Theoretical Analysis of Coupling Constant, Longitudinal Modes, and Threshold Gain for Coupled-Cavity Semiconductor Lasers

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Abstract

A theoretical analysis of the coupled-cavity semiconductor laser requires simultaneous consideration of the gain and loss in the two cavities after taking into account their mutual optical feedback.

Is wavelength-independent (and hence the same for all Fabry-perot (FP)), in a coupled-cavity laser the effective facet loss becomes different.

1. Introduction
The basic concept of using the coupled-cavity scheme for longitudinal-mode selection is known from the early work on gas lasers and has been widely used for mode selection [1]. In the case of semiconductor lasers, considerable work was carried out by coupling the semiconductor laser to an external cavity [2]. Monolithically integrated coupled-cavity device were considered occasionally in relation to their specific properties such as optical disability, optical amplification, and longitudinal-mode selectivity [1,2]. Recently they have attracted wide attention in an attempt to obtain single-frequency semiconductor lasers that are useful for optical communications in the (1.55 nm) wavelength region [3]. At the same time, the interest in semiconductor lasers coupled to a small-length external cavity has revived. To distinguish them from monolithic coupled-cavity devices such lasers are often referred to as external-cavity semiconductor lasers [2,3].

2. Theory

The mechanism of mode selectivity in coupled-cavity semiconductor lasers can be understood by referring to Fig.(1) [4]. The effect of feedback from the external cavity can be modeled through an effective wavelength-dependent reflectivity of the facet facing the external cavity. As a result, the cavity losses are different for different Fabry-Perot (FP) modes of the laser cavity. In general the loss profile is periodic as shown schematically in the bottom part of Fig.(1). The mode selected by the coupled-cavity device is the FP mode that has the lowest cavity loss and is closest to the peak of the gain profile. Because of the periodic nature of the loss profile, other FP modes with relatively low cavity losses may exist. Such modes discriminated by the gain roll-off because of their large separation from each other [4]. The power carried by these side modes is typically 20-30 dB below that of the main mode occurring in the vicinity of the gain peak [5].
Figure (1) Schematic illustration of longitudinal-mode selectivity in a coupled-cavity laser to drawing shows the two cavities.

The effect of external cavity is to make the effective reflectivity of the facet wavelength dependent (middle drawing). Bottom drawing shows the resulting periodic loss profile superimposed on the gain profile. The main mode is the FP mode closest to the gain peak with the lowest loss, other FP mode become side modes suppressed because of their relatively higher losses.

A general analysis of coupled-cavity semiconductor lasers is extremely complicated, so it is necessary to make several simplifying assumptions. We assume that the field distribution associated with foundamental lateral and transverse mode of the wave guide is unaffected by intercavity coupling and that only axial propagation in each cavity needs to be considered. This reduces the problem to one dimension. However, that the coupling depends upon the mode width and other related parameters, and mode-conversion losses occur as to optical mode leaves cavity 1, diffracts in the air inside the gas, and then reenters cavity 2. Such losses would be incorporated through an effective gap loss ($\alpha_g$) [6].

In this paper we introduce the concept of coupling constant and then obtain the longitudinal modes and their respective threshold gains for the coupled system.

2-1 Coupling Constant

The first step in the analysis is to determine the extent of coupling between the two cavity. We consider the general case applicable for both active-active and active-passive devices [7]. Figure (2) shows the geometry and notion. The coupling between the cavities is governed by an airgap of width $L_g$. The air gap itself forms a third FP cavity, and the intercavity coupling is affected by the loss and phase shift experienced by the optical field while traversing the gap. In the scattering matrix approach the fields in the two cavities are related by:

$$
\begin{pmatrix}
E'_1 \\
E'_2
\end{pmatrix} =
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
E_1 \\
E_2
\end{pmatrix}
$$

(1)
The scattering matrix elements can be obtained by considering multiple reflections inside the gap.

\[
S_{11} = r_1 \frac{r_2 (1 - r_1^2) t_g}{1 - r_1 r_2 t_g} \quad \text{.......................................................... (2)}
\]

\[
S_{22} = r_2 \frac{r_1 (1 - r_2^2) t_g}{1 - r_1 r_2 t_g} \quad \text{.......................................................... (3)}
\]

\[
S_{12} = S_{21} = \frac{[r_1 (1 - r_1^2)(1 - r_2^2)]^{1/2}}{1 - r_1 r_2 t_g} \quad \text{.......................................................... (4)}
\]

where:

\[
t_g = \exp(2i\beta_g L_g) = \exp(2iK_o L_g)\exp(-\alpha_g L_g) \quad \text{.......................................................... (5)}
\]

Here \(\beta_g = K_o + \frac{i\alpha_g}{2}\) and accounts for the phase shift and the loss inside the gap. Further, \(r_1\) and \(r_2\) are the reflection coefficients at the two facets forming the gap; i.e., \(r_n = (\mu_n - 1)/(\mu_n + 1)\) where \(\mu_1\) and \(\mu_2\) are the mode indices in the two effective reflection cavities.
coefficients are $S_{11}$ and $S_{22}$ are the effective transmission coefficients from cavity 1 to cavity 2 and vice versa. It is useful to define a complex coupling parameter:

$$C = C \exp(i\theta) = \left( \frac{S_{12}S_{21}}{S_{11}S_{22}} \right)^{1/2} \ ...................... (6)$$

where:

$C$: governs the strength of mutual coupling and $\theta$ is the coupling phase.

The magnitudes of $C$ and $\theta$ depend on a large number of device parameters. The simplest case occurs for a semiconductor laser coupled to an external mirror. In this case the coupling element is just the laser-air interface. Since $L_g = 0$, $t_g = 1$. Furthermore $\mu_2 = 1$, and therefore $r_2 = 0$. Using these values in eqs. (2)-(6), we find that $C = \left(1 - r_1^2\right)^{1/2} / r_1$ and $\theta = \pi/2$.

For a C3 laser, the two cavities have nearly equal indices of refraction, and therefore $r_1 = r_2 = r$. Using eqs. (2)-(6), we now obtain:

$$C \exp(i\theta) = \left( \frac{1 - r_1^2}{r} \right) \left( \frac{t_1^{1/2}}{1 - t_1} \right) \ ...................... (7)$$

2-2 Longitudinal Modes and Threshold Gain

In this section we obtain an eigenvalue equation whose solutions yield the wavelength and threshold gain associated with the longitudinal modes of the coupled system \cite{8}. A simple way to do this is to consider the relationship between $E_1$ and $E_1'$ using Fig. (2). The field $E_1'$ results from reflection of $E_1$ and transmission of $E_2$ and given by (see eq. (1)).

$$E_1' = S_{11}E_1 + S_{12}E_2 \ ...................... (8)$$

On the other hand, the round trip through cavity 1 results in the relation

$$E_1 = r_1' \exp(2i\beta_1L_1)E_1' = r_1' + E_1' \ ...................... (9)$$

where, the complex propagation constant:

$$\beta_n = \mu_n K_o - i\alpha_n / 2 \quad (n = 1, 2) \ ...................... (10)$$

Govern wave propagation in $n$th cavity, $K_o = \frac{2\pi}{\lambda}$ where $\lambda$ is the device wavelength, and $\alpha_n$ is the mode gain. The mode gain is related to the material gain $g_n$ by:

$$\bar{\alpha}_n = \Gamma g_n - \alpha_n^{int} \ ...................... (11)$$
where:
\( \Gamma \): is the confinement factor and \( \alpha_{\text{int}} \) is the internal loss. For an active-active device, \( g_2 = 0 \) and
\( \bar{\alpha}_2 \): accounts for the absorption loss in the passive cavity.

Eqs.(8) and (9) can be combined to obtain the relation:

\[
\left( 1 - r'_1 t_{11} S_{11} \right) E_1 = r'_1 t_{12} S_{12} E_2 \tag{12}
\]

Similar considerations for cavity 2 lead to:

\[
\left( 1 - r'_2 t_{22} S_{22} \right) E_2 = r'_2 t_{21} S_{21} E_1 \tag{13}
\]

These two homogeneous equations have nontrivial solutions only if the secular condition:

\[
\left( 1 - r'_1 t_{11} S_{11} \right) \left( 1 - r'_2 t_{22} S_{22} \right) = r'_1 r'_2 t_{12} S_{12} S_{21} \tag{14}
\]

Is satisfied Eq.(14) is desired eigen value equation for the coupled system and has been extensively. In the absence of coupling, \( S_{12}=0, S_{nn}=r_n \), and we recover the threshold condition for uncoupled cavities. Note that:

\[
t_n = \exp(2i\beta_n L_n) = \exp(2i\mu_n K_n L_n) \exp(-\bar{\alpha}_n L_n) \tag{15}
\]

Incorporates the phase shift and gain (or loss) experienced by the optical field during a round trip in each cavity. The loss and phase shift inside the gap are included through \( tg \) and \( Sij \) as given by eqs.(2)-(5). The eigen value eq.(14) is applicable for all kinds of coupled-cavity device with arbitrary reflectivities at four interfaces (see Fig.(2)).

Before proceeding it is useful to introduce the concept of an effective-mirror reflectivity. In many cases the role of one cavity (say cavity 2) is provide a control through which a signal FP mode of cavity 1 is selected. The effect of cavity 2 on mode selectivity can be treated by an effective reflectivity for the laser facet facing cavity 2. Eq.(14) can be written in the equivalent form of:

\[
\left( 1 - r'_1 R_{\text{eff}} t_{11} \right) = 0 \tag{16}
\]

where, the effective reflectivity:
\[ R_{\text{eff}} = S_{11} + \frac{r_2^* t_2 S_{12} S_{21}}{1 - r_2^* t_2 S_{22}} = S_{11} \left( 1 + \frac{\bar{f}_2}{1 - f_2} \right) \]  (17)

and \( f_2 = r_2^* t_2 S_{22} \) and is the fraction of the amplitude coupled back into the laser cavity after a round trip in cavity 2. Eq.(16) suggests that as far as mode selectivity is concerned, the couple-cavity laser is equivalent to a single-cavity laser with facet reflection coefficients \( r_1^* \) and \( \text{Reff} \). Although eq. (16) is formally exact, coupling so that a change in \( t_1 \) (through operating conditions of cavity 1) does not affect \( \text{Reff} \) through a change in the feedback fraction \( f_2 \). This is often the case for active-passive devices. In the case of active-active devices, the effective reflectivity concept is valid when cavity 2 is based below threshold.

3. Results

The evolutions of \( C \) and \( \theta \) requires Knowledge of the gap loss \( \alpha_g \). These losses rise mainly from diffraction-spreading of the beam inside the gap and have been estimated using a simple diffraction-spreading model perpendicular to the junction plane \(^{[2]}\). Figure (3) shows the variation of \( C \) and \( \theta \) with gap width using this model after taking \( |r|^2 = 0.31 \) as the cleaved-facet reflectivity. Both \( C \) and \( \theta \) vary considerably with small changes in \( L_g \). The in-phase coupling (\( \theta = 0 \)) occurs whenever \( L_g = \frac{m \lambda}{2} \) (m is an integer), and \( C \) also goes through a maximum for that value of \( L_g \).
To illustrate the extent of mode selectivity offered by the coupled-cavity mechanism, we consider solutions of the eigenvalue eq.(14) for a specific C3-type active-active device for
which $r_1 = r'_1 = r_2 = r'_2 \cong 0.56$. We assume that cavity 2 is biased below threshold such that $\bar{\alpha}_2 = 0$ (in the absence of coupling) and that it does not change significantly with coupling. Eq.(14) is used to obtain $\bar{\alpha}_1$, and the wavelength $\lambda = 2\pi/K_o$ corresponding to various longitudinal modes. Figure (4) shows the longitudinal modes and their respective gains for a specific gap width ($L_g = 1.55\mu$m) and for two sets of cavity lengths $L_1$ and $L_2$ corresponding to long-long and long-short geometries, since the concept of effective reflectivity is approximately valied, $|R_{\text{eff}}|$ versus $\lambda$ is also shown.
4. Conclusion

We are studied and analysed the effects of coupling constant, longitudinal modes, and threshold gain on coupled-cavity semiconductors which the coupling constant is depended on coupling phase, effective gap loss, and the air gap of width. The other side the longitudinal modes and threshold are depended on the effective reflectivity and the lengths of cavities where this lengths and the effective reflectivity will limit the type of modes (i.e. long-long and long-short geometries).

5. References


