COMPUTATIONAL METHOD FOR UNSTEADY MOTION OF TWO-DIMENSIONAL AIRFOIL

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ABSTRACT

A numerical method is developed for calculation of the wake geometry and aerodynamic forces on two-dimensional airfoil under going an arbitrary unsteady motion in an inviscid incompressible flow (panel method). The method is applied to sudden change in airfoil incidence angle and airfoil oscillations at high reduced frequency. The effect of non-linear wake on the unsteady aerodynamic properties and oscillatory amplitude on wake rollup and aerodynamic forces has been studied. The results of the present method shows good accuracy as compared with flat plate and for unsteady motion with heaving and pitching oscillation the present method also shows good trend with the experimental results taken from published data. The method shows good results for a wide range of unsteady motion of a two-dimensional airfoil.

الخلاصة:

تم تطوير طريقة عددية لحساب القوى الهوائية وشكل الاعقاب خلف مطيار ثنائي الابعاد واقع تحت تاثير حركة غير متوازنة لجريان غير لزج وغير قابل للانضغاط (طريقة البوابات). طبقت الطريقة على المطيار الذي يبدا بحركة فجائية وبزاوية هجوم معينة كذلك المطيار الذي يتحرك بتذبذب ذو قيمة اهتزازية عالية. ان تاثير التصرف اللاخطي للاعقاب على الخواص الهوائية وقيم العليا للحركة الاهتزازية وتاثيراتها على الالتفاف الحاصل بالاعقاب خلف المطيار والقوى الهوائية المتولدة حيث تمت دراستها. اوضحت الطريقة المستخدمة حاليا نتائج جيدة اذا ما قورنت مع الاسطح المستوية كذلك فان الحركة الاهتزازية الرفعية والانحنائية اظهرت تطابق جيد من حيث التصرف مع النتائج العملية المتحصلة من بحوث سابقة. ان الطريقة الحالية تظهر نتائج جيدة ولمديات واسعة في الحركة الغير مستقرة لمطيار ثنائي البعد.

KEYWORDS

Airfoil Oscillations, Panel Method, Unsteady Aerodynamics, Aerodynamic Coefficients, Hydrofoils, Lift, Pressure Distribution

INTRODUCTION:

One of the basic assumptions of airfoil theory deal with the presence of stagnation point at the sharp trailing edge, what is commonly called Kutta Condition. While the existence of this condition is well established for steady non-separated flow situation, its validity for time-dependent cases is still controversial **(Hess & Smith 1966)**.

It can be shown that Kutta Condition can be applied for force and moment prediction in unsteady small amplitude non-separated high Reynolds Number flows. As a first approximation it is reasonable assume that the flow can be regarded as inviscid so long as the flow in the region of the trailing edge is not separated.

For general unsteady motion it is possible to obtain numerical solutions by imposing either the condition of finite velocities about the trailing edge or the condition of zero loading about the trailing edge **(Basu and Hancock 1978).**

The most comprehensive numerical solution is due to **(Gesing 1968)** and is based on (**Hess & Smith 1966)** procedure to solve two-dimensional airfoil in steady incompressible flow by using source and vortex distribution on the surface and by using Kutta condition invoked velocities equal in both magnitude and direction at midpoint of the two trailing edge elements. The method discrete vortex shedding from the trailing with strength equal to the negative vortex strength of the airfoil, it found that the approach gives good behavior of the wake and loads when the airfoil move in unsteady motion. **(Basu and Hancock 1978)** developed a numerical method to calculate two-dimensional airfoil under going an arbitrary unsteady motion in an inviscid incompressible flow. The method of **Gesing** is modified by adding additional panel at the trailing edge with length and angle depends on the solution of airfoil vorticity and by application of zero loads at the trailing edge, so that, a system of non-linear equation is produced due to this element and with iteration technique is used to solve these equations. Good results obtained with this modification as compared with **Gesing**. **(Chen and Sheu 1980)** used the interior singularities to solve the unsteady incompressible inviscid airfoil. Same approach of **Basu and Hancock** for oscillations motion analysis although results have been presented for a sudden change in incidence at high frequency oscillation and entry into a sharp-edge gust, the method is completely general. **(Kats and Weihs 1981)** used a thin airfoil theory to solve the unsteady motion, as compared with other published experimental data. It is found that when the trailing edge displacement is small the range of linearized theory calculations using Kutta condition can be extended far beyond reduced frequency larger than 1. **(Poling and Telionis 1986)** presented two cases of unsteady flow field over a NACA 0012 airfoil at an angle of attack, the results indicates that the unsteady Kutta condition proposed by **Gesing** is examined and some evidence is provided for its support.

In the present work will adapt the method stated by **(Katz and Plotkine 1991)** which created to solve the unsteady flow about thin (Flat Plate) airfoil, and then coupled with **Hess & Smith** method to solve thickness problem (e.g. NACA 0012). Some modification presented to solve airfoil load coefficients which illustrated in numerical procedure.

MATHEMATICAL MODEL:

The flow field is assumed to be potential (inviscied and irrotational) and incompressible. In that case velocity potential satisfies the laplace equation:

$$\Delta\phi = 0 \qquad (1)$$

The equation is the same, both for steady and unsteady flows. Owing to that, methods for steady cases can be applied for the solution of unsteady flow problems, as well. Unsteadiness is introduced by the unsteady boundary condition:

$$\vec{Q}.\vec{n} = 0 \quad (2)$$

Of the Kelvin theorem:

$$\frac{D\Gamma}{Dt} = 0 \quad (3)$$

And the unsteady form of the Bernoulli equation:

$$\frac{p - p_\infty}{0.5\rho_\infty} = V_\infty^2 - Q^2 - 2\frac{\partial \phi}{\partial t} \quad (4)$$

In order to define the aerodynamic characteristics of the airfoil, two models should be established: airfoil model and wake model.

The airfoil model is modeled by **(Hess & Smith 1966)** method, which enables to model airfoil with different shape by using constant source and vortex panel on the airfoil surface.

Numerical modeling of the wake must be done carefully due to its high influence on the lift force generation. To satisfy the unsteady Kutta Condition at the trailing edge, the pressure coefficient for the unsteady flow is defined as:

$$C_p = 1 - \frac{Q^2}{V_\infty} - \frac{2}{V_\infty}\frac{\partial \phi}{\partial t} \quad (5)$$

According to that, the difference between the upper and lower surface pressure coefficients at the trailing edge is:

$$\Delta C_p = -\frac{Q_u^2 - Q_l^2}{V_\infty^2} - \frac{2}{V_\infty^2}\frac{\partial}{\partial t}(\phi_u - \phi_l) = 0 \quad (6)$$

Where u and l are the upper and lower surface value;

The potential difference is:

$$\Gamma = \phi_u - \phi_l \quad (7)$$

The Kutta condition can be expressed as the uniqueness of the pressure coefficients at the trailing edge, which mathematically expressed, takes the form:

$$\frac{\partial \Gamma}{\partial t} = -V_\infty \gamma_{TE} \quad (8)$$

From this equation it can be clearly seen that the variation of airfoil circulation in time can be compensated by releasing vortices of magnitude γ_{TE}.

NUMERICAL METHOD:

The solution for the flow about an airfoil under going an arbitrary time-dependent motion which started at t = 0 is calculated at successive intervals of time.

$$t_k \; (t_0 = 0, k=1,2,3.....) \quad (9)$$

By a method based on **(Hess and Smith 1966)** approach to solve steady linear incompressible flow about airfoil at time t_k, the model of non-linear unsteady incompressible flow is shown in **Fig (1).**

The airfoil contour at time t_k is replaced by N straight-line elements. A uniform source distribution $(\sigma_i)_k$ and a uniform vorticity distribution $(\gamma)_k$ are placed on i^{th} element (i =1,2,…..,N) where $(\sigma_i)_k$ varies from element to element but $(\gamma)_k$ is the same for all elements on airfoil and the subscript k refer to the time t_k. The over all circulation (Γ_k) can be calculated from $(\gamma)_k$ * (airfoil perimeter).

The unsteady motion of the airfoil is characterized by the presence of the trailing vortex wake emanating from the trailing edge of the airfoil. The vorticity shed during any time t_k is equal in magnitude and opposite in sign to the change in circulation about the airfoil during t_k.

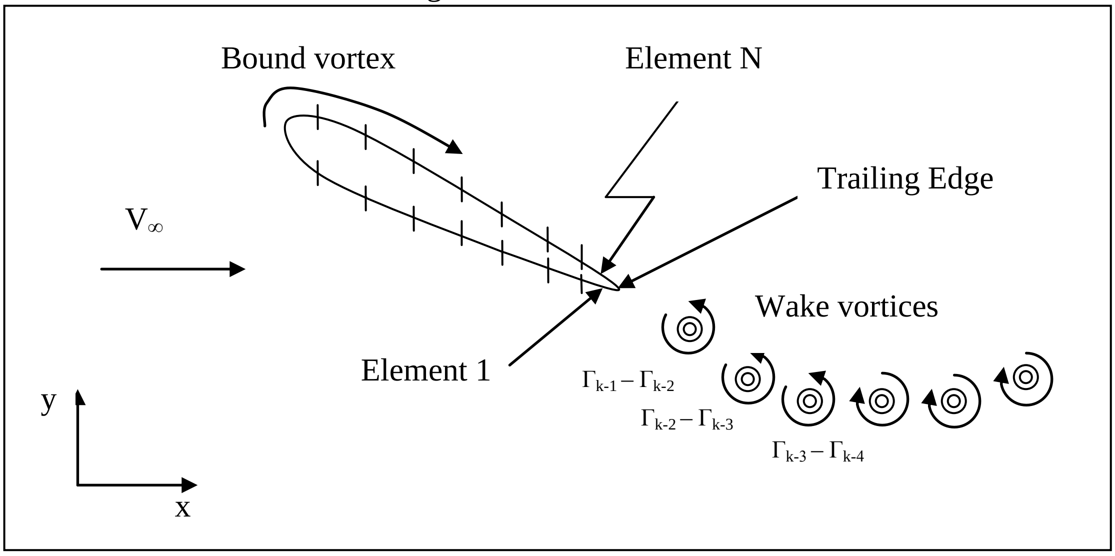


Fig (1): The unsteady motion of a two-dimensional airfoil**.**

Once the vorticity is shed it moves as a fluid particle subjected firstly to the onset flow and secondly to the perturbation velocity due to the airfoil and then to the induced velocity due to the remainder of the shed vorticity, this procedure is implemented as indicated in **Fig (2).** From any initial stage the prescribed movement of the airfoil during next t_k is carried out and the vorticity shed during t_k is computed from the simultaneous application of the usual Kutta condition at the trailing edge of the airfoil and the vorticity conservation low mentioned above. This vorticity is placed in the fluid as a line vortex at a location representative of the average fluid velocity over the trailing edge locus during t_k. Finally, all previously shed line vortices are moved to new locations using a predictor-corrector method. As shown in the **Fig (2).**

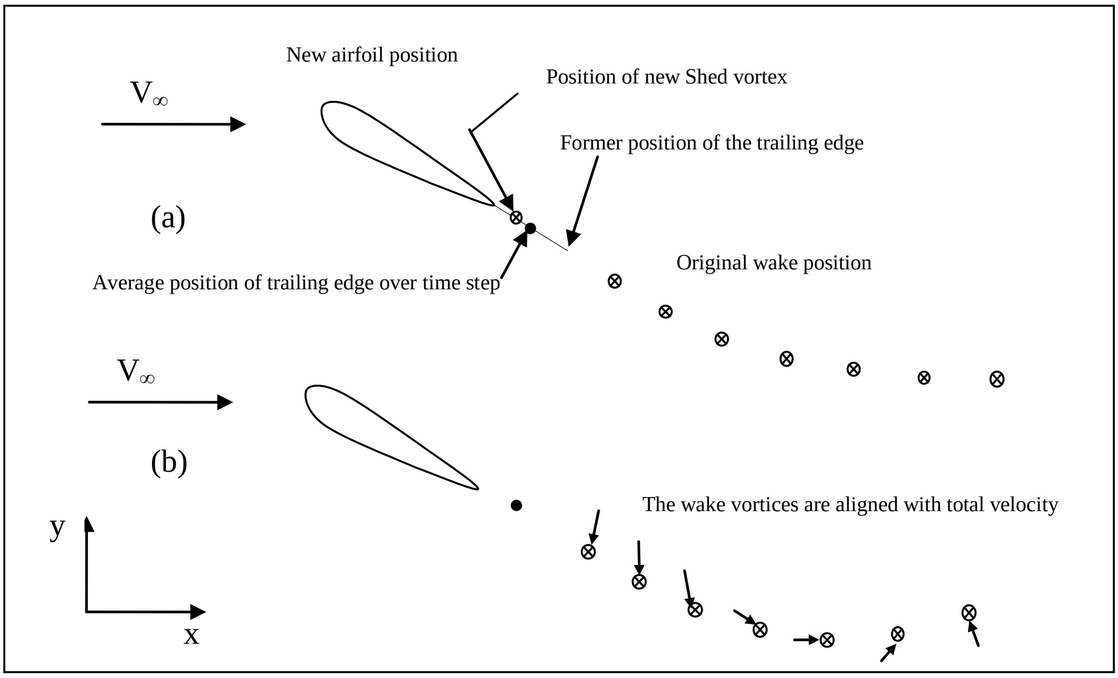


Fig (2): (a) Airfoil Moves. (b) Wake Moves **(from Hess J. L. 1975)**

The solution can be summarized at time t_k, there are N+2 unknowns $(\sigma_i)_k$ (i =1,2,3,………,N) , $(\gamma)_k$ and the Γ_W. The basic set of equations can be formulated as follows;

1- the condition of zero normal velocity (Q_n) at the external midpoint of each airfoil element **j** at time t_k such that,

$$\left(Q_{n\,j}\right)_{k=0} = 0 \tag{10}$$

2- the condition of equal velocities at the midpoint of the two elements on the airfoil on either side of the trailing edge is,

$$(Q_{t1})_k = (Q_{tN})_k \tag{11}$$

Where Q_{t1}, Q_{tN}: is the total tangential velocity at the midpoint of element 1 and N at time t_k.

3- the vorticity conservation low (the Kelvin's Conditions),

$$\Gamma_W = \Gamma_k - \Gamma_{k-1} \tag{12}$$

Where Γ_W = is the Wake Circulation.
Γ_k = is the circulation about airfoil at time t_k.

Since the problem is concerned with incompressible flow the formula for the induced velocities by source and vorticity distributions are the same as for the steady case. Thus the experience gained with the steady **(Hess and Smith 1966)** method carries over to the unsteady problem.

4- The boundary condition of the equations represents the normal velocity component due to the motion of the airfoil which is known from kinematics equation. Since the strength of the other wake vortices is known from the previous time steps. So that, their effect on the normal velocities will be included in the boundary condition also.

5- Solving the set of equations and once the source and vorticity strengths and wake vortex have been determined the velocity distribution on the airfoil or at any point in the flow field is known. The unsteady Bernoulli's equations namely **(Basu and Hancock 1978)**.

$$C_P = 1 - \frac{Q^2}{V_\infty} - \frac{2}{V_\infty^2}\frac{\partial \phi}{\partial t} \tag{13}$$

Where ***Q*:** is the total velocity on the outer airfoil surface and ϕ is the velocity potential.

6- The forces and moments are obtained by direct integration of the pressure distributions. In calculation of unsteady pressure coefficient, $(\frac{\partial \phi}{\partial t})$ has to be determined. In the present numerical method the value of $(\frac{\partial \phi}{\partial t})$ at the midpoint of the j^th^ element at time t_k is approximated by;

$$(\frac{\partial \phi}{\partial t_j})_k = \{(\phi_j)_k - (\phi_j)_{k-1}\}/(t_k - t_{k-1}) \tag{14}$$

7- The velocity potential (ϕ) is obtained by integrating the velocity field along the x-axis from upstream of the airfoil and then around the airfoil surface

8- Once the solution at time t_k has been determined, the model stepped for time t_{k+1}, with the wake pattern calculated from the solution at time t_k. The distributed vorticity on the wake element at time t_k is now assumed to be concentrated into a vortex strength (Γ_W) at time t_{k+1} situated at ,

$$\begin{aligned} x_{k+1} &= (x_{TE})_k + (U_W)_k \Delta t \\ y_{k+1} &= (y_{TE})_k + (V_W)_k \Delta t \end{aligned} \tag{15}$$

The resultant velocity at the center of each of the other concentrated vortices in the wake is calculated from the solution at time t_k. Then the position of that vortex at time t_{k+1} follows directly.

A computer program has been developed in **FORTRAN** power Station, and is applied to,

i. Sudden Change in Angle of Attack:

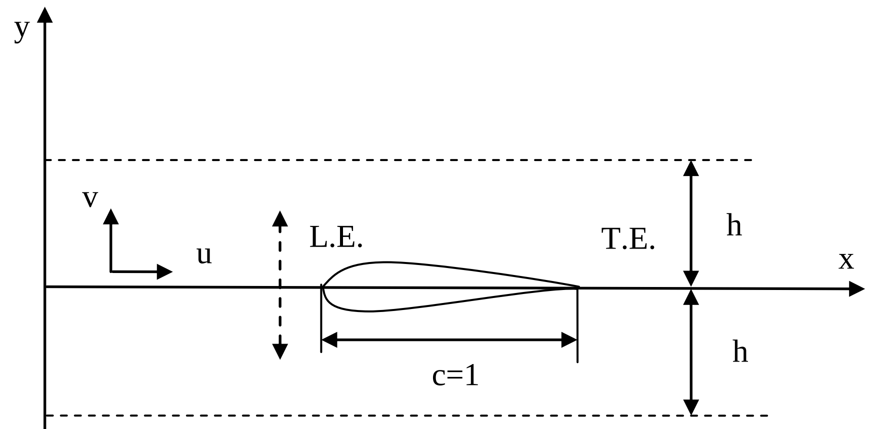
The analytic solution of impulsive incidence (for flat plate thin airfoil) was studied by Wagner in 1925 and illustrated in **(Yuong 2006),** who provided what is now referred to as the Wagner Function. The Wagner Function has else where been used as a validation of unsteady panel method (e.g. **Katz and Plotkin 1991**). The lift coefficient ratio $\frac{c_l}{c_l(t=\infty)} = 1 + \phi(t)$ with $\phi(t)$ is an approximation to the Wagner function,

$$\phi(t) = -0.165e^{-0.091t} - 0.335e^{-0.6t} \quad (16)$$

This equation is valid for angle of attack of 5 deg.

ii. Heaving Oscillation:

The general equations of motion and parametric nomenclature for the configuration used to simulate numerical solution are illustrated in **Fig (3).** The equations of motion are shown. From the equations illustrated in the figure the airfoil will oscillate up and down the x–axis with amplitude equal to h. this oscillation create a velocity on the boundaries of the airfoil showed in terms of u/V_∞ and v/V_∞. It must be note that these velocities must be in concluded in the boundary condition of the zero normal velocity. The equations show a harmonic behavior so that, it expected the results has these effects.



$$\frac{y(t)}{c} = \frac{h}{c} \cos(k * t)$$

$$\frac{u}{V_\infty} = 1 \quad , \quad \frac{v}{V_\infty} = -\frac{h}{c} k \sin(k * t)$$

Fig (3): Equation of motion of heaving oscillation and nomenclature.

iii. Pitching Oscillation:

The pitching motion and its equations are illustrated in fig (4). The figure shows that the airfoil will translate with pitching motion about a pivot point called x_{cp}. The position of this point is changed to study its effect on the airfoil wake and forces coefficients.

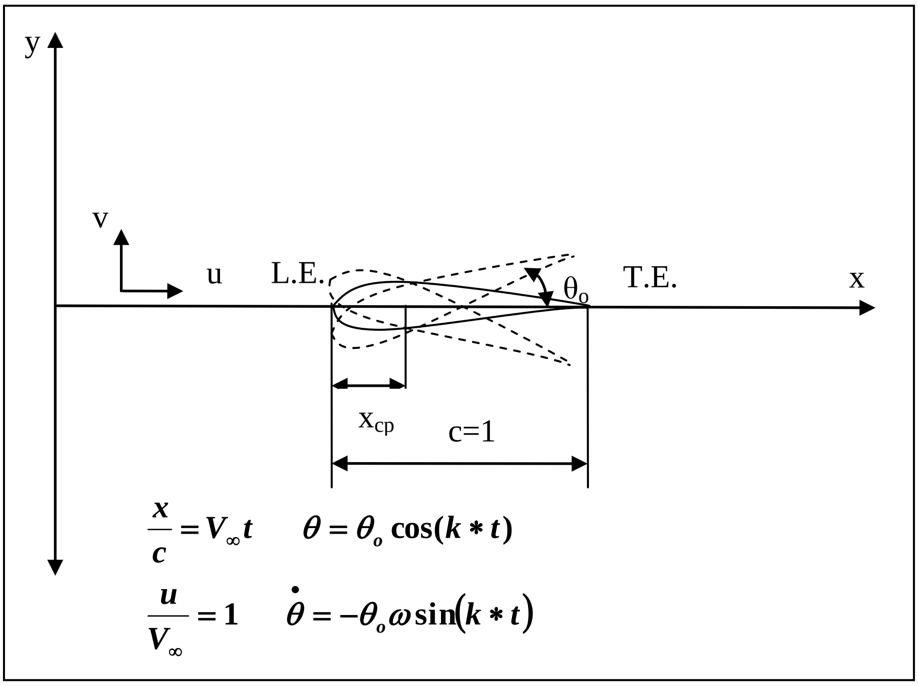


Fig (4): Equation of motion of Pitching oscillation and nomenclature**.**

NACA 0008 thick symmetrical airfoil is used in pitching oscillation for a mean incidence angle of 0 deg. At high reduced frequency k=20 with amplitude of θ_o=0.01 rad., So that, the results can be compared with those obtained from (**Basu and Hancock 1978)**.

RESULTS AND DISCUSSION:

For sudden change in angle of attack calculation presented in **Fig (5)** is at angle of attack equal to 5^o and $\frac{\Delta t V_\infty}{c} = 0.25$. The results for computation of lumped vortex method (**LVM**) which is used to find the unsteady flow over thin airfoils is also shown in the figure for comparison. The **LVM** depends on using lumped vortex on the quarter chord and Kutta condition was valid at third quarter chord of the airfoil. The wake rollup calculation and forces such that at each time interval (dt) a vortex shed from the Trailing edge where the strength of each vortex is equal in magnitude and opposite in sign to the change in circulation of the airfoil bound circulation. This procedure is continued for the next time steps. The results obtained is in good accuracy as compared with the other approaches, and it could be conclude that at time t=0 there are an impulse lift which arises from instantaneous change in ϕ with time as shown in pressure coefficient **Eq (5).** This change in ϕ with time due to acceleration of the flow becomes smaller with reduced influence of the starting vortex.

Fig (6) show that there is a drag force, this force could be divided into two components, first is the wake induced downwash and the second is due to fluid acceleration these two components calculated and presented in figure.

While examining the wake vorticity as presented in **Fig (7),** it can be observed that the first vortices are the strongest and that all vortices have a counterclockwise values. Also if the wake is allowed to rollup, due to the velocity field induced by the wake and the airfoil, the shape is as shown in **Fig (8).** It could be concluding that the present method gives at least good behavior and trends for sudden change in angle of attack motion of the airfoil.

For heaving oscillation **(Katz and Plotkine 1991)**, presented a wake shape visualizations for a flat plate under going heaving oscillation (Plunging motion), corresponding solution were performed with the present Method for comparison. Assume that thickness of the airfoil is NACA 0001 (i.e. 1% thickness) instead of flat plate.

Three results were performed with k = 8.5, k = 2.1 and k =0.6 with amplitude h = 0.019. Fig (10,11 and 12), show a very close agreement with **(Kats and Plotkine 1991)** results in terms of the shape of the wake and the degree of self induced roll-up.

Overall the present method demonstrated excellent qualitative and quantitative agreement with published Panel Method data.

Other comparisons are performed between experimental wake visualization of airfoil NACA 0012 oscillating in different reduced frequencies and amplitudes with computational procedure discussed in the previous section and illustrated in **Fig (13).**

Also figures shows wake behavior of the airfoil with lines represents positive vortices(clock wise) and lines which represents negative vortices (counter clock wise). So that, with simple procedure solution depending on linear equations the non-linear behavior of the problem was solved with good accuracy.

For the airfoil thickness NACA 0008 the calculation of heaving oscillation for the mean angle of incidence 0 deg with an amplitude h=0.018 c and reduced frequency k = 8.5.

The lift and drag coefficients for the upper conditions are presented in **Fig (14).** Due to the strong non-linear effects of the discrete vortices in the wake, the variations of lift and drag distribution are not simply harmonic.

The comparison shows good trends between experimental and computations. So that, it could be find complex and nonlinear problems by a linear solution with good accuracy.

For pitching oscillation, generally speaking, the non-linearity in the problem arises from the wake, it is non-linear process to find the position and the shape of the wake, the wake pattern in the **Fig (15),** shows the rollup of the wake vorticity into discrete vortices of opposite sign.

Fig (16) show the wake rollup for NACA 0008 thick symmetrical airfoil in pitching oscillating with various oscillatory amplitudes θ_o and various oscillatory positions x_{cp} on mean chamber line at reduced frequency 20 respectively. It could be shown that the effect of oscillatory amplitude $\theta_o$on the wake rollup is significant if the airfoil is oscillating about the position between the leading edge x_{cp} =0 and the mid-point on the mean camber line x_{cp} =c/2.

The lift and drag forces coefficients are predicted and simulated in **Fig (17).** The variation in lift and drag are not simply harmonic behavior due to the skew ness and the phase shift in the neighborhood of the peak values.

The vorticity distributions will influence the vortex shedding from the trailing edge and overall lift and drag distributions on the airfoil surface.

CONCLUSIONS:

The numerical method out lined in this paper leads to the calculation of the inviscid flow field about an airfoil undergoing an arbitrary time-dependent motion. If it is assumed that the flow remains attached and that it separates at the trailing edge of the airfoil. Although results have been presented for a sudden change in incidence and a high frequency oscillation the method is completely general.

The wake rollup calculations show good agreement with the available flow visualization data. The effects of the wake rollup on the calculation of unsteady lift and drag coefficients are significant.

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Nomenclature:

c	Chord length (m)
C_d	Drag coefficient per unit span $D/(q_\infty c)$
C_l	Lift coefficient per unit span $L/(q_\infty c)$
C_p	Pressure coefficient
h	Amplitude level in heaving oscillation (m)
k	Reduced frequency, $\omega c/ 2V_\infty$
Q	Total velocity on the airfoil surface (m/s)
Q_n	Total normal velocity on the airfoil surface (m/s)
Q_t	Tangential velocity on the airfoil surface (m/s)
t	Time (s)
u	horizontal kinematic velocity (m)
U_w	Wake influence velocity in x-direction (m/s)
v	Vertical kinematic velocity (m/s)
V_∞	Free stream velocity (m/s)
V_w	Wake influence velocity in y-direction (m/s)
x	Horizontal displacement of airfoil in terms of c
x_{cp}	Pivot point at which the airfoil pitching in terms of c
x_{TE}	Airfoil trailing edge displacement in horizontal displacement
y	Vertical displacement of airfoil in terms of c
y_{TE}	Airfoil tailing edge displacement in vertical oscillation
Γ_w	Wake vorticity
Γ_k	Vorticity due to airfoil motion at k^{th} step
ϕ_∞	Uniform flow velocity potential
γ	Vorticity strength distribution
α	Angle of attack
ω	Circular frequency
θ_o	Amplitude angle in pitching oscillation
θ	Pitching angle of oscillating airfoil
$\dot{\theta}$	Angular velocity of pitching oscillation airfoil
ϕ	Velocity potential and Wagner function

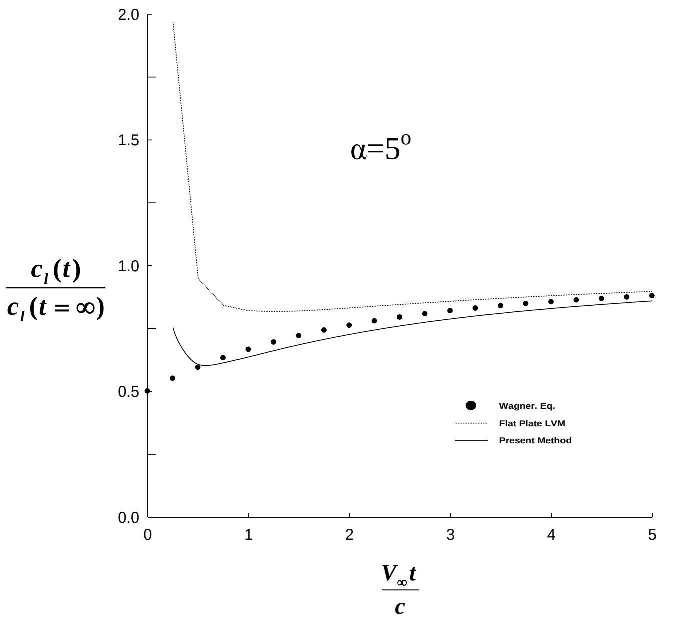


Fig (5): Variation of lift after initiation of sudden forward motion of two-dimensional airfoil.

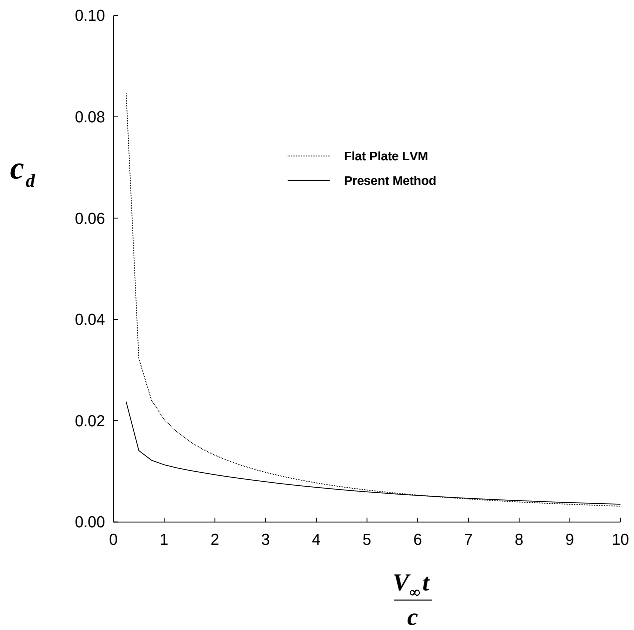


Fig (6): Variation of drag after initiation of sudden forward motion of two-dimensional airfoil.

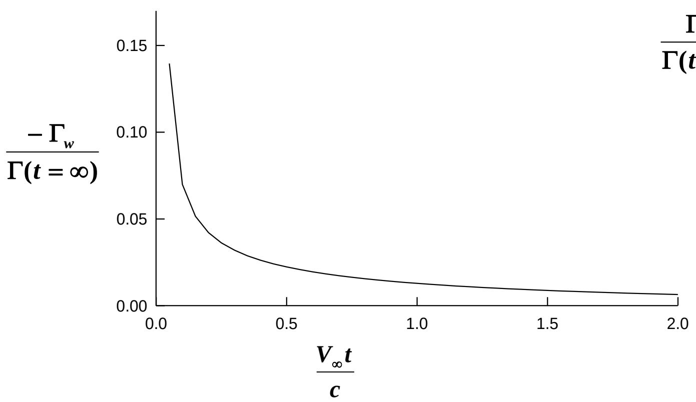


Fig (7): Wake circulation behined the airfoil which was suddenly set into motion.

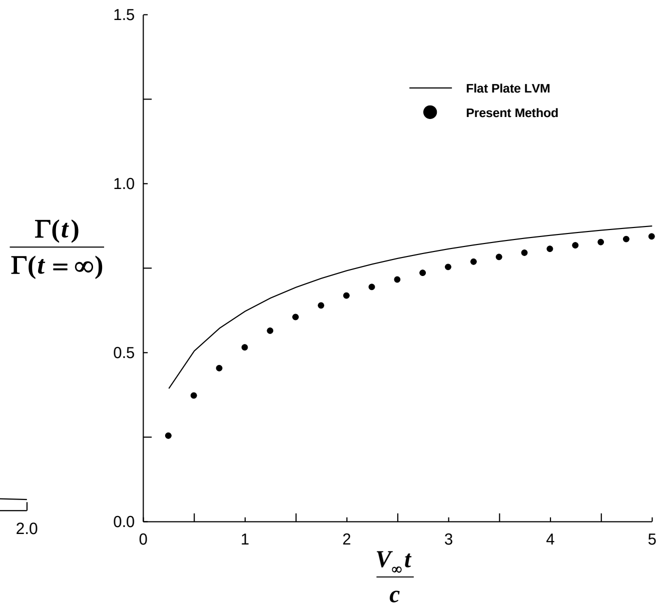


Fig (8): Variation of airfoil circulation after initiation of sudden forward motion of two-dimensional airfoil.

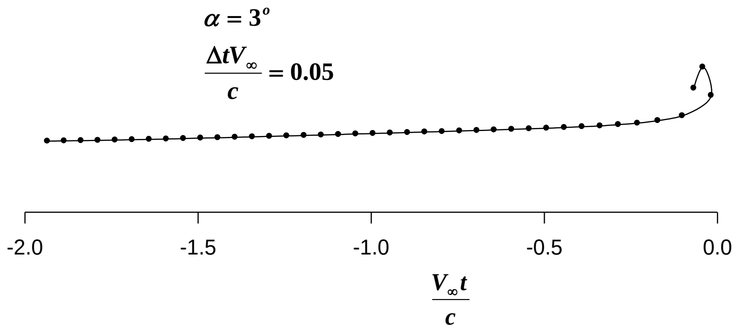
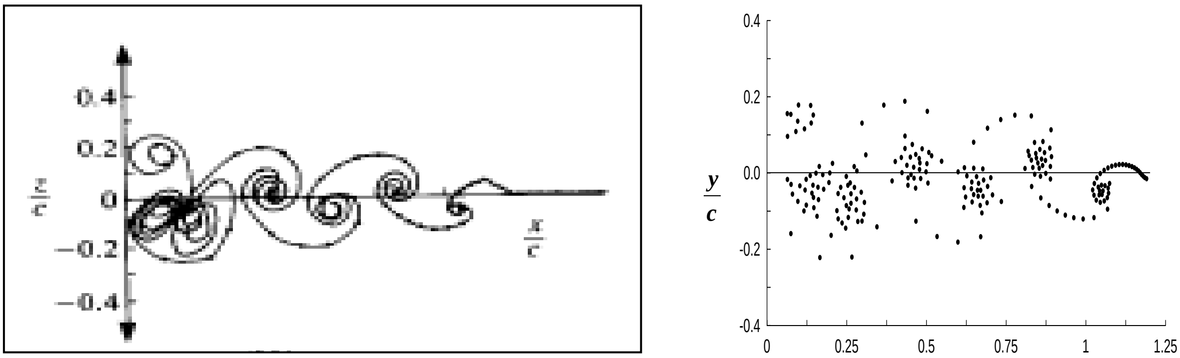
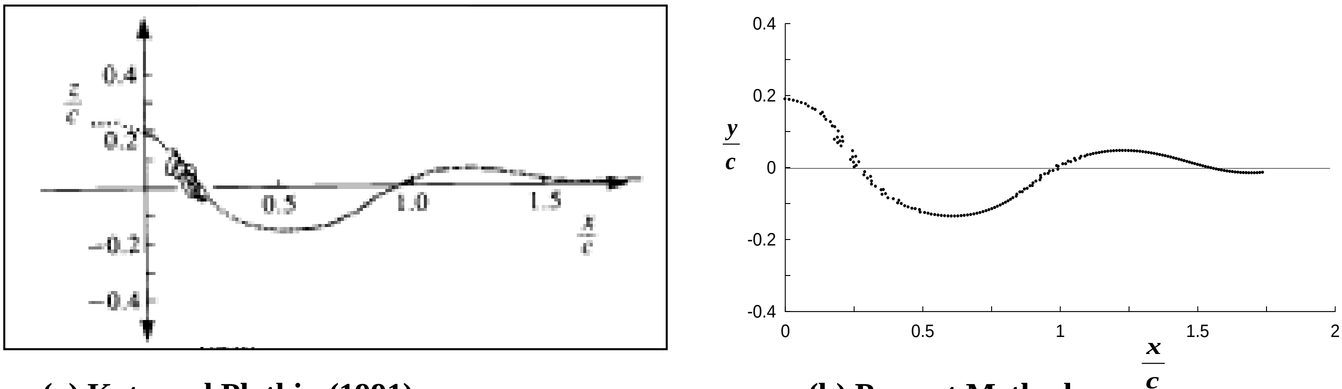


Fig (9): Wake rollup behind a two-dimensional airfoil which was suddenly set into motion.



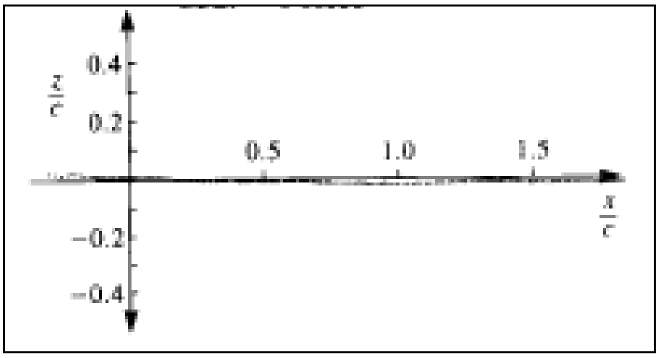
(a) Katz and Plotkin (1991) **(b) Present Method** $\frac{x}{c}$

Fig (10): Wake pattern calculated from **Katz and Plotkin (1991)** and corresponding present calculations for heaving oscillation of $k = \frac{\omega c}{2V_\infty}$ =8.5 and h=0.019.

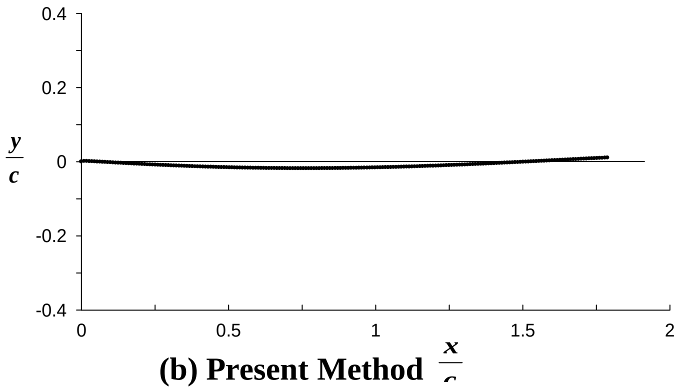


(a) Katz and Plotkin (1991) **(b) Present Method**

Fig (11): Wake pattern calculated for heaving oscillation of $k = \frac{\omega c}{2V_\infty}$ =2.1 and h=0.019.

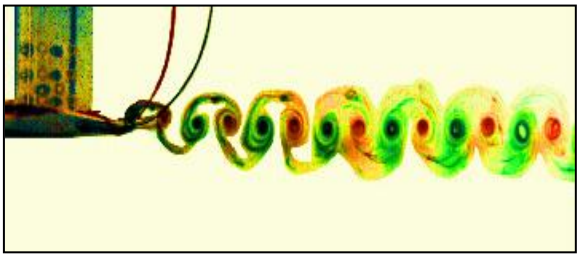


(a) Katz and Plotkin (1991)

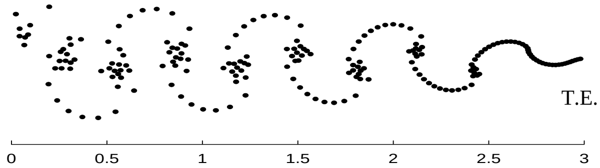


(b) Present Method

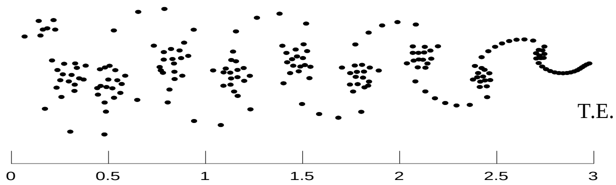
Fig (12): Wake pattern calculated for heaving oscillation of $k = \frac{\omega c}{2V_\infty}$ =0.6 and h=0.019.



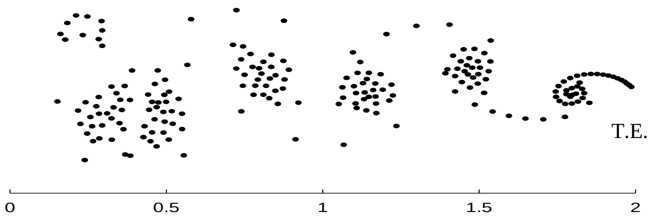
k=7.85
h=0.0125



k=7.85
h=0.025



k=7.85
h=0.05



(a) Young J. (2006)

(b) Present Method

Fig (13): NACA 0012 Wake pattern visualization from **Young J. (2006)** as compared with computed wake for heaving oscillations at various amplitudes .

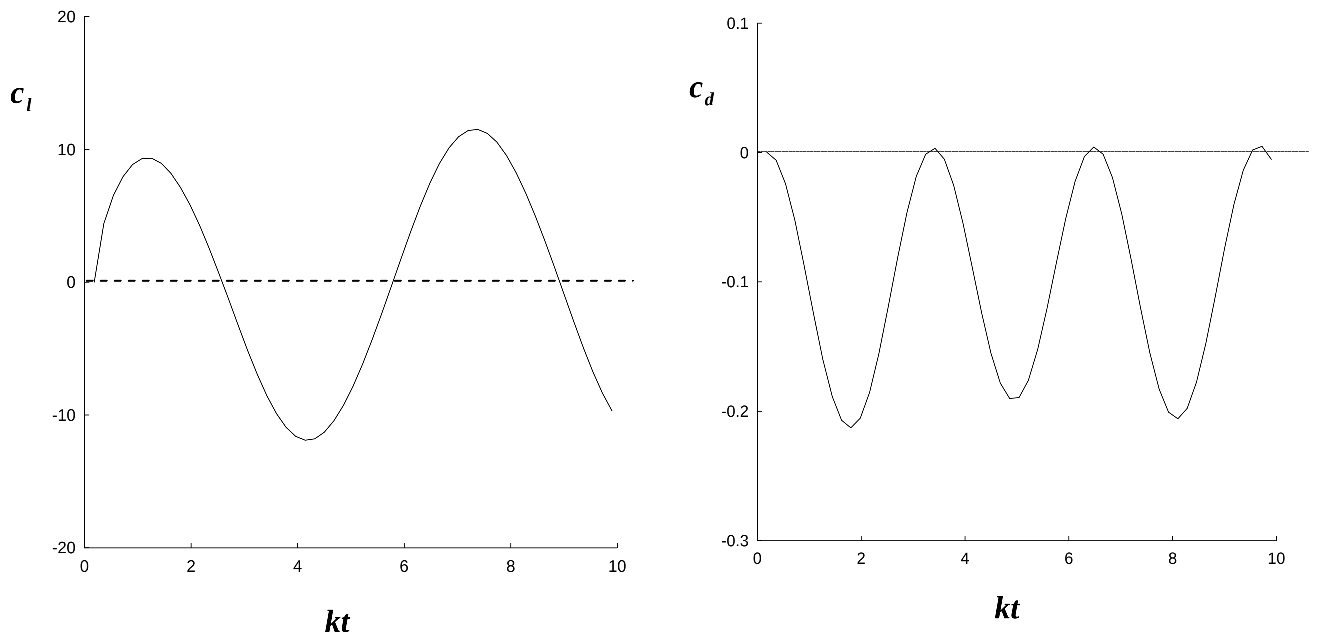


Fig (14): Lift and Drag coefficients of NACA 0008 airfoil oscillating with Heaving motion k=8.5, h=0.018.

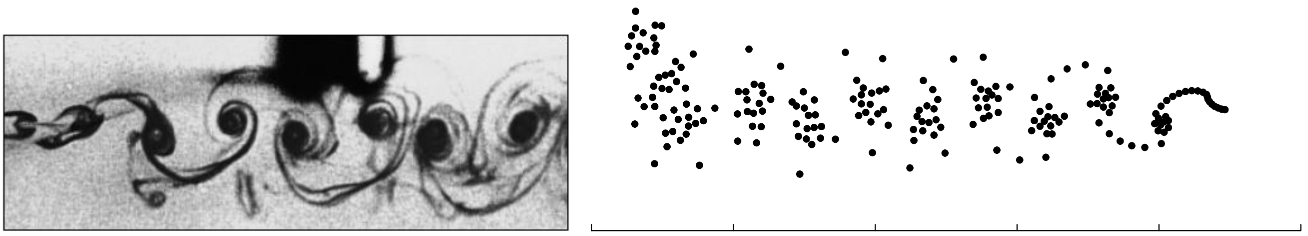


Fig (15): Wake Pattern for pitching oscillation as compared with **Basu and Hancock (1978)**, airfoil with k=20, θ_o=0.01 rad.

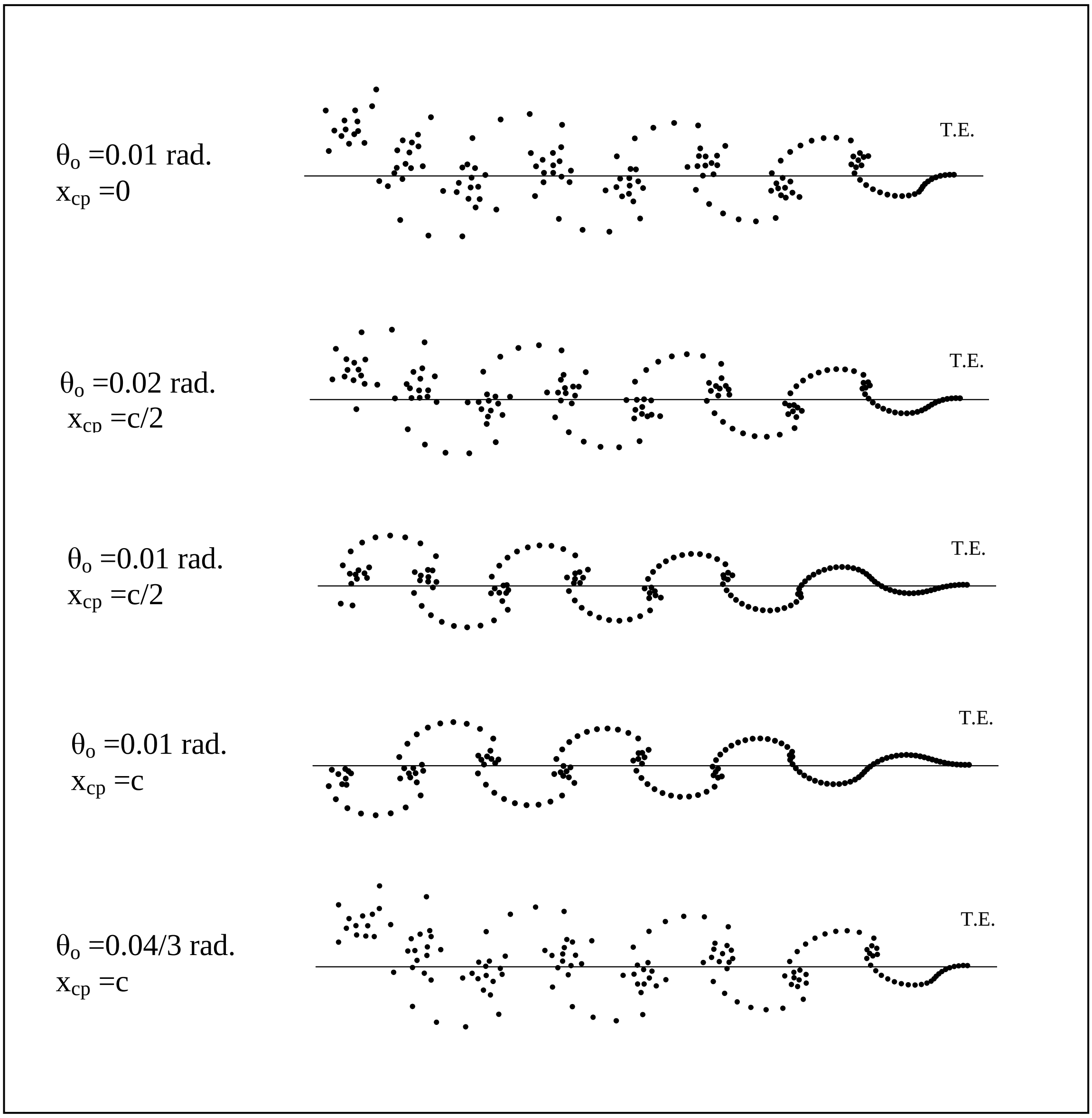


Fig (16): Wake patterns for NACA 0008 thick symmetrical aerofoil in pitching oscillation at reduced frequency k=20.

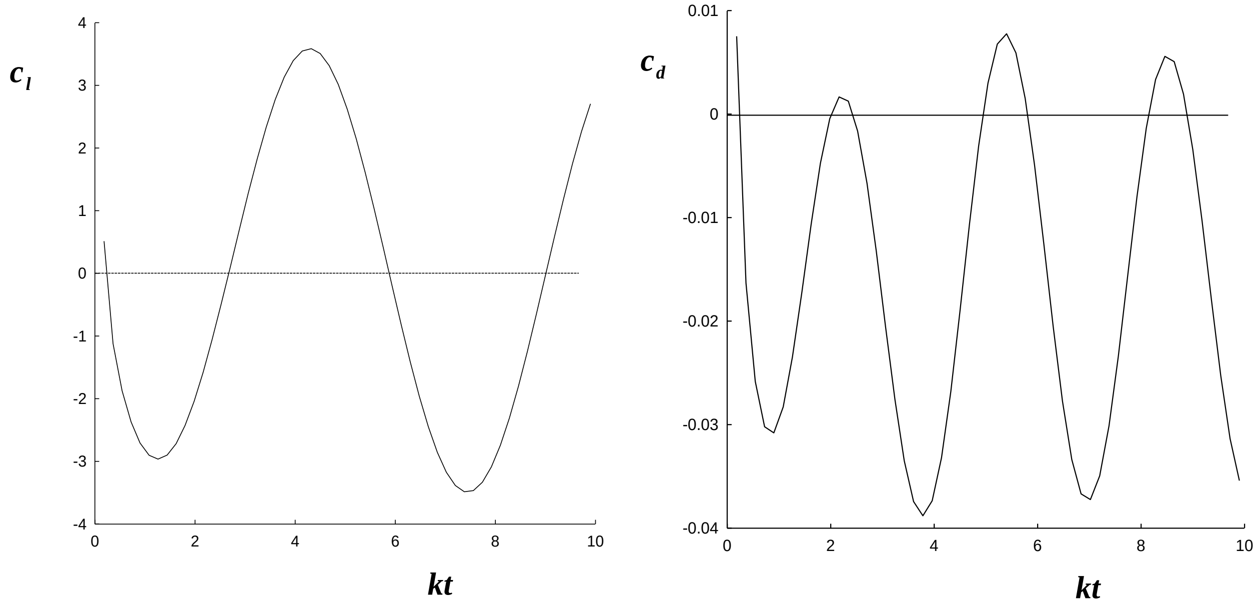


Fig (17): Lift and Drag coefficients of NACA 0008 airfoil oscillating with pitching motion k=20, θ_o =0.01 rad.