Optimum Solving SHEPWM Equations for Single Phase Inverter Using Resultant Method

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Abstract

This paper represents new method to determine the optimum switching angles for Selective Harmonic Eliminated PWM (SHEPWM) inverter. Such switching angles are defined by a set of nonlinear equations to be solved using the Resultant method. This is done by first converting these equations that specify the harmonic elimination problem into an equivalent set of polynomial equations. Then, using the mathematical theory of Resultants, all solutions to this equivalent problem can be found without the need for any initial guess. The complete solutions for unipolar SHEPWM switching pattern which produce the fundamental while not generating specifically chosen harmonics are investigated.

الحل الأمثل لمعادلات تضمين عرض النبضة بحذف التوافقيات الأنتقائي لعاكس أحادي الطور بأستخدام طريقة المحصلة

الخلاصة

البحث الحالي يمثل طريقة جديدة لتحديد زوايا التشغيل المثالية لعاكس يعمل بتضمين عرض النبضة بحذف التوافقيات الأنتقائي. هده الزوايا تُعرّف عن طريق مجموعة معادلات غير خطية يتمّ حلها بأستخدام طريقة المحصلة. ويتمّ ذلك أولا" بتحويل تلك المعادلات المخصصة لحذف التوافقيات الى معادلات متعددة الحدود. وباستخدام النظرية الرياضية لهده الطريقة يمكن الحصول على كل الحلول للمعادلات المكافئة بدون الحاجة الى أي تخمين أبتدائي للشروط. تمّ التحقق من الحلول الكاملة لنموذج التشغيل أحادي القطبية لتضمين عرض النبضة بحذف التوافقيات الأنتقائي والذي يوّلدّ التوافقية الأساس بينما يحذف بعض التوافقيات المحددة.

Key-Words: Harmonic Elimination, PWM, Resultant Theory.

I. Introduction

The optimum technique is that technique which minimizes the harmonic content of the inverter output voltage. The best compromise between efficiency and quality of inverter operation is achieved by the optimal switching pattern with the lowest total harmonic distortion (THD).

In this paper, it is shown how the complete solution (i.e., *all* possible solutions) to the problem considered in [1,2] is obtained. Specifically, in [1,2] the harmonic elimination problem was formulated as a set of transcendental equations that must be solved to determine the times (angles) in an electrical cycle for turning the switches on and off in a full bridge inverter so as to produce a desired fundamental amplitude while eliminating, for example, the 3^{rd} and 5^{th} harmonics.

These transcendental equations are then solved using iterative numerical

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techniques to compute the switching angles. Challenging approaches have been reported by several papers [3-7] which try to modify its numerical process. Some of them have found to have multiple solutions in three phase cases [3-6] and this fact deepens its numerical aspects. The Walsh function method [7] also has been proposed to simplify the process. Recently, on-line computation methods have been proposed to make the technique a more flexible and interactive one. Here a method is presented that not only obtains these solutions, but also another (different) set of the switching angles, and this other set of switching angles actually generates a smaller harmonic distortion due to the 7th and 9th harmonics.

II. H-Bridge Inverter

Basically, a full-bridge single-phase inverter is known as an H-bridge inverter, which is illustrated in Figure 1.

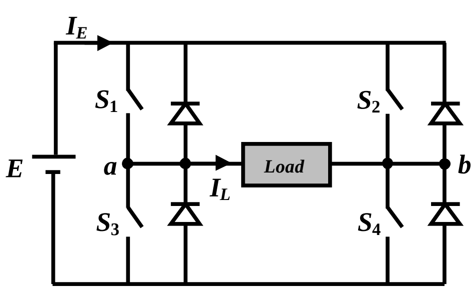

Fig. 1: H-bridge Inverter

The full bridge inverter can provide either Bipolar or Unipolar output voltage switching. The Unipolar inverter is optimum for harmonic elimination more than the Bipolar inverter. Therefore the Unipolar scheme is the optimum technique [8].
The Unipolar inverter circuit consists of four main switches and four freewheeling diodes. According to four-switch combination, three output voltage levels, +*E*, -*E*, and 0, can be synthesized for the voltage across *a* and *b* [9]. Figure 2 shows the unipolar waveform output from H-bridge inverter.

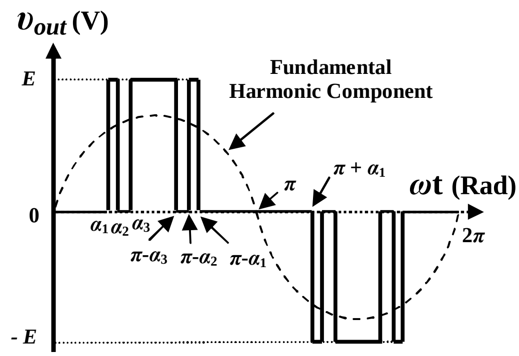

Fig. 2: Unipolar Switching Scheme

III. Optimized SHEPWM Switching Angles

The optimized unipolar waveform shown in Figure 2 is assumed to be the quarter-wave symmetric.
The Fourier series of the general quarter-wave symmetric H-bridge inverter output waveform is written as follows: [3]

$$u_{out}(wt) = u_{ab} = \sum_{n=1}^{\infty} \frac{4E}{np} \left[\sum_{k=1}^{K} (-1)^{(k-1)} \cos(na_k) \right] \sin(nwt) \qquad (1)$$

where α_k is the optimized switching angles, which must satisfy the following condition: $0 \le \alpha_1 \le \alpha_2 \le ... \alpha_k \ldots \le \alpha_K \le \pi/2$.
The amplitude of all odd harmonic components including fundamental one, are given by:

$$h_n = \frac{4E}{np} \sum_{k=1}^{K} (-1)^{(k-1)} \cos(na_k) \qquad (2)$$

where: *n* is the harmonic order and *K* is the number of switching angles per quarter cycle. The amplitude of DC component and all even harmonics equal zero. Thus, only the odd harmonics in the quarter-wave symmetric waveform need to be eliminated. The switching angles of the waveform will be adjusted to get the lowest output voltage THD.

IV. Solving SHEPWM Equations

A. Numerical Methods

Eq. 2 consists of nonlinear equations and transcendental in nature. As a result, many people have utilized numerical iterative techniques in order to solve these equations. For example, *Jian Sun* had been used the Newton-Raphson numerical technique [10]. Another numerical technique one might use is Gauss-Seidel, although this particular numerical technique is not as robust as Newton-Raphson.

Unfortunately, numerical iterative techniques have their drawbacks:

1. These techniques require an initial guess in order to work. However, if the initial guess is not good enough, a solution will not be found.
2. They will only find one solution, if one exists.
3. They needed large time for calculation. This time increased with increasing the degree of freedom of the nonlinear equations.

The obvious drawback here is that more than one solution might exist to the problem at hand.

Until now, numerical iterative techniques seemed to be the only viable method to solve the aforementioned nonlinear harmonic equations. However, the next section will introduce Resultant theory. Using Resultant theory, all solutions to these nonlinear equations can be found without the need for an initial guess.

B. Resultant Theory

When the Unipolar SHEPWM switching scheme is implemented using *K* switching angles, Eq. 2 can be used to derive *K* different harmonic equations. In other words, *K* switching angles will be used to control the values of *K* different harmonics.

By making some simple changes of variables and simplifying for transcendental equations, these equations can be transformed into a set of polynomial equations. Then, Resultant theory can be utilized to find all solutions to the harmonic equations without the need for an initial guess.

An example application of Resultant theory will be given in the next section by considering an H-bridge inverter. In this example, the value of the output voltage fundamental will be controlled while the 3rd and 5th order harmonics are eliminated.

V. Transcendental SHEPWM Equations

Eq. 2 gives the values of the odd sine harmonics corresponding to the unipolar switching scheme with *K* switching angles. If three switching angles are used instead, it can be shown that the corresponding equation is:

$$h_n = \frac{4E}{np}\left[\cos(na_1) - \cos(na_2) + \cos(na_3)\right] \qquad (3)$$

If one wants to control the peak value of the output voltage to be V_1 and eliminate the 3rd and 5th order harmonics, the resulting harmonic equations are:

$$\frac{4E}{p}\left[\cos(a_1) - \cos(a_2) + \cos(a_3)\right] = V_1 \qquad (4)$$

$$\cos(3a_1) - \cos(3a_2) + \cos(3a_3) = 0 \qquad (5)$$

$$\cos(5a_1) - \cos(5a_2) + \cos(5a_3) = 0 \qquad (6)$$

One can also rewrite Eq. 4 as:

$$\cos(a_1) - \cos(a_2) + \cos(a_3) = m \qquad (7)$$

where the parameter m acts as the modulation index and:

$$m = p\, V_1 / 4E \qquad (8)$$

It should be pointed out that a square wave of amplitude E results in the maximum peak value of the fundamental [11].

VI. Solutions to the SHEPWM Equations by Resultant Theory

For the transcendental harmonic equations given in Eqs. (4)-(6), consider the following changes of variables:

$x_1 = \cos(\alpha_1)$
$x_2 = \cos(\alpha_2)$
$x_3 = \cos(\alpha_3)$ (9)

Also, consider the following trigonometric identities:

$$\cos(3a) = -3\cos(a) + 4\cos^3(a) \qquad (10)$$

$$\cos(5a) = 5\cos(a) - 20\cos^3(a) + 16\cos^5(a) \qquad (11)$$

Applying the results given in Eqs. (9)- (11) to the transcendental harmonic Eqs. (5)-(7), one obtains the following polynomials:

$$p_1(x_1, x_2, x_3) = 0 = x_1 - x_2 + x_3 - m \qquad (12)$$

$$p_3(x_1, x_2, x_3) = 0 = \sum_{n=1}^{3} (-1)^{n-1}\left(-3x_n + 4x_n^{\,3}\right) \qquad (13)$$

$$p_5(x_1, x_2, x_3) = 0 = \sum_{n=1}^{3} (-1)^{n-1}\left(5x_n - 20x_n^{\,3} + 16x_n^{\,5}\right) \qquad (14)$$

It should be noted that unipolar switching requires: $0 \le a_1 \le a_2 \le a_3 \le p/2$, where the units of the switching angles are radians. Therefore, the new variables: x_1, x_2 and x_3 must satisfy: $0 \le x_3 \le x_2 \le x_1 \le 1$.

Eqs. (12)-(14) are polynomial equations in the variables x_1, x_2 and x_3. Resultant method using Elimination theory [12] can now be used to solve polynomials p_1, p_3 and p_5 for the common roots of these three equations.

In general, to solve the harmonic equations by Resultant theory, they must be changed as it was shown before into polynomials. First, change the variables:

$x_1 = \cos(\alpha_1)$
$x_2 = \cos(\alpha_2)$
.
.
.
$x_K = \cos(\alpha_K)$ (15)

Applying the results given in Eqs. (15) and the trigonometric identities $\cos(3\alpha)$, $\cos(5\alpha)$, $\cos(7\alpha)$,…, $\cos(n\alpha)$ to the transcendental harmonic Eqs. 2, the following polynomials: $p_1(x_1,x_2,\ldots,x_K)$, $p_3(x_1,x_2,\ldots,x_K)$,…, $p_K(x_1,x_2,\ldots,x_K)$ can be found. For these polynomial equations, the following situation must be satisfied: $0 \le a_1 \le a_2 \le \ldots\ldots \le a_K \le p/2$. So that the variables x_1, x_2,…, x_K must satisfy: $0 \le x_K \le \ldots\ldots \le x_2 \le x_1 \le 1$.

Now, the transcendental harmonic equations have been changed into polynomial equations in the variables x_1, x_2,…, x_K. Resultant theory can be used to solve these polynomial equations to find the optimized switching angles.

A. Solutions to Polynomials Using Resultant Theory

The polynomials p_1, p_3 and p_5 [See Eqs. (12)-(14)] are functions of the variables x_1, x_2 and x_3. Using p_1 to solve for x_1 in terms of the other two variables, one gets:

$x_1 = m - x_2 - x_3$ (16)

Substituting this result into p_3 and p_5, one gets:

$$p_3(x_2,x_3) = \left(-3(m-x_2-x_3)+4(m-x_2-x_3)^3\right) - \left(-3x_2+4x_2^3\right)+\left(-3x_3+4x_3^3\right) \quad (17)$$

$$p_5(x_2,x_3) = \begin{pmatrix} 5(m-x_2-x_3)-20(m-x_2-x_3)^3 \\ -16(m-x_2-x_3)^5 \end{pmatrix} - \left(5x_2-20x_2^3+16x_2^5\right) + \left(5x_3-20x_3^3+16x_3^5\right) \quad (18)$$

After x_1 has been trivially eliminated, one can now apply Resultant theory to eliminate x_2. It should be noted that, for Eqs. 17 and 18, it turns out that the degree of polynomials $p_3(x_2,x_3)$ and $p_5(x_2,x_3)$ in the variable x_2 is two and four, respectively.

All Resultant calculations were found by using the *Resultant* command in the software package Mathematica. After factoring and then eliminating redundant factors and unnecessary numerical constants, the Resultant of the two polynomials in Eqs. 17 and 18 was found to be as in the Appendix (A), where:

$$res(x_3) = res\left(p_3(x_2,x_3),\, p_5(x_2,x_3),\, x_2\right) \quad (19)$$

Since the polynomial $res(x_3)$ is only a function of one variable, one can begin the process of finding the appropriate switching angles by the following steps:

1. Given the value for the parameter m, solve for the roots of $res(x_3) = 0$.

2. Keep the roots for which: $0 \le \mathrm{Re}(x_3) \le 1$, where Re refers to the real part of a possibly complex root. Denote these roots as $\{x_{3k}\}$.

3. For each member of the set $\{x_{3k}\}$. Substitute it into $p_3(x_2,x_3)$ and solve for the roots of $p_3(x_2,x_{3k}) = 0$.

4. Keep the roots for which: $0 \le \mathrm{Re}(x_{3k}) \le \mathrm{Re}(x_2) \le 1$. Denote the set of remaining roots as $\{x_{2l},x_{3l}\}$.

5. For each member of the set $\{x_{2l},x_{3l}\}$, compute m-x_{2l}-x_{3l} to find the values for x_1.

6. Keep the roots for which: $0 \le \mathrm{Re}(x_{3l}) \le \mathrm{Re}(x_{2l}) \le \mathrm{Re}(x_1) \le 1$. Denote the set of remaining roots as: $\{(x_{1n}, x_{2n}, x_{3n})\}$.

7. For each member of the set $\{(x_{1n}, x_{2n}, x_{3n})\}$, keep just the real parts of x_{1n}, x_{2n}, and x_{3n}. Denote these triples as $\{(\hat{x}_{1n}, \hat{x}_{2n}, \hat{x}_{3n})\}$.

8. Using Eqs.13 and 14, compute:

$$\sqrt{\left(\frac{p_3(\hat{x}_{1n},\hat{x}_{2n},\hat{x}_{3n})}{3}\right)^2 + \left(\frac{p_5(\hat{x}_{1n},\hat{x}_{2n},\hat{x}_{3n})}{5}\right)^2} \quad (20)$$

9. If the result is less than some arbitrarily small tolerance level ***ε*,** the switching angles are given by:

$$\{(a_{1n}, a_{2n}, a_{3n})\} = \left\{\cos^{-1}(\hat{x}_{1n}), \cos^{-1}(\hat{x}_{2n}), \cos^{-1}(\hat{x}_{3n})\right\} \quad (21)$$

Eq. 20 gives an indication of the harmonic distortion due to the 3rd and 5th order harmonics. The values given by Eq. 20 can be controlled such that they are always below some arbitrarily small number ε. For the work presented in this paper, this tolerance level was set at 0.001 times the current value of m.

VII. Minimization of the 3rd and 5th Harmonic Components

For those values of m for which $p_3(x_1,x_2)$, $p_5(x_1,x_2)$ do not have common zeros satisfying $0 \le x_1 \le 1$, $0 \le x_2 \le 1$, the next best thing is to minimize the error:

$$e(x_1,x_2)=\frac{1}{9}p_3^{\,2}(x_1,x_2)+\frac{1}{25}p_5^{\,2}(x_1,x_2) \qquad (22)$$

This was accomplished by simply computing the values of $e(i\Delta x, j\Delta y)$ for i, $j = 0, \ldots, 1000$ with $\Delta x = 0.001$, $\Delta y = 0.001$ and then choosing the minimum value.

VIII. Simulation Results

The computer software package Mathematica was used to perform all of the above calculations as a first part. The second part of the theoretical calculations involved organizing and analyzing all of the collected switching angles. For this purpose, the software package MATLAB was utilized. Using MATLAB, the collected switching angles were organized into look-up tables to be used later in simulations. Also, MATLAB was used to generate plots of the switching angles and THD versus *m*.

The THD mathematically calculated by:

$$THD=\frac{\sqrt{\sum_{n=2}^{\infty}h_n^{\,2}}}{h_1} \qquad (23)$$

For 3-switching Unipolar SHEPWM inverter $m\in[0\text{-}0.83]$ there is solutions for the Eq. 3. Consequently, for these range of *m*, the switching angles were determined by minimizing the error in Eq. 22. Figure 3 shows a plot of the resulting minimum error vs. *m* for these value of *m* in single phase inverter with 3-switching angles (K=3). Figure 3 shows, when $m\in[0\text{-}0.83]$, the error goes to zero, because these values correspond to the boundary of the exact solutions of Eq. 3. However, note that, the minimum error in the interval higher than 0.83 is too large to make the corresponding switching angles for this interval of any use.

The optimized Unipolar SHEPWM switching angles can be represented as in Figure 4 with the variation of the number of switching angles *K* form 2 to 7. We can show that increasing of *K* causes decreasing of the *m* range.

Evaluation of the inverter performance can be calculated from the performance factor THD in Eq. 23. Figure 5 illustrate the relationship between this factor and *m* with different values of *K*. We can see that decreasing of *m* has direct effects, causing an increase in the harmonics amplitude of the inverter. This increase leads to increase the harmonic currents and torque pulsations of the motor fed from. Increasing of harmonic currents causes increasing of motor copper losses as heat, and they act as the main increasing in the THD. The selection of high *K* can cancel the negative effect of *m* decreasing. As a result, THD decreases with increasing *K* (spatially at low *m*) and increase with decreasing *m*.

The voltage harmonic spectra for Unipolar SHEPWM waveform are given in Figure 6 with number of switching angles *K* = (2-7) to eliminate (1-6) low order harmonics, where the number of harmonics to be eliminated = K–1.

This Figure shows that, increasing of *K* will cause increase in the number of low order eliminated harmonics, which causes to push more harmonic energy into high frequency regions, therefore low frequency harmonics are well attenuated. It can be seen, that the variation of *K* values affect the location of the harmonics in the spectrum, (i.e. the first significant component in the inverter output for K=7 is equal to 15 or 750 Hz).

Increasing of *K* causes to increase the motor impedance with frequency ($X=2\pi fL$), therefore, the harmonic currents will decrease for constant harmonic voltages amplitude as shown in Figure 7. The induction motor can be represented as a good low pass filter. As a result the increasing of K is the way to reduce the effects of reducing *m* when small values of voltages and frequencies are required in the output of the inverter.

The inverter is loaded by single-phase Permanent Spilt Capacitor (PSC) induction motor with the following ratings: Rate power is 175Watt, rated current is 1.22A, rated speed is 1275Rpm and rated supply voltage is 220V.

All the optimized switching angles and the first seven odd harmonic amplitudes are illustrated in Table (1).

IX. Conclusions and Suggestions

A full solution to the problem eliminating the 3^{rd}, 5^{th},…, 13^{th} harmonics in a unipolar SHEPWM inverter has been given. Specifically, Resultant theory was used to completely characterize for each *m* when a solution existed and when it did not (in contrast to numerical techniques such as Newton-Raphson).

The results show that the switching angles computed accurately, eliminate the selected harmonics of the desired fundamental amplitudes. When low order harmonics are eliminated through the modulation of the inverter, only higher-order harmonics will appear at the output, and need to be attenuated by the filter to get nearly sinusoidal output. The cut frequency of the filter can thus be increased, this lead to significant reduction in the filter size.

However, increasing the number of switching angles will lead to polynomial equations of higher degree [with respect to Eqs. 17 and 18]. Therefore Resultant theory will be not effective of solving these polynomials.

One suggestion for future work would be to extend the SHEPWM switching scheme to include more than 7-switching angles per quarter cycle.

X. References

[1] H. S. Patel and R. G. Hoft, "**Generalized harmonic elimination and voltage control in thyristor converters: Part I-harmonic elimination**," IEEE Trans. on Ind. Appl., Vol. 9, pp. 310-317, May/June 1973.

[2] H. S. Patel and R. G. Hoft, "**Generalized harmonic elimination and voltage control in thyristor converters: Part II-voltage control technique**," IEEE Trans. on Ind. Appl., Vol. 10, pp. 666-673, Sept. /Oct. 1974.

[3] P. N. Enjeti, P. D. Ziogas, and J. F. Lindsay, "**Programmed PWM Techniques to Eliminate Harmonics: A Critical Evaluation**," IEEE Trans. Ind. Applicat., Vol. 26, pp. 302–316, Mar./Apr. 1990.

[4] A. Pollmann, "**A digital pulse width modulator employing advance modulation techniques**," IEEE Trans. Ind. Applicat., Vol. 19, pp. 409-414, May/June 1983.

[5] T. Kato., "**Precise PWM waveform analysis of inverter for selected harmonic elimination**," in Proc. IEEE IAS Ann. Meeting, 1986, pp. 611-616.

[6] Toshiji Kato, "**Sequential Homotopy-Based Computation of Multiple Solutions for Selected Harmonic Elimination in PWM Inverters**," IEEE Trans. on Circuits and Sys.-1: Fund. Theory and Applicat. Vol., 46, No. 5, May 1999.

[7] T. Jun Liang, M. Oconnell and G. Hoft, "**Inverter Harmonic Reduction Using Walsh Function Harmonic Elimination Method**," IEEE Trans. Power Elec., Vol. 12, No. 6, Nov., 1997.

[8] K. S. Krikor and Jamal A. Mohammed, "**PWM Strategies for Inverter-Fed Induction Motors - A Comparative Study**," Engineering and Technology, Vol. 21, No. 11, 2002.

[9] J. M. Jacob, **Power Electronics: Principles & Applications**, 2nd Edition, Vikas Publishing House Pvt. Ltd., Singapore, 2004.

[10] J. Sun, H. Grotstollen, "**Solving Nonlinear Equations for Selective Harmonic Eliminated PWM using Predicted Initial Values**," in Proc. IECON 1992, pp. 259-264.

[11] H. Rashid, **Power Electronics**, 2nd Edition, Prentice-Hall, Inc. 1994.

[12] S. Lang, **Introduction to Algebraic Geometry**, 3rd Edition, Addison-Wesely Publishing Company, Inc., 1972.

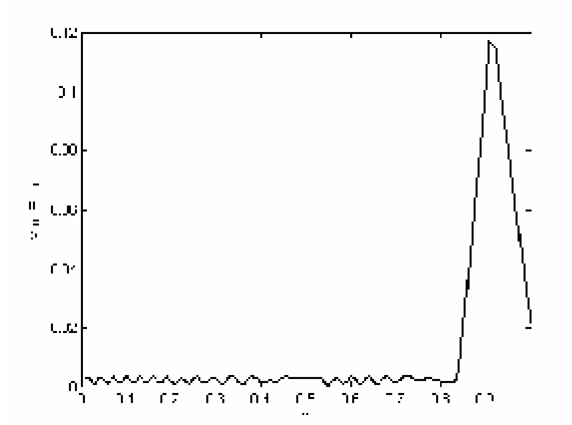

Fig. 3: Error Minimizing for Unipolar SHEPWM Single Phase Inverter

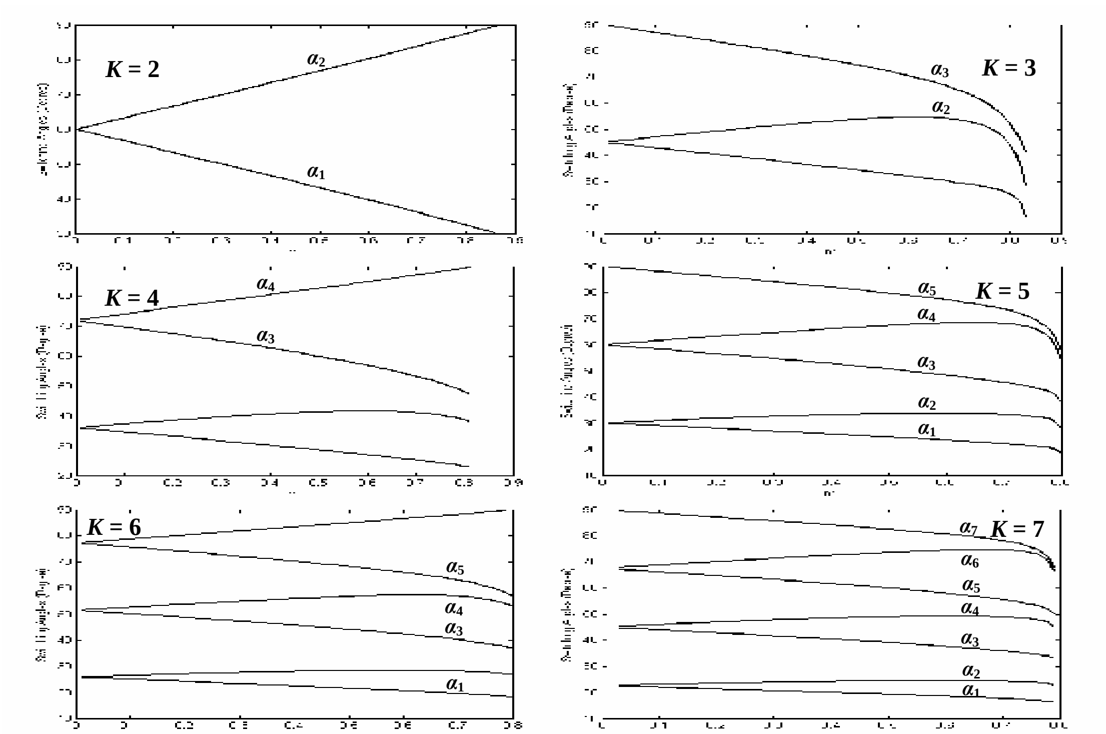

Fig. 4: the Solutions of Switching Angles vs. *m* with different values of *K* for Unipolar SHEPWM Inverter

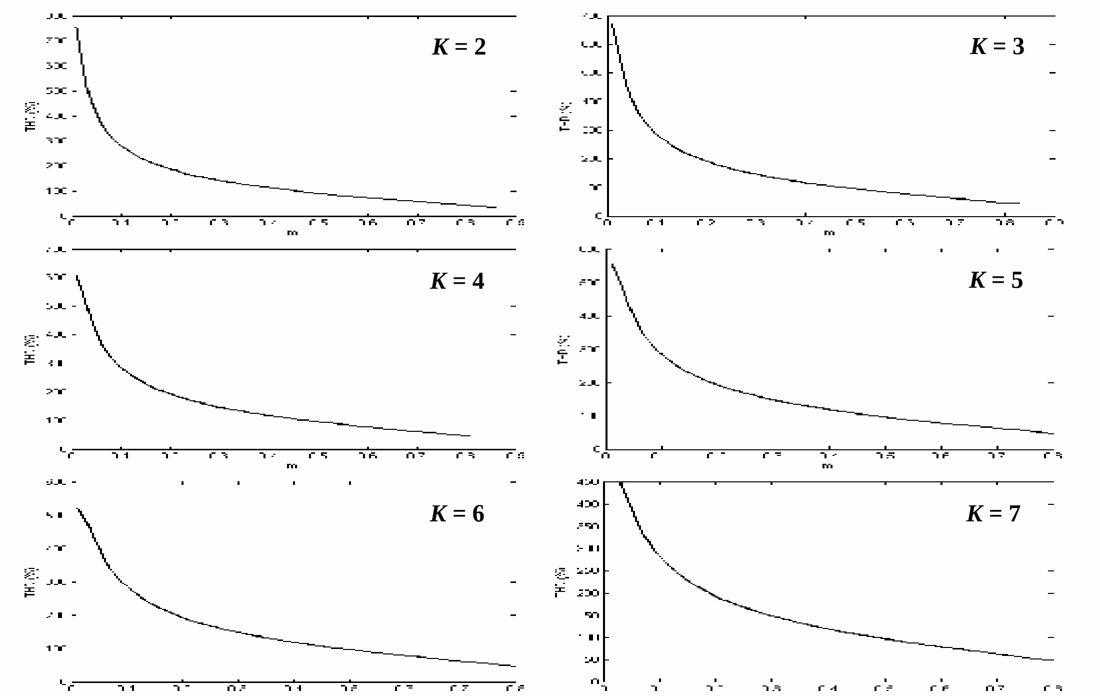

Fig. 5: the Voltage THD vs. *m* with different values of *K* for Unipolar SHEPWM Inverter

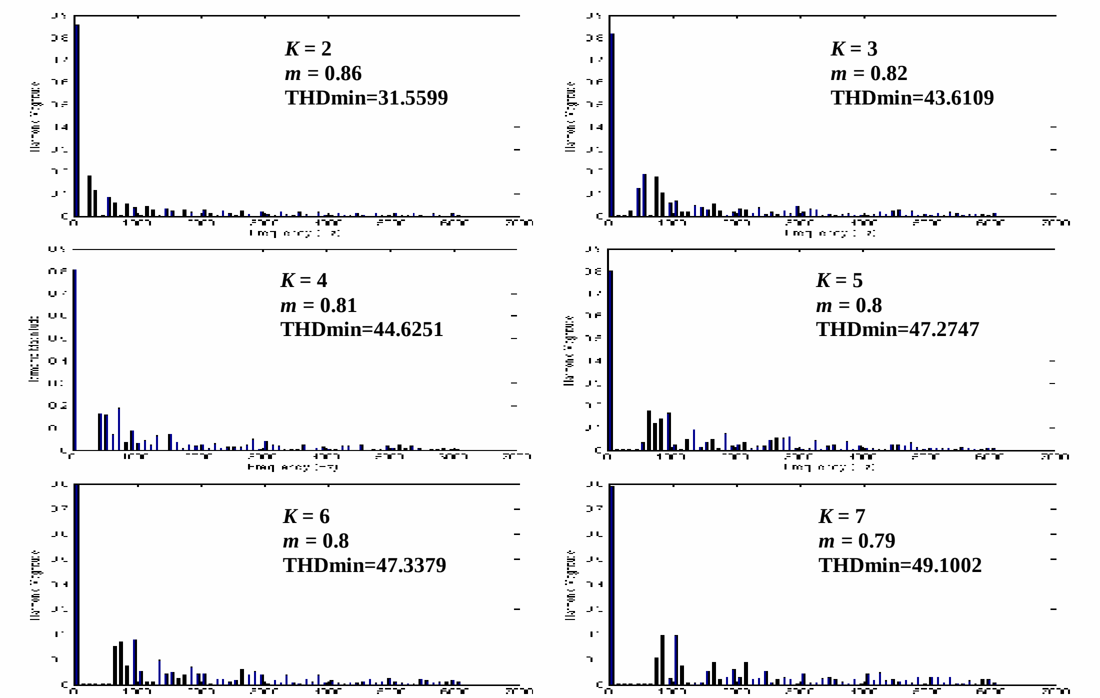

Fig. 6: the Output Voltage Spectra with different values of *K* for Unipolar SHEPWM Inverter at lowest THD

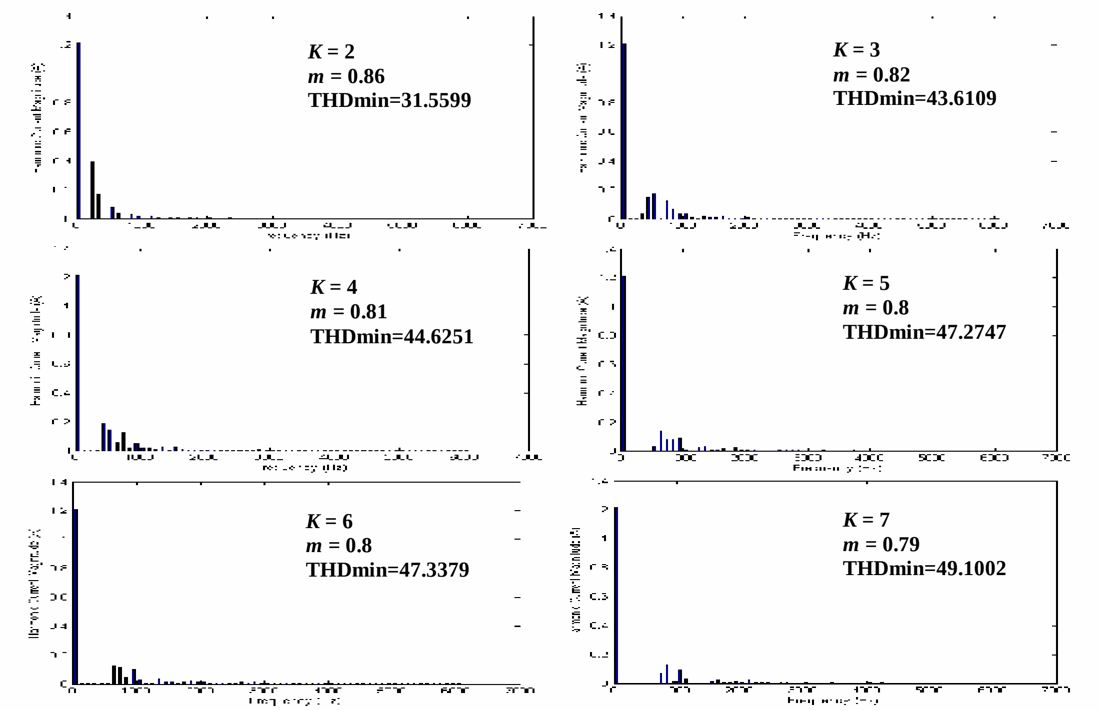

Fig. 7: the Line Current Spectra of Motor Fed by Unipolar SHEPWM Inverter with Different Values of *K* at Lowest THD

Table 1: Switching Angles, Normalized Voltage Harmonics and THD with *K*

Switching No. (*K*)		2	3	4	5	6	7
α (Degree)	α_1	30.2299	21.8958	22.9250	18.8804	18.2243	16.3179
	α_2	89.7701	36.1960	38.2119	28.0493	26.7161	22.7210
	α_3		45.6422	47.3323	38.1820	36.9936	32.9286
	α_4			89.8262	54.7979	53.1178	45.0800
	α_5				58.2133	56.9332	50.0789
	α_6					89.9573	66.3199
	α_7						67.7067
Harmonics (p.u.)	V_1	0.86	0.83	0.81	0.80	0.80	0.79
	V_3	0	0	0	0	0	0
	V_5	0.1792	0	0	0	0	0
	V_7	0.1177	0.1192	0	0	0	0
	V_9	0	0.0476	0.1642	0	0	0
	V_{11}	0.0847	0.1517	0.1556	0.0338	0	0
	V_{13}	0.0605	0.2166	0.0696	0.1789	0.1520	0
THD_{min}(%)		31.5599	43.6109	44.6251	47.2747	47.3379	49.1002
THD_{max}(%)		754.8863	671.5544	607.9097	555.3122	521.5931	446.6963

Appendix A

$$res(x_3) = res\left(p_3(x_2, x_3), p_3(x_2, x_3), x_2\right)$$

$$\begin{aligned}
&= 983040\, m^2 x_3 - 24084480\, m^4 x_3 + 184811520 m^6 x_3 - 463994880 m^8 x_3 \\
&+ 440401920 m^{10} x_3 - 25829120 m^{12} x_3 - 1966080 m\, x_3^2 + 96337920 m^3 x_3^2 \\
&- 1108869120\, m^5 x_3^2 + 3711959040 \mathrm{m}^7\, x3^2 - 4404019200\, m^9\, x3^2 \\
&+ 1509949440\, m^{11}\, x3^2 - 145489920\, m^2\, x_3^3 + 2577530880\, m^4\, x_3^3 \\
&- 12858163200\, \mathrm{m}^6\, x_3^3 + 19597885440\, m^8\, x_3^3 - 8556380160\, m^{10}\, x_3^3 \\
&+ 167772160\, m^{12}\, x_3^3 + 98304000\, m\, x_3^4 - 2917662720\, m^3\, x_3^4 \\
&+ 25181552640\, \mathrm{m}^5\, x_3^4 - 51086622720\, m^7\, x_3^4 + 30198988800\, m^9\, x_3^4 \\
&- 2013265920\, m^{11}\, x_3^4 + 1773404160\, m^2\, x_3^5 - 29829365760\, m^4\, x_3^5 \\
&+ 86853550080\, m^6 x_3^5 - 72603402240\, m^8\, x_3^5 + 10670309376\, m^{10} x_3^5 \\
&- 668467200\, m\, x_3^6 + 20730347520\, m^3\, x_3^6 - 101858672640\, m^5\, x_3^6 \\
&+ 121802588160\, m^7\, x_3^6 - 32883343360\, m^9\, x_3^6 - 7856455680\, m^2 x_3^7 \\
&+ 83456163840\, m^4\, x_3^7 - 142816051200\, m^6\, x_3^7 + 64927825920\, m^8\, x3^7 \\
&+ 1808793600\, m\, x3^8 - 44669337600\, m^3\, x3^8 + 118027714560\, m^5\, x3^8 \\
&- 84557168640\, m^7 x_3^8 + 13306429440\, m^2\, x_3^9 - 70715965440\, m^4\, x_3^9 \\
&+ 71303168000\, m^6\, x_3^9 - 2202009600\, m x_3^{10} + 30198988800\, m^3 x_3^{10} \\
&- 36440113152\, m^5\, x_3^{10} - 7549747200\, m^2\, x_3^{11} + 10066329600\, m^4\, x_3^{11} \\
&+ 1006632960\, m\, x_3^{12} - 1342177280\, m^3\, x_3^{12}
\end{aligned}$$