as long as they are Lipschitz .In the presence of the disturbances that are discontinuous ,additional terms will appear in the tracking bonds.

Numeical Example :

In this section we consider an example of a two-link manipulator with flexible joints. It is used to illustrate the effectiveness of the algorithm. Computer simulations are carried out to indicate the transient and steady state performance. The simulation is conducted by means of the fourth order Rung kutta method. Here we consider a two-link manipulator as shown in fig.(1).

Both joints are assumed flexible as illustrated in fig.(2) and to be linear. The dynamics of this manipulator takes the following form [10].

$$M(q)\ddot{q} + \beta(q,\dot{q}) + g(q) + k(q-\theta) = 0$$
$$R\ddot{\theta} - k(q-\theta) = u \qquad \text{.............(25)}$$

In the above equations, $q = (q_1, q_2)^T$ contains the link angles, $\theta = (\theta_1, \theta_2)^T$ the rotor angles, and $u = (u_1, u_2)^T$ the two inputs to the joint motors. The manipulator inertia matrix is

$$M(q) = \begin{bmatrix} L_2^2 m_2 + 2L_1 L_2 m_2 \cos(q_2) + L_1^2(m_1 + m_2) & L_2^2 m_2 + L_1 L_2 m_2 \cos(q_2) \\ L_2^2 m_2 + L_1 L_2 m_2 \cos(q_2) & L_2^2 m_2 \end{bmatrix}$$

And the coriolis , centrifugal and gravity terms are combined as

$$\beta(q,\dot{q}) + g(q) = \begin{bmatrix} m_2 L_1 L_2 \dot{q}_2 (2\dot{q}_1 + \dot{q}_2)\sin q_2 + g m_2 L_2 \cos(q_1 + q_2) + \\ g L_1 (m_1 + m_2)\cos q_2 \\ m_2 L_1 L_2 \dot{q}_1 \sin q_2 + g m_2 L_2 \cos(q_1 + q_2) \end{bmatrix}$$

where m_1, m_2 denote the masses of the up arm and low arm L_1, L_2 denote the length of the upper link and lower link respectively ,*g* is the gravity constant . In this simulation , the parameter values are chosen as :

$m_1 = 0.5kg$, $m_2 = 0.5kg$,$L_1 = 1$ m $L_2 = 0.8$ m , $g = 9.8\ m/\sec^2$

and

$$R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \quad kg.m^2$$,the inertia matrix of the joints .

$$k = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} = \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix} \quad kg.m/\sec^2$$,the joint stiffness

The robot is idle at $q_1 = -1.57$ rad and $q_2 = 2.967$ rad.

The control objective is to force the manipulator to track the desired trajectory as given by [9]

$q_{1d}(t) = -1.57 + 0.916(1 - \cos 1.26t)$ rad for $t \le 2.5$ sec

$= 0.261$ rad for $t > 2.5$ sec

$q_{2d}(t) = 2.967 - 1.047(1 - \cos 1.26t)$ rad for $t \le 2.5$ sec

$= 0.8726$ rad for $t > 2.5$ sec

then assume that, a PD feedback control law is designed to ensure the stability of the system in the following form

$$u = k_1'(q - q_d) - k_2'\dot{q} + k_3(\dot{\theta} - \dot{q}) + v \qquad \text{.........(26)}$$

With

$$k_1' = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \quad k_2' = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \quad k_3 = \begin{bmatrix} 200 & 0 \\ 0 & 200 \end{bmatrix}$$

Where v is the learning control input added to improve the motion tracking. the learning law can be derived as follow

$$\begin{aligned} v_{1,k+1} &= A_{11}v_{1,k} + A_{12}v_{1,k-1} + A_{13}v_{1,k-2} + B_{11}(\ddot{q}_{1d} - \ddot{q}_{1,k}) + B_{12}(\ddot{q}_{1d} - \ddot{q}_{1,k-1}) \\ &\quad + B_{13}(\ddot{q}_{1d} - \ddot{q}_{1,k-2}) \\ v_{2,k+1} &= A_{21}v_{2,k} + A_{22}v_{2,k-1} + A_{23}v_{2,k-2} + B_{21}(\ddot{q}_{2d} - \ddot{q}_{2,k}) + B_{22}(\ddot{q}_{2d} - \ddot{q}_{2,k-1}) \\ &\quad + B_{23}(\ddot{q}_{2d} - \ddot{q}_{2,k-2}) \end{aligned}$$

where $A_{1i}, A_{2i}, B_{1i}, B_{2i}$, $i=1,2,3$ are the gain that are chosen to satisfy the condition of eq.8 and eq.9 so that

$A_{11} = 1.2,\quad A_{12} = -0.15,\quad A_{13} = -0.05,\quad A_{21} = 1.2,\quad A_{22} = -0.15,$
$A_{23} = -0.05,$
$B_{11} = 1.065m_1,\quad B_{12} = -0.1m_1,\quad B_{13} = 0.035m_1,\quad B_{21} = 1.065m_1,$
$B_{22} = -0.1m_1,$
$B_{23} = 0.035m_1$

Example:

In this section, three cases are given to illustrate the effectiveness of the proposed learning control algorithm , in each case the performance of the third. Order learning control is compared with that of the first order learning control algorithm proposed in [9] which used PD feedback with learning control part uses one past error history data.

6.1 Case when the dynamics of the plant is known :

In this case we take the exact values of the masses of the up and low arms and assume that they are to be known. Simulation results of the motion leaning control for flexible joint robot are obtained in fig.(3) and fig.(4) which

show the error tracking performance for different trials for q_1 and q_2.Comparison of the performance of our proposed method with that in [9] indicates that a smaller tracking error is obtained by applying our proposed method.

6.2 Effect of error in initial settings:

In this case the robustness of the proposed learning control algorithm against error in initial settings is studied. For this case we take the same settings of the control gains being in case one.

Suppose we have an error in the initial settings, say $q_1(0)=0.002$ and $q_2(0)=0.002$,as shown in fig.(5) and fig.(6). We can observe, from the computer simulation that the tracking performance recover in ten iterations when the third order learning control method was used while a similar performance was achieved after 14 iterations in first order method in [9].

6.3 Effect of disturbances :

Consider the robustness with respect to disturbances. Suppose that there is an external disturbance which increases the mass m_2 by *20%* ,fig.(7) and fig.(8) contain the results for the case when there is a disturbance, error in initial settings and measurement noise. We can observed from the result obtained that the third order method is faster than the first order method in [9]. It was shown that the trajectories approach neighborhoods of the desired one in a certain sense, even if error of initialization, error in measurements and disturbances during operation exist to some extent at every attempt of operation. Investigating these figures highlights the main conclusion “the proposed algorithm has a smaller tracking error and it is robust against error in initial settings and disturbances”.

Conclusions :

It has been shown that an iterative learning control method can be used to improve the tracking performance of robot manipulators with flexible joints . It was shown that the link angles of a robot will track the specified trajectory with bounded errors and theses bounds can be reduced by properly choosing the learning control parameters . In contrast to other known methods the proposed learning control scheme can utilize more than one past error history contained in the trajectories generated at prior iterations. A convergence proof is given and a numerical example is presented to show the validity of the algorithm in control the robot manipulator with flexible joints. It was shown that the proposed method is robust against variations in initial setting and disturbances. Tracking error is reduced due to the use of the proposed learning control algorithm which makes the output to be close to the desired trajectory. It gives a good tracking performance.

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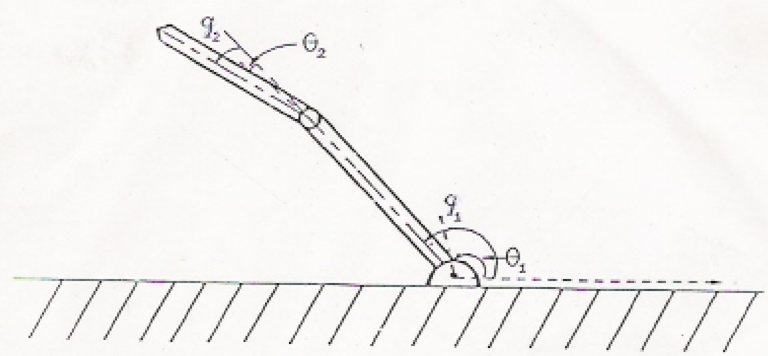


Fig 1: A two-link manipulator with flexible joints

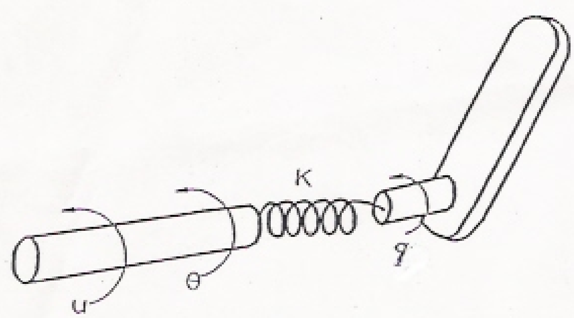


Fig 2: Flexible joint transmission

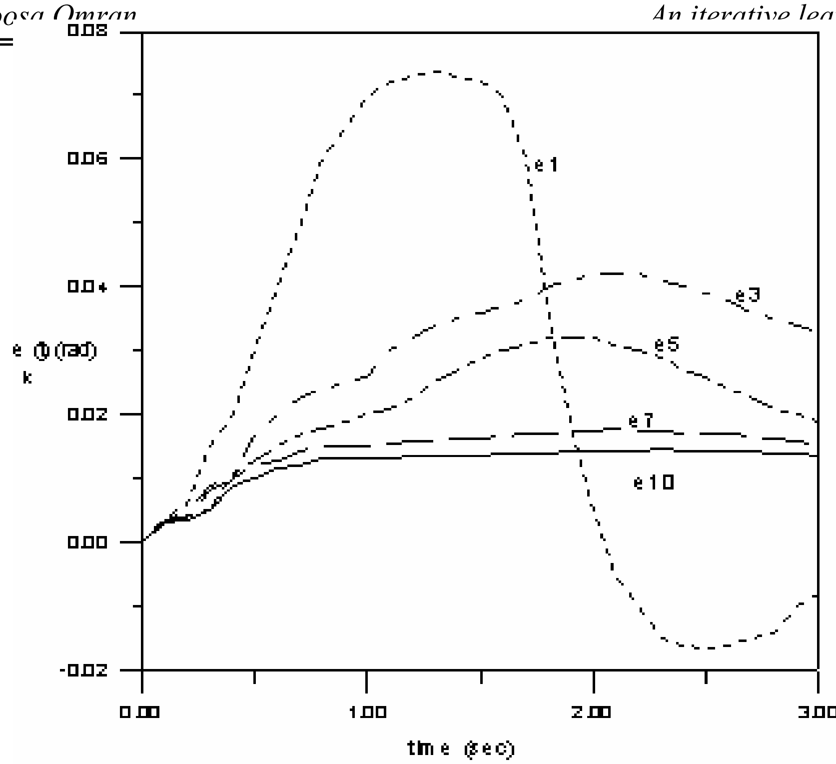


Fig (3) Error tracking performance for q1

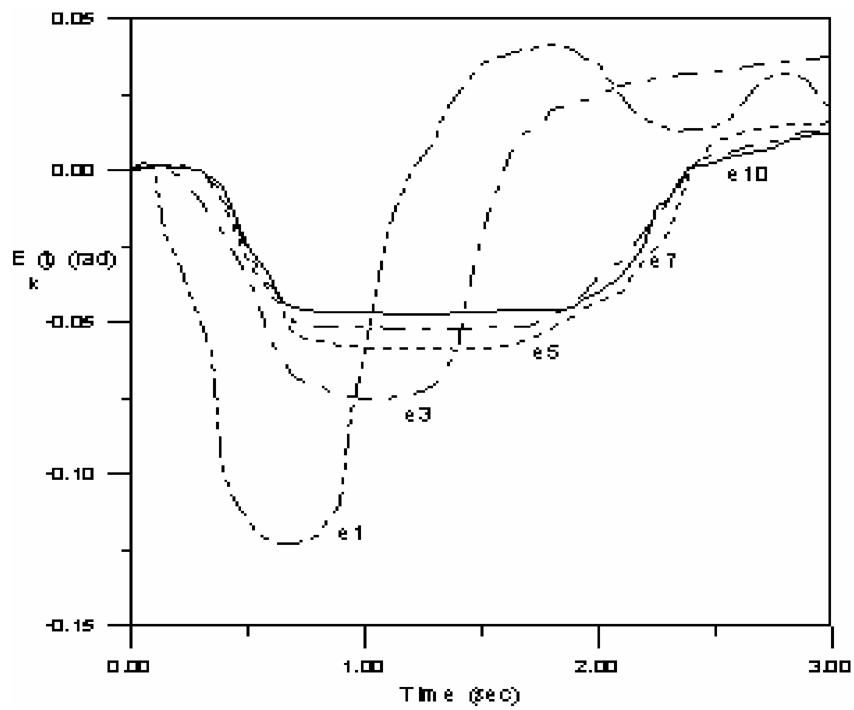


Fig (4) Error tracking performance for q2

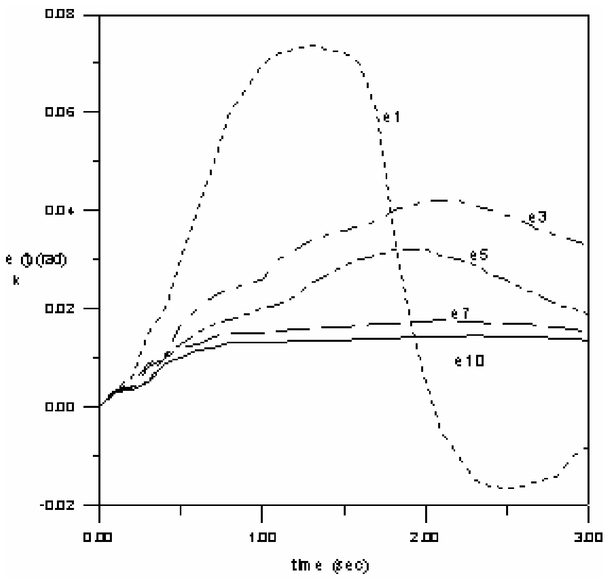


Fig (5) Error tracking performance for q1
in the presence of error in initial settings

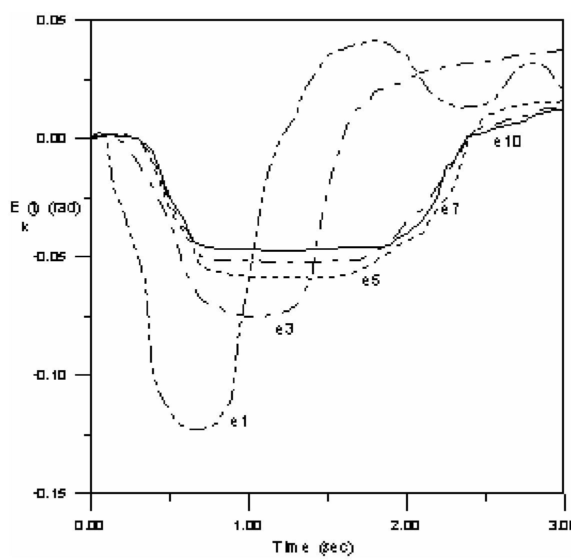
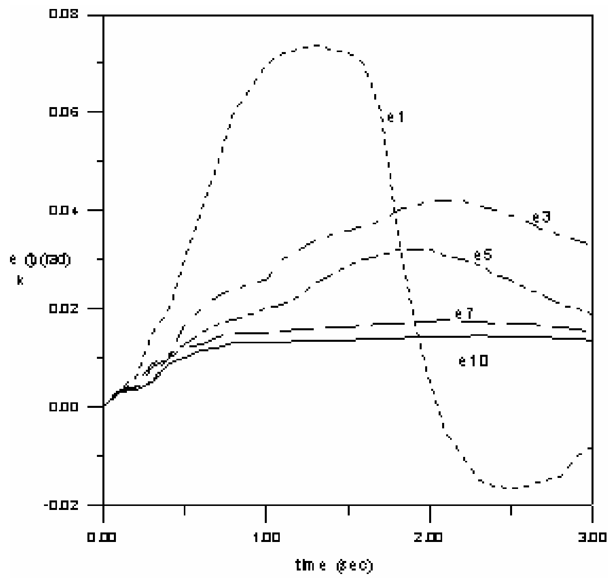


Fig (6) Error tracking performance for q2
in the presence of error in initial settings



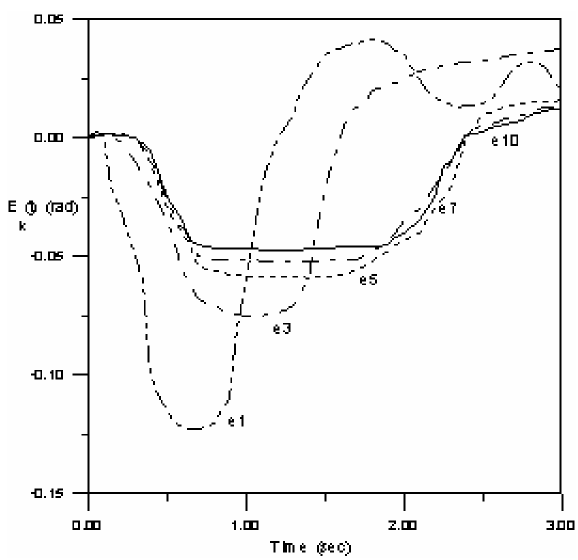


Fig (8) Error tracking performance for q2
In the presence of disturbance,noise,and error
in initial settings