Finding the Area for Irregular Shapes Using Boundary Tracing Technique

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Abstract
In this research an attempt for calculating the area for any irregular shape had been done by merge image processing technology with mathematic theorem to obtain the estimated area value or real value, this is carried out by applying the boundary tracing method to the set formed from the digital image of an object in order to obtain the characteristics vector which is very important in the recognition of an object. In this method, only the pixels with contour (boundary) lines were traced by scanning the set for a given order, then an algorithm that use the coordinates of the boundary pixels were applied to find the area by sub dividing the object area into finite polygons depending on the points that obtained by tracing the contour of the object. The results achieved comparing with previous methods give a good agreement.

Introduction
Physical attributes, such as density, mass, surface area and volume, have often been used to calculate water loss, heat transfer, quantity of pesticide applications, respiration rates, evaluation of fruit growth and quality, ripeness index to forecast optimum harvest time, grading and so on (Hahn and Sanchez, 2000[4]; Lee et al., 2006[8]; Lorestani and Tabatabaeefar, 2006[10]; Topuz et al., 2005[13]; Wilhelm et al., 2005)[15]. Among all these attributes, surface area is one of the most important factors in all these application fields.
Research work has been done to determine the relationship between surface area and more easily measured attributes such as mass, volume and 2-Dmeasures (Forbes and Tattersfield, 1999[3]; Hahn and Sanchez, 2000[4]; Lee et al., 2006[8]; Sabliov et al., 2002[12]; Wang and Nguang, 2007)[14]. In recent years, the search to find rapid and non-destructive techniques for measurement of these physical attributes for size sorting, quality grading etc have attracted many researches. Different mathematical models and numerical methods have been
applied to extract a representation of surface area and volume. Machine vision and image processing techniques have been found increasingly useful in the fruit industry, especially for applications in quality inspection and shape sorting. Researches in this area indicate the feasibility of using such systems to improve product quality while freeing people from the traditional hand sorting of agricultural materials. Currently, machine vision is the most effective tool for external feature measurements such as color intensity, color homogeneity, bruises, size, shape and stem identification (Forbes and Tattersfield, 1999[3]; Jafari et al., 2006[5]; Lee et al., 1999[7]; Lorestani et al., 2006[10]; Sabliov et al., 2002[12]). The use of machine vision is gaining interest for determination of physical attributes of fruits and irregular shaped objects, because it is a non-destructive method requiring image analyses and image processing operations. Forbes and Tattersfield (1999)[3] developed a combined machine vision and artificial neural network technique for the estimation of pear volume from 2-D digital images. The RMS percentage error using a single digital image was 3%. This error was reduced to 1.9% when the volume was estimated from sets of four images. Lorestani et al. (2006) [10] developed a fuzzy logic based algorithm for sorting of Golden delicious apples. Features such as color and size were measured through a data acquisition system consisted of apples sorter, illumination chamber, webcam and a PC. Grading results obtained in this manner showed 91.2 and 95.2% agreements for off-line and online cases, respectively, with that of human expert. Hahn and Sanchez (2000)[4] developed an imaging algorithm to measure the volume of non-circular shaped agricultural produce such as carrots. Both Sabliov et al. (2002)[12] and Wang and Nguang (2007)[14] used image processing techniques to compute the volume and surface area of axi-symmetric agricultural products. Leen (2003)[3] presents two algorithms for shape-based porosity measures. Specifically, he describes two innovative attributes called perimeter porosity and area porosity as feature descriptors. Perimeter porosity refers to how “porous” or “weak” an image object is, while area porosity determines the degree of “branchiness” of an object. He shows the applicability of the two measures through a SAR sea ice image classification example.

M. Khojastechnazh et.al.2009[6] developed in his paper, an accurate image processing algorithm for determination of volume and surface area of orange. The proposed machine vision system consists of two CCD cameras, an appropriate lighting system and a personal computer. The total volume and surface area of the orange is approximated as the sum of all elementary frustums. The difference between the computed volumes and surface areas obtained by the image processing method and measured by water displacement and tape method, respectively, are not statistically significant at the 5% level. The Bland-Altman results show that the orange size has no effect on the accuracy of estimated volume and surface area found by the image processing technique.
D.J. Lee et al. (2006)[8] A machine vision system that uses a nondestructive method to measure volume and surface area of objects with irregular shapes is presented in that paper. The system first takes a series of silhouettes of the object from different directions by rotating the object at a fixed angular interval. The boundary points of each image are then extracted to construct a silhouette. A three dimensional wire-frame model of the object can be reconstructed by integrating silhouettes obtained from different view angles. Surface area and volume can then be measured by means of surface fitting and approximation on the wire-frame model.

Polygon is a number of coplanar (Lying in the same plane) line segments, each connected end to end to form a closed shape. Irregular shape / irregular polygon shaped meaning each side could be a different length and each interior angle could be different.

If you know the x, y coordinates of the vertices (corners) of the shape, there is a method for finding the area directly (area of a polygon, coordinate geometry). This works for all polygon types (regular, irregular, convex, concave). [1]

**Algorithm (1)**

1. Make a table with the x, y coordinates of each vertex. Start at any vertex and go around the polygon in either direction. Add the starting vertex again at the end. You should get a table that looks like the leftmost gray box in the figure above.

2. Combine the first two rows by:
   a) Multiplying the first row x by the second row y.
   b) Multiplying the first row y by the second row x.
   c) Subtract the second product form the first.

3. Repeat this for rows 2 and 3, then rows 3 and 4 and so on.

4. Add these results, make it positive if required, and divide by two.

1- Coordinates
The first step is to turn each vertex (corner) into a coordinate, as shown in Fig (1):

Fig (1) irregular shape with its corners set up with coordinates

2- Area under one line segment

Now, for each line segment, work out the area down to the x-axis. As shown in Fig (2).
Fig (2) area for each segment

3- Average the two heights, then multiply by the width

Example: For the shape highlighted above, we take the two heights (the "y" coordinates 2.28 and 4.71) and work out the average height:

\[(2.28+4.71)/2 = 3.495\]

Work out the width (the difference between the "x" coordinates 2.66 and 0.72)

\[2.66-0.72 = 1.94\]

The area under that line segment is width × height

\[1.94 \times 3.495 = 6.7803\]

4- Add Them All Up

Add when they go forwards (positive width), and subtract when they go backwards (negative width).

If you always go clockwise around the polygon, and always subtract the second "x" coordinate from the first, it works out naturally, like this:

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Avg. Height</th>
<th>Width (+/-)</th>
<th>Area (+/-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
<td></td>
</tr>
<tr>
<td>0.72</td>
<td>2.28</td>
<td>2.66</td>
<td>4.71</td>
<td>3.495</td>
</tr>
<tr>
<td>2.66</td>
<td>4.71</td>
<td>5</td>
<td>3.5</td>
<td>4.105</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>3.63</td>
<td>2.52</td>
<td>3.01</td>
</tr>
<tr>
<td>3.63</td>
<td>2.52</td>
<td>4</td>
<td>1.6</td>
<td>2.06</td>
</tr>
<tr>
<td>4</td>
<td>1.6</td>
<td>1.9</td>
<td>1</td>
<td>1.3</td>
</tr>
<tr>
<td>1.9</td>
<td>1</td>
<td>0.72</td>
<td>2.28</td>
<td>1.64</td>
</tr>
</tbody>
</table>

Total: 8.3593

And it looks like Fig (3):
**Derivation of the general formula:**

This procedure for finding the area may be put in a general form that may be applied for any general shape.

First, number the vertices in order, going either clockwise or counter-clockwise, starting at any vertex.

The area is then given by the following formula:

$$\left| (x_2 - x_1) \times \frac{(y_2 + y_1)}{2} + (x_3 - x_2) \times \frac{(y_3 + y_2)}{2} + (x_4 - x_3) \times \frac{(y_4 + y_3)}{2} + (x_1 - x_4) \times \frac{(y_1 + y_4)}{2} \right|$$
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\[
\text{area} = \frac{1}{2} | \sum_{i=1}^{n} (x_i y_{i+1} - x_{i+1} y_i) |
\]

Where \( x_1 \) is the x coordinate of vertex 1 and \( y_n \) is the y coordinates of the nth vertex etc. Notice that the in the last term, the expression wraps around back to the first vertex again.

Adjust the quadrilateral ABCD by dragging any vertex. The area is calculated using this method as you drag. A detailed explanation follows the diagram.

The above diagram shows how to do this manually and applies algorithm (1).

5- Candidate system

In the previous algorithm, the basic point for calculating the area depend on finding the coordinate of the convex of the irregular shape, which may be found by tracing the boundary for any irregular shape. This is done by designing a simple algorithm depends on reading a digital image containing an irregular
shape to be traced and merge algorithm(1) with image technique to reach a
goal(calculate area of any irregular shaped automatically).

**Calculate the Area Using Trace Boundary Technique**

In this work, we use the image processing technique of object recognition through
traceing its boundary to find all the pixels coordinates and use these to calculate
the area depend on the algorithm 1. Since statistical or geometrical or
characteristics vectors are obtained with the information about boundaries, so to
trace the boundary pixels of any object we first construct a chain code [17] of the
object boundary. This algorithm use a 4-neighbour chain code. It contains an
initial point \((x_{\text{ini}},y_{\text{ini}})\), an end point \((x_{\text{end}},y_{\text{end}})\), and a set of 4-direction leading
from the current point to the next on boundary. Given an image contains an
irregular object, convert it to binary that contain the object with the pixels labeled
with value 0 or ‘black’ and its background labeled with value 1 or ‘white’, the
following algorithm find the boundary:[16]

**Algorithm(2)**

a. Find the topmost, leftmost pixel with the label 0(zero). This is the initial point
of the chain code.
b. Construct the chain code by moving to the leftmost 4-neighbor that shares the
same label until the next neighbor has reaches the initial point of the chain code.
c. The number of directions or moves of the chain code is the boundary.

**Experimental Results**

In order to evaluate this work, three regular shape were selected namely (Circle
with diameter 36mm, Square with dimension 24mm and equal sides Triangle
with side equal to 24mm) as shown in fig.(4). The area of these shapes may be
calculated manually with their traditional geometrical Laws, which are shown in
Table -1.

To make a comparison between the geometrical method and the algorithm of this
work, the geometrical shapes were taken as a digital image using scanner. The
calculated area by this method for circle as an example is \(1.3555 \times 10^5\) (pixel)\(^2\). A
scale ratio of 36mm/417pixel is recognized for this image, therefore the real area
of the circle will be

\[
\text{area of circle} = 1.3555 \times 10^5 \text{ (pixel)}^2 \times \left( \frac{36\text{mm}}{417\text{pixel}} \right)^2 = 1010.258\text{mm}^2
\]

The error between the two methods is \(\frac{1017.87 - 1010.258}{1017.87} = 0.7478\%\) the results for
the other two shapes are shown in table (2).
Table (1) Results of area calculated geometrically and by algorithm 2

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area by geometrical calculation (mm)$^2$</th>
<th>Area calculated using algorithm2 (pixel)$^2$</th>
<th>Scale ratio</th>
<th>Area calculated by algorithm in (mm)$^2$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle with d=36</td>
<td>1017.87</td>
<td>1.3555x10$^3$</td>
<td>$\frac{36 \text{ mm}}{417 \text{ pixel}}$</td>
<td>1010.258</td>
<td>0.7478%</td>
</tr>
<tr>
<td>Square with L=24</td>
<td>576</td>
<td>7.4188x10$^4$</td>
<td>$\frac{24 \text{ mm}}{272 \text{ pixel}}$</td>
<td>577.58</td>
<td>0.2743%</td>
</tr>
<tr>
<td>Equal sides Triangle with L=24</td>
<td>249.415</td>
<td>2.6838x10$^4$</td>
<td>$\frac{24 \text{ mm}}{248.2606 \text{ pixel}}$</td>
<td>250.817</td>
<td>0.562%</td>
</tr>
</tbody>
</table>

To generalize this method for the other irregular shapes, the Iraq map image with size 1140x1260 shown in figure (5) has been taken as an example, the algorithm applied to it, and the resultant area is 604291 (pixel)$^2$. Where

\[
Scale \ ratio = \frac{1140}{189} = 6.03174 \ \text{pixel/mm}
\]
\[
Area = 604291 \text{ pixel}^2 \times \left( \frac{189 \text{ mm}}{1140 \text{ pixel}} \right)^2 \times \left( \frac{100 \text{ km}}{20 \text{ mm}} \right)^2 = 415240 \text{ km}^2
\]

Conclusions

From previous application can described the following point:

1- Application is more efficient to calculated area for any irregular shaped by given the estimated value near to real value.

2- It is more flexible for mat-staff and survey engineers for estimating real area for geometrical maps and lands.

3- All result reached - compared with manual calculated –is more fixed and with high speed.

4- The error in using this approach is very small and negligible, where the error ranked between 0.27% - 0.74% for the selected shape in this work.
References


