Locally T-Semi Connected Spaces

الفضاءات شبه المتصلة – T المحلية

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Abstract :

In this paper, we introduce the concept, of locally T-semi connected space, which generalizes the concept properties of locally T-semi connected spaces are proved.

المستخلص :

في هذا البحث، قدمنا مفهوم الفضاءات - T المحلية والتي تعتبر تعميم الى مفهوم الفضاءات شبه المتصلة المحلية عندما يكون المؤثر T هو المؤثر المحايد. قد برهنت عدة خصائص للفضاءات شبه المتصلة –**T** المحلية.

1- Introduction:

In a recent paper [1], [2], [3], we study and introduce concept T-semi connected spaces. In this paper, we the concept of locally T-semi connected spaces

Where T is operator associated with the topology t defined on a non-empty set X.

Throughout this paper, we use the following notations: cl(A) denotes the usual closure and int(A) denotes the interior of a set A [4].

2- Basic Definitions and Results:

In the section, we recall and introduce the basic definitions needed in this work.

Definition(2.1) [1]:

Semi- open set ; a sub set A of a topological spaces X is called Semi- open set if and if $A \subseteq \tilde{A}^{\circ}$.

Definition(2.1) [2]:

Let (X, τ) be topological space, let $P(X)$ be the power set of X, let $T: P(X) \rightarrow P(X)$be function, we say that T is an operator associated with the topology t on X if $U \subseteq T(U)$ for every open set U in X, the triple (X, τ , T) is called an operator topological space.

Definition(2.3):

Let (X, τ , T) be an operator topological space[2]. We say that X is locally T-semi connected at the point $x \in X$ if and only if every T-semi open set U [3].
Containing x, there exist T-semi connected open set A, [3]. Such that $x \in A \subseteq U$. (X, τ , T) is called locally T-semi connected if and only if it is locally T-semi connected at every point of X.

Remarks (2.4):

Every locally T-semi connected space is locally T-semi connected, [3].

Definition(2.5) [5]:

A function F from a space X onto space y is called monotone if the inverse image every sub continuum in y is continuum in x (continuum is compact, connected T2 spaces).

Definition(2.6) :

Let (X, τ, T) be an operator topological space. We say that T is a monotone operator[2]. Let $x \in X$, T-semi component [6], of x denoted by T-S. C(x), is the union of all T-semi component subsets of X containing x.

Remarks (2.7):

(i) T-S. C(x), is T-semi connected.
(ii) Each T-semi component T-S. C(x), is a point of X form a partition of X .
(iii) The set of all T-semi component of a point of X form of X.
(iv) Each T-S. C(x), is T-semi closed.

3. Main Results :

In this section, we state and prove several properties and characterizations of locally T-semi connected spaces are given.

Theorem (3.1) :

Let (X, τ, T) be an operator topological space. Where T is monotone operator then X is locally T-semi connected if and only if each T-semi component of T-semi open set is open.

Proof:

Suppose that(X, τ, T) is locally T-semi connected Let $A \subseteq X$ be T-semi open and B be T-semi component of A. if $y \in A$ therefore, there is a T-semi connected open set U such that $y \in U \subseteq A$ since B is T-semi component of y and U is a T-semi connected we have that $y \in U \subseteq B$ therefore B is open conversely if $x \in X$ and A is T-semi open set containing x , let B be a T-semi component of A such that $x \in B$ since B is a T-semi connected open set, $x \in B \subseteq A$ so X is locally T-semi connected.

Definition(2.3) :

Let $f: (X, \tau, T) \to (Y, \sigma, L)$ be a function from an operator topological space, (X, τ, T) to an operator topological space (Y, σ, L) we say that f is (T,L)- semi continuous if for each L-semi open set V in Y, $f^{-1}(V)$ is T-semi open in X.

Theorem (3.3) :

If $f: (X, \tau, T) \to (Y, \sigma, L)$is (T,L)- semi continuous function and onto, and if X is a T-semi connected, then Y is L-semi connected.

Proof:

Suppose that Y is not L-semi connected and let A, B be an L- separation of Y such that $Y = A \cup B = A \cap \left(L - scl(B)\right) = \emptyset$ it follows that A and B are L-semi open and L-semi closed sets in Y it follows that $f^{-1}(A) \cup f^{-1}(B) = X$. $f^{-1}(A)$ and $f^{-1}(B)$ are T-semi open and T-semi closed in X therefore we obtain that X is not T-semi connected which is a contradiction hence Y is L-semi connected.

Theorem (3.4) :

Let $f: (X, \tau, T) \to (Y, \sigma, L)$ be a (T,L)- semi continuous and open function [4] and $A \subseteq X$ be an open set. If A is a T-semi connected set, then $f(A)$ is an L-semi connected set.

Proof:

Since A is T-semi connected and open in X , then $(A, \tau A)$ is also T-semi connected (τA is the relative topology on A) but $f/A : (A, \tau A) \to (f(A), f(A))$ is an onto and (T,L)- semi continuous function so $f(A)$ is L-semi connected.

4. References :

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