V-SEMI-CONNECTED SPACES

Aqeel Ketab Mezaal Al-khafaji

Department of Computer Sciences, College of Sciences for womenBabylon University,aqeelketab@yahoo.com

Abstract:

This paper introduces a new topological concept of v-semi-connected spaces which depends on the works ofHamza[6] and Dorsett[4].This space is stronger than the semi-connected space, that is every v-semi-connected space is a semi-connected space but the converse is not always true as well as giving the condition that make the converse is true which is the semi-normality. We prove thatthis property is a topological property.

Key words: semi open, semi closed, connected spaces, semi-connected spaces.

Introduction:

This paper contains two sections, section one includes the fundamental topological concepts that we needed in this work.

In section two we give the main results of this paperwhich is a new a topological concept, v-semi-connected spaces with examples and four results. By a proper subset of a set X we mean that a non-empty A of X such that A≠X.

1. Fundamental Concepts and Fundamental Theorems.

1.1Definition[7]

A subset S of a topological space X is said to be a semi-openif and only if S$\subset cl\ int(S)$.

1.2Definition [7]

A subset S of a topological space X is called a semi-closed if and only if S^c is semi-open.

1.3 Definition[3]

Let X and Y be topological spaces and $f: X \longrightarrow Y$ .f is called:

i) **Irresolute** if and only if for each semi-open set V in Y, f^{-1}(V) is semi-open in X.ii)**Semi-open**(resp. **semi-closed**) if and only if for each semi-open (resp. semi-closed)V in X , $f(V)$ is semi-open(resp. semi-closed) in Y.

1.4 Definition [8]

Let X and Y be topological spaces and f be a function from X into Y then f is called **homeomorphism** if and only if f is bijective, and f and f^{-1} are continuous.

1.5 Definition [3]

Let X and Y be topological spaces and f be a function from X into Y. f is called **irresolute-homeomorphism** if and only if f is bijective and both f,f^{-1} areirresolute .

1.6 Theorem [2]

Any homeomorphism is an irresolute-homeomorphism.

1.7 Theorem:[1]

Let $f : X \rightarrow Y$ *be* abijective function from a topological space X into a topological space Y. Then the following statements are equivalent:

(i)*The inverse function* $f^{-1} : Y \rightarrow X$ *is irresolute.*

$(ii) f$ *is a semi* $-$ *open.*

$(iii) f\ is\ a\ \ semi-closed.$

1.8 Definition [4]

A topological space X is called a semi-connected if is not the union of two non-empty disjoint semi-open sets. A subset $B \subset X$ is semi-connected if it is semi-connected as a subspace of X.

1.9 Remark:

The onlysemi-open and semi-closed subsets in a semi-connected space X are X and the empty set ∅.

1.10 Lemma [4]

A topological space X is **not semi-connected** if and only if any of the following statements holds:

(i) X is the union of two non-empty disjoint semi-open sets.

(ii) X is the union of two non-empty disjoint semi-closed sets.

1.11 Definition[6]

A topological space X is said to be a V-connected space if and only if for any disjoint proper subsets G and H of X,there exists an open proper subset U of X such that G∪H ⊆ U.

1.12Theorem[6]

(i) A $\mathrm{V-connected\ space\ is\ connected.}$

(ii) A V $-$ connectedness is a topological property.

1.13Definition[5]

A topological space X is called **semi-normal** if and only if for every pair R and S of disjoint semi-closed subsets of X,there existsemi-open sets G and H such that$R \subseteq G, S \subseteq H\ and\ G \cap H = \emptyset$.

2. The main results

2.1 Definition:

A topological space X is called a**V-semi-connected space** if and only if for any disjoint semi-closed proper subsets $A\ and\ B$ of X, there exists asemi-open proper subset U of X such that $A \cup B \subseteq U$.

2.2 Examples:

(i) Let X = {1,2} and $\tau = \{\emptyset, X, \{1\}\}$. Then all semi-open set in X is$\{\emptyset, X, \{1\}\}$ and all semi-closed in X is$\{\emptyset, X, \{2\}\}$. X is V-semi-connected space.

(ii) A discrete topological(X, ID) of more than one point is not V-semi-connected. If P is a point in X, then $\{P\}\ and\ X \setminus \{P\}$ are disjoint semi-closed proper subsets of X. But there is no an semi-open proper subset of X contains $(\{P\} \cup X \setminus \{P\}) = X$.

2.3 Theorem:

A V-semi-connected space is semi-connected.

Proof:

Let X bea V-semi-connected space. Suppose that X is not semi-connected.$Then\ X = A \cup B$ where$A\ and\ B$ are non-emptysemi-closed, disjoint subsets of X [Lemma 1.0].Note that each of $A\ and\ B$ is a proper subset of X.Which a contradiction for X is a V-semi-connected space. Then X must be semi-connected ■

2.4 Remark:

The converse of the above theorem need not be true.

The following example will show that:

2.5 Example:

Consider the topological space $(X,\tau)$where, $X = \{1,2,3\}\ and\ \tau = \{\emptyset, X, \{1\}, \{1,2\}\}$. Then all semi-open sets in $X = \{\emptyset, X, \{1\}, \{1,2\}, \{1,3\}\}$ and all semi-closed sets in $X = \{\emptyset, X, \{2,3\}, \{3\}, \{2\}\}$. X is semi-connected space ,but not a V-semi-connected space.

The following Theorem give a condition to make the converse of Theorem2.2 is true.

2.6 Theorem:

A V-semi-connected space a topological property.

Proof:

Let X be a V-semi-connected space and Y be a topological space which is a homeomorphic to X .Then there exists a homeomorphism $f: X \longrightarrow Y$. Let $F\ and\ G$ are disjoint semi-closed proper subsets of Y. Then $f^{-1}(F) and\ f^{-1}(G)$ are disjoint semi-closed proper subsets of X.(By Definition1.3 andTheorem1.6).

Since X be a V-semi-connected space, then there exists a semi-open subset $U\ of\ X\ such\ that: f^{-1}(F) \cup f^{-1}(G) \subseteq U \Rightarrow f\big(f^{-1}(F) \cup f^{-1}(G)\big) \subseteq f(U) \Rightarrow f\big(f^{-1}(F)\big) \cup f(f^{-1}(G)) \subseteq f(U) \Rightarrow F \cup G \subseteq f(U)\ (for\ \ f\ \ is\ onto)$.

Since $f\ is\ a$ semi-open (Theorem1.7), then $f(U)$ is a semi-open subset of Y, also $f(U)$ is a proper subset of Y. Thus Y is a V-semi-connected space ■

2.7 Theorem:

If X is semi-normal connected space, then it is a V-semi-connected space.

Proof:

Let $G\ and\ H$ be disjoint semi-closed proper subsets of X. Then there exist semi-open sets $U\ and\ V\ such\ that\ G \subseteq U\ and\ H \subseteq V$.(since X is a semi-normal).$\Rightarrow G \cup H \subseteq U \cup V = W$.

Note that $W\ is$ semi-open(any union of semi-open set is semi-open[7]) and proper subset of X.(If $W = X, then\ X$ will be not semi-connected).Thus X is a V-semi-connected space ■

REFERENCES

1-**ALhindawe,A.L**.,On Some Types of Semi-Topological Groups, M.Sc. thesis, Baghdad University, Iraq.(**1998)**.

2-**ALkutaibi,S.H**., On some types of Identifications, *Journal of sciences*. College of Education.Tikrit University, Vol. **4** No**. 2(1997).157-163.**

3-**Crossely, S.G.** and **Hilderbrand, S.K**., Semi-Closed Sets and Semi-Continuity, Texas J. Sci**22(1971), 99-112.**

4-**Dorsett, C**.,Semi-Connected Spaces, Indian J. Mech. Math., **17(1) (1979), 57-61**.

5-**Dorsett, C.,** Semi-Normal Spaces, *Kyungpook Math*. J.,**25(2) (1985) 173-8o.**

6-**Hamza ,A.M**. and **Husain J.A**, V-Connected Spaces, *Journal of Babylon University*, Vol. **17** No .**3(2009).886-889**.

7-**Levine, N**., Semi-Open Sets and Semi-Continuity in Topological Spaces, *Amer. Math*. Monthly**, 70(1963), 36-41**.

8-**Munkres, J.R**., Topology, **2nd** Ed. , PHI , **(2009).**

الفضاءات شبه المتصلة من النوع –V

عقيل كتاب مزعل الخفاجي
قسم علوم الحاسوب– كلية العلوم للبنات– جامعة بابل

<u>الملخص:</u>

في هذا البحث قدمنا مفهوم الفضاءات شبه المترابطة من النوع–V وقد اعتمدنا على عمل كل من حمزة [6],ودورست[3].وهذا المفهوم هو اقوى من الفضاءات شبه المترابطة ولكن العكس غير صحيح وبالإضافة الى ذلك فقد اعطينا الشرط الذي يجعل العكس صحيح وهو أن يكون الفضاء شبه طبيعيا. برهنا ان هذه الخاصية هي خاصيه تبولوجية.