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Bayesian Estimators of the parameter and Reliability Function of Inverse Rayleigh Distribution"A comparison study”

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Abstract

In this paper, Bayesian estimator for the parameter and reliability function of inverse Rayleigh distribution (IRD) were obtained Under three types of loss function, namely, square error loss function (SELF), Modified Square error loss function (MSELF) and Precautionary loss function (PLF), taking into consideration the informative and non- informative prior. The performance of such estimators was assessed on the basis of mean square error (MSE) criterion by performing a Monte Carlo simulation technique.

Keywords: Inverse Rayleigh distribution, loss function, Risk function prior information, posterior distribution.

1. Introduction

In reliability studies, most of the life time distributions used are characterized by a monotone failure rate, however, one parameter inverse Rayleigh distribution has also been used as a failure time distribution. Recently, many researchers interested in studying Inverse Rayleigh distribution in various aspects, for example: Mukherjee and Saran (1984) demonstrated that for a given parameter θ the distribution is increasing (or decreasing) failure rate according as the distribution variety is less than or more than $1.069543/\sqrt{\theta}$. [1]. Gharraph (1993) 0btained five measures of the parameter of (IRD). Moreover, he estimated the parameter by using different methods of estimation [2]. Mukherjee and maiti (1996) derived percentile estimator of the parameter θ and its asymptotic efficiency [3]. Abdel-Monem (2003) discussed some estimation and prediction results for (IRD) [4]. EL-Helbawy and Abdel-Monem (2005) used Bayesian approach to estimate the parameter of IRD under four loss functions [5]. Soleman et-al. (2010) discussed Bayesian and non-Bayesian estimation of the parameter of IRD along with Bayesian prediction on the basis of lower Record values. [6].

2. Inverse Rayleigh Distribution (IRD)

The continuous random variable t is said to have inverse Rayleigh distribution with scale parameter θ if it has the probability density function

$$f(t,\theta)=\frac{2\theta}{t^3}\exp\left(-\frac{\theta}{t^2}\right),\quad t>0,\theta>0 \tag{1}$$

The corresponding cumulative distribution function is

$$F(t,\theta)=\exp\left(-\frac{\theta}{t^2}\right),\quad t>0,\theta>0 \tag{2}$$

Therefore, the reliably function of IRD is given by

$$R(t,\theta)=1-F(t,\theta)=1-\exp\left(-\frac{\theta}{t^2}\right),\quad t>0,\theta>0 \tag{3}$$

A variance and higher order moments do not exist for this distribution, moreover, it can be shown that IRD is special case of inverse Weibull distribution with parameters θ,β when $\beta=2$ [7].

3. Prior information's

A convenient choice of priors is indispensable for Bayesian analysis. Many researchers choose priors on the basis of their subjective beliefs and knowledge's**.** However, if enough information about parameter is presented, we should use informative priors; otherwise, it is better to employ vague or non-informative priors. In this paper, we consider the general rule developed by Jeffrey (1961) to obtain the non –informative prior. He established that the single unknown parameter θ which is regarded as a random variable follows such a distribution that is proportional to the square root of Fisher information $I(\theta)$, that is $g(\theta)\propto\sqrt{I(\theta)}$ where $g(\theta)$ denotes the prior information. Equivalently,

$$g(\theta)=c\sqrt{I(\theta)} \tag{4}$$

Where C is a constant of proportionality and,

$$I(\theta)=-nE\left[\frac{\partial^2 lnf(t,\theta)}{\partial\theta^2}\right] \tag{5}$$

It follows that:

$$g(\theta)=c\sqrt{-nE\left[\frac{\partial^2 lnf(t,\theta)}{\partial\theta^2}\right]} \tag{6}$$

4. Posterior density of inverse Rayleigh parameter based on Jeffrey's prior information

From equation (1), we have:

$$lnf(t,\theta)=ln2+ln\theta-3lnt-\frac{\theta}{t^2}$$

$$\frac{\partial lnf(t,\theta)}{\partial\theta}=\frac{1}{\theta}-\frac{1}{t^2}$$

It follows that $\frac{\partial^2 lnf(t,\theta)}{\partial\theta^2}=-\frac{1}{\theta^2}$

$$g(\theta) = \frac{c}{\theta}\sqrt{n} \tag{7}$$

From Bayesian perspective, the posterior density denoted by $h(\theta|\underline{t})$ can be determined by combining the prior distribution $g(\theta)$ with the likelihood function $L(\underline{t}\ |\theta)$, as follows:

$$h(\theta|\underline{t}) = \frac{g(\theta)L(\underline{t}|\theta)}{\int_{\theta} g(\theta)L(\underline{t}|\theta)} \tag{8}$$

Let $t = (t_1, t_2, \dots \dots t_n)$ be a random sample drawn from inverse Rayleigh distribution, then the likelihood function is:

$$L(\underline{t}|\theta) == \prod_{i=1}^{n} f(t_i, \theta)$$

$$= 2^n \theta^n \prod_{i=1}^{n} \frac{1}{t_i^{\,3}} exp\left[-\theta \sum \frac{1}{t_i^{\,2}}\right] \tag{9}$$

Let $h_1(\theta|t)$ denote the posterior density based on Jeffrey's prior distribution $g_1(\theta)$ for inverse Rayleigh parameter θ, then by substituting equations (7) and (9) into equation (8) with simplification, we get:

$$h_1(\theta|\underline{t}) = \frac{\theta^{n-1}e^{-\theta T}}{\int_0^{\infty} \theta^{n-1}e^{-\theta T}d\theta}$$

Where $T = \sum_{i=1}^{n} \frac{1}{t_i^{\,2}}$

It follows that

$$h_1(\theta|\underline{t}) = \frac{T^n \theta^{n-1}e^{-\theta T}}{\Gamma n} \tag{10}$$

The posterior density function in equation (10) is recognized as the density of gamma distribution, that is:

$\theta|t\sim$ gamma $(n, \frac{1}{T})$

Hence, $E(\theta|\underline{t}) = \frac{n}{T}$ (11)

5. Posterior density of inverse Rayleigh parameter based on exponential prior distribution

Assuming that the inverse Rayleigh parameter θ follows exponential prior distribution with parameter, that is:

$$g_2(\theta) = \lambda e^{-\lambda\theta}\ , \lambda > 0, \theta > 0 \tag{12}$$

Where $g_2(\theta)$ denotes the exponential prior distribution of the inverse Rayleigh parameter θ.

By substituting equations (9) and (12) into equation (8) with simplification, we get:

$$h_2(\theta|t) = \frac{p^{n+1}\theta^n e^{-\theta p}}{\Gamma(n+1)} \tag{13}$$

Where $h_2(\theta|t)$ denotes the posterior density based on exponential prior distribution and $p = T + \lambda$.

From equation (13), it can be easily noted that $\theta|\underline{t}$ distributed as gamma with parameter $\left(n+1, \frac{1}{p}\right)$. It follows that:

$$E(\theta|\underline{t}) = \frac{n+1}{p} \tag{14}$$

6. Types of loss functions

From Bayesian viewpoint, [8]. The essential step in the estimation and prediction problems represented by choosing the loss function. In fact, there is no specific analytical procedure to determine the suitable loss function to be employed. In most of studies concerning Bayesian estimation problem, the researchers consider the underlying loss function to be squared error loss function (SELF) which is symmetric in nature. However, in many cases, using the squared error loss function is not appropriate, especially in those cases where the losses are not symmetric. Accordingly, in order to make the statistical inferences more practical and applicable, we often need to choose an asymmetric loss function.

In this paper, we consider both symmetric and asymmetric loss functions for better realization of Bayesian analysis. In particular, the following loss functions have been considered assuming that $\hat{\theta}$ is an estimate of θ, and $L(\hat{\theta}, \theta)$ symbolizes the loss function.

i) The Squared error loss function (SELF) is defined as:

$$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2 \tag{15}$$

ii) Modified Squared error loss function (MSELF) is defined as:

$$L(\hat{\theta}, \theta) = \theta^r(\hat{\theta} - \theta)^2 \quad r=1, 2, 3 \tag{16}$$

Iii\) Precautionary loss function (PLF) is defined as:

$$L(\hat{\theta}, \theta) = \frac{(\theta - \hat{\theta})^2}{\hat{\theta}} \tag{17}$$

7. Bayesian Estimation

The Bayes estimator of the parameter θ is the value of θ that minimizes the posterior expectation known as the risk function and denotes $R(\hat{\theta}, \theta)$, that is:

$$R(\hat{\theta}, \theta) = E[L(\hat{\theta}, \theta)] = \int_\theta L(\hat{\theta}, \theta) h(\theta|\underline{t}) d\theta \tag{18}$$

Where $h(\theta|\underline{t})$ is the posterior density of $\theta|t$

8. Bayes estimator of the parameter θ of IRD under SELF

In general, if SELF is chosen, then according to equation (18), we have:

$$R(\hat{\theta}, \theta) = \int_\theta (\hat{\theta} - \theta)^2 h(\theta|\underline{t}) d\theta$$

$$= \hat{\theta}^2 \int_\theta h(\theta|\underline{t})\, d\theta - 2\hat{\theta} \int_\theta \theta h(\theta|\underline{t}) d\theta + \int_\theta \theta^2 h(\theta|\underline{t}) d\theta$$

It follows that:

$R(\hat{\theta},\theta) = \hat{\theta}^2 - 2\hat{\theta}E\,(\theta|t) + E(\theta^2|t)$

By differentiating $R(\hat{\theta},\theta)$ with respect to $\hat{\theta}$ and setting the resultant derivative equal to zero, we get:

$2\hat{\theta} - 2E(\theta|t) = 0$

By Solving for $\hat{\theta}$, we obtain the Bayes estimator of θ under SELF denoted by

$$\hat{\theta}_{sq} = E(\theta|t) \tag{19}$$

From equation (11) and on the basis of non-informative prior, the Bayes estimator of inverse Rayleigh parameter θ denoted as $\hat{\theta}_{sq(J)}$ is given by.

$$\hat{\theta}_{sq_{(J)}} = \frac{n}{T} \tag{20}$$

Where $\hat{\theta}_{sq_{(J)}}$ denotes the Bayes estimate of θ based on Jeffrey's prior information. If the inverse Rayleigh parameter follows the exponential prior distribution, then by equation (14), we conclude that

$$\hat{\theta}_{sq_{(E)}} = \frac{n+1}{p} \tag{21}$$

Where $\hat{\theta}_{sq_{(E)}}$ denotes the Bayes estimate of θ based on exponential prior distribution.

9. Bayes estimator of the Inverse Rayleigh parameter $\boldsymbol{\theta}$ under MSELF

By substituting $L(\hat{\theta},\theta)$ given in equation (16) into equation (18), we get

$R(\hat{\theta},\theta) = \int_0^\infty \theta^r (\hat{\theta}-\theta)^2 h(\theta|\underline{t})d\theta$

By evaluating the integral, we get

$R(\hat{\theta},\theta) = \hat{\theta}^2 E(\theta^r|\underline{t}) - 2\hat{\theta}E\,(\theta^{r+1}|\underline{t}) + E(\theta^{r+2}|\underline{t})$

By differentiating $R(\hat{\theta},\theta)$ with respect to $\hat{\theta}$, then equating the resultant derivative to zero and solving for $\hat{\theta}$, we get the Bayes estimator of θ under Modified squared error loss function denoted by $\hat{\theta}_{Msq}$ as follows.

$$\hat{\theta}_{Msq} = \frac{E(\theta^{r+1}|\underline{t})}{E(\theta^r|\underline{t})} \quad r=1,2, \tag{22}$$

If k is positive integer, it is well known that

$$E(\theta^K|\underline{t}) = \int_0^\infty \theta^k h(\theta|\underline{t})d\theta \tag{23}$$

On the basis of Jeffrey's prior information and by substituting $h(\theta|\underline{t})$ by $h_1(\theta|\underline{t})$ given in equation (10), then evaluating the integral in (23); it can easily be shown

$$E(\theta^K|\underline{t}) = \frac{\Gamma(k+n)}{T^k\Gamma n} \tag{24}$$

Therefore, the Bayes estimator given in equation (22) can be simplified to be:

$$\hat{\theta}_{Msq(J)} = \frac{n+r}{T} \qquad (25)$$

If r=1 we get

$$\hat{\theta}_{Msq(J_1)} = \frac{n+1}{T} \qquad (26)$$

If r=2 we get

$$\hat{\theta}_{Msq(J_2)} = \frac{n+2}{T} \qquad (27)$$

If r=3 then we obtain,

$$\hat{\theta}_{Msq(J_3)} = \frac{n+3}{T} \qquad (28)$$

On the basis of exponential prior distribution and by replacing $h(\theta|\underline{t})$ in equation (23) by $h_2(\theta|\underline{t})$ given in equation (13), we get:

$$E(\theta^K|\underline{t}) = \frac{\Gamma(k+n+1)}{p^k\Gamma(n+1)} \quad , \quad k=1,2 \qquad (29)$$

Where $p = T + \lambda$

Therefore, the Bayes estimator given in equation (22) can be simplified to be:

$$\hat{\theta}_{Msq(E)} = \frac{n+r+1}{p} \qquad (30)$$

For r=1 we get:

$$\hat{\theta}_{Msq(E_1)} = \frac{n+2}{p} \qquad (31)$$

For r=2 we get:

$$\hat{\theta}_{Msq(E_2)} = \frac{n+3}{p} \qquad (32)$$

For r=3 we get:

$$\hat{\theta}_{Msq(E_3)} = \frac{n+4}{p} \qquad (33)$$

10. Bayes estimator of the Inverse Rayleigh parameter θ under precautionary loss function

By substituting $L(\hat{\theta},\theta)$ presented in equation (17) into equation (18), we get:

$$R(\hat{\theta},\theta) = \int_0^\infty \frac{(\theta-\hat{\theta})^2}{\hat{\theta}} h(\theta|t)d\theta$$

By evaluating the integral, we get:

$R(\hat{\theta},\theta)$ $=\frac{1}{\hat{\theta}}E(\theta^2|\underline{t})-2E(\theta|\underline{t})+\hat{\theta}$

Differentiating $R(\hat{\theta},\theta)$ with respect to $\hat{\theta}$ and setting the derivative equal to zero then solving for $\hat{\theta}$, we get the Bayes estimator of θ under precautionary loss function denoted as $\hat{\theta}_p$, that is:

$$\hat{\theta}_p=\sqrt{E(\theta^2|t)} \qquad (34)$$

On the basis of Jeffrey's prior information, the Bayes estimator of the inverse Rayleigh parameter θ can be obtained by assuming that k=2 in equation (24) to get $E(\theta^2|\underline{t})$, then substituting it into equation (34), so we obtain,

$$\hat{\theta}_{P_{(J)}}=\frac{\sqrt{n(n+1)}}{T} \qquad (35)$$

On the same manner, by putting k=2 in equation (29), then substituting) it into equation (34), we get the Bayes estimator of inverse Rayleigh parameter θ under precautionary loss function and exponential prior distribution , denoted as $\hat{\theta}_{P_{(E)}}$ and given by:

$$\hat{\theta}_{P_{(E)}}=\frac{\sqrt{(n+2)(n+1)}}{p} \qquad (36)$$

11. Bayes estimator of the reliability function R(t) of IRD under SELF

Let us assume that k is any positive integer, then,

$E[R(t)^k|\underline{t}]=\int R(t)^k h(\theta|\underline{t})\,d\theta$

For IRD and according to equation (3), we have,

$$E[R(t)^k|\underline{t}]=\int_0^\infty\left(1-e^{\frac{-\theta}{t^2}}\right)^k h(\theta|\underline{t})\,d\theta \qquad (37)$$

For Jeffrey's prior information, we substitute from $h(\theta|\underline{t})$ in equation (37) by $h_1(\theta|\underline{t})$ given in equation (10), then in order to evaluate the integral in equation (37), we develop the following formula:

$$\left(1-e^{\frac{-\theta}{t^2}}\right)^k=\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}e^{\frac{-\theta(k-j)}{t^2}} \qquad (38)$$

The integral in equation (37) should be reduced to:

$$E[R(t)^k|\underline{t}]=\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}\int_0^\infty e^{\frac{-\theta(k-j)}{t^2}}h_1(\theta|\underline{t})d\theta$$

This implies that:

$$E[R(t)^k|\underline{t}]=\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}\left[\frac{Tt^2}{Tt^2+k-j}\right]^n \qquad (39)$$

where $T=\sum_{i=1}^{n}\frac{1}{t_i^{\,2}}$

For exponential prior distribution, it can be shown in the same manner that:

$E[R(t)^k|\underline{t}] = \sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}\int_0^\infty e^{\frac{-\theta(k-j)}{t^2}}\, h_2\,(\theta|\underline{t})d\theta$,

which implies that:

$$E[R(t)^k|\underline{t}] = \sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}\left[\frac{pt^2}{pt^2+k-j}\right]^{n+1} \quad (40)$$

where p=T+λ

Under SELF and according to equation (19), we conclude that:

$\hat{R}(t)_{sq} = E[R(t)|t]$

For Jeffrey's prior information, putting k=1 in equation (39), we get:

$$\hat{R}(t)_{sq_{(J)}} = 1-\left[\frac{Tt^2}{Tt^2+1}\right]^n \quad (41)$$

For exponential prior distribution, putting k=1 in equation (40), we get:

$$\hat{R}(t)_{sq_{(E)}} = 1-\left[\frac{pt^2}{pt^2+1}\right]^{n+1} \quad (42)$$

12. Bayes estimator of the reliability function R(t) of IRD under MSELF

From equation (22) it is clear that the estimator of the reliability function R(t) under MSELF is given by:

$\hat{R}(t)_{MS} = \frac{E[R(t)^{r+1}|\underline{t}]}{E[R(t)^r|\underline{t}]}$, r=1,2,…

For IRD, with Jeffry's prior information and r=1 then,

$\hat{R}(t)_{MS(J_1)} = \frac{E[R(t)^2|\underline{t}]}{E[R(t)\ \ |\underline{t}]}$.

By putting k=2, k=1 in equation (39) we obtain $E\left[R(t)^2|\underline{t}\right], E\left[R(t)|\underline{t}\right]$ respectively. This implies that:

$$\hat{R}(t)_{MS(J_1)} = \frac{1-2(\frac{Tt^2}{Tt^2+1})^n+(\frac{Tt^2}{Tt^2+2})^n}{1-(\frac{T}{Tt^2+1})^n} \quad (43)$$

For r=2, it follows that:

$\hat{R}(t)_{MS(J_2)} = \frac{E[R(t)^3|\underline{t}]}{E[R(t)^2|\underline{t}]}$, putting k=3 in equation (39) we obtain

$E[R(t)^3|\underline{t}]$. Hence,

$$\hat{R}(t)_{MS(J_2)} = \frac{1-3(\frac{Tt^2}{Tt^2+1})^n+3(\frac{Tt^2}{Tt^2+2})^n-(\frac{Tt^2}{Tt^2+3})^n}{1-2(\frac{Tt^2}{Tt^2+1})^n+(\frac{Tt^2}{Tt^2+2})^n} \quad (44)$$

For r=3 it follows that:

$\hat{R}(t)_{MS(J_3)} = \frac{E[R(t)^4|\underline{t}]}{E[R(t)^3|\underline{t}]}$, putting k=4 in equation (39), we get

$E[R(t)^4|\underline{t}]$, and hence

$$\hat{R}(t)_{MS(J_3)} = \frac{1-4(\frac{Tt^2}{Tt^2+1})^n+6(\frac{Tt^2}{Tt^2+2})^n-4(\frac{Tt^2}{Tt^2+3})^n+(\frac{Tt^2}{Tt^2+4})^n}{1-3(\frac{Tt^2}{Tt^2+1})^n+3(\frac{Tt^2}{Tt^2+2})^n-(\frac{Tt^2}{Tt^2+3})^n} \quad (45)$$

Similarly, for exponential prior distribution and according to equations (22) and (40), it can be shown that the Bayes estimators of the reliability function R(t) under MSELF for r=1,2,3 are given in respective

$$\hat{R}(t)_{MS(E_1)} = \frac{1-2(\frac{pt^2}{pt^2+1})^{n+1}+(\frac{pt^2}{pt^2+2})^{n+1}}{1-(\frac{T}{pt^2+1})^{n+1}} \quad (46)$$

$$\hat{R}(t)_{MS(E_2)} = \frac{1-3(\frac{pt^2}{pt^2+1})^{n+1}+3(\frac{Tt^2}{pt^2+2})^{n+1}-(\frac{Tt^2}{pt^2+3})^{n+1}}{1-2(\frac{Tt^2}{pt^2+1})^{n+1}+(\frac{Tt^2}{pt^2+2})^{n+1}} \quad (47)$$

$$\hat{R}(t)_{MS(E_3)} = \frac{1-4(\frac{pt^2}{pt^2+1})^{n+1}+6(\frac{pt^2}{pt^2+2})^{n+1}-4(\frac{pt^2}{pt^2+3})^{n+1}+(\frac{pt^2}{pt^2+4})^{n+1}}{1-3(\frac{pt^2}{pt^2+1})^{n+1}+3(\frac{pt^2}{pt^2+2})^{n+1}-(\frac{pt^2}{pt^2+3})^{n+1}} \quad (48)$$

13. Bayes estimator of the reliability function R(t) of IRD under precautionary loss function

From equation (34), it is clear that the estimator of the reliability function R(t) under precautionary loss function is given by:

$\hat{R}(t)_p = \sqrt{E[R(t)^2|t]}$

For Jeffrey's prior information, putting k=2 in equation (39), we get:

$$\hat{R}(t)_{p_{(J)}} = \sqrt{1-2(\frac{Tt^2}{Tt^2+1})^n+(\frac{Tt^2}{Tt^2+2})^n} \quad (49)$$

On the basis of exponential prior distribution, putting k=2 in equation (40), we get:

$$\hat{R}(t)_{p_{(E)}} = \sqrt{1-2(\frac{Pt^2}{Pt^2+1})^{n+1}+(\frac{pt^2}{pt^2+2})^{n+1}} \quad (50)$$

14. Simulation Study

In our simulation study, L=2000 sample of size n=10, 50, 100 and 200 were generated in order to represent, small, moderate, large and very large sample sizes from inverse Rayleigh distribution with two values of the scale parameter ($\Theta = 0.5 , \Theta = 1.5$) (.The) scale parameter λ of exponential prior was chosen to be (λ=0.5, λ=1) and assumed the values of r in modified square error loss function to be r=1, r=2 and r=3(. The) criterion mean square error (MSE) was employed to compare the performance of different methods of estimation of the scale parameter and reliability function of IRD where

$$MSE(\hat{\theta}) = \frac{1}{L}\sum_{i=1}^{L}(\hat{\theta}_i - \theta)^2 \quad (51)$$

$$MSE\left[\hat{R}(t)\right] = \frac{1}{L}\sum_{i=1}^{L}[\hat{R}_i(t) - R(t)]^2 \quad (52)$$

The results are presented in the following tables.

Table1. (MSE) for parameter θ by using Jeffrey's prior information at $\theta = 0.5$.

Estimator \ **n**	**10**	**50**	**100**	**200**
Sq	0.0043	0.0001081	0.000026174	0.000006512
MS r=1	0.0061	0.0001186	0.000027468	0.000006676
MS r=2	0.0086	0.0001334	0.000029277	0.000006903
MS r=3	0.0119	0.0001524	0.000031602	0.000007193
Pr	0.0215	0.0049	0.0025	0.0012
Best	Sq	Sq	Sq	Sq

Table 2. (MSE) values of the Reliability function estimators by using Jeffrey's prior information at $\theta = 0.5$.

Estimator \ **n**	**10**	**50**	**100**	**200**
Sq	0.000295	0.0000099	0.0000024714	0.0000006220
MS r=1	0.000353	0.0000103	0.0000025315	0.0000006297
MS r=2	0.000436	0.0000111	0.0000026240	0.0000006416
MS r=3	0.000541	0.0000120	0.0000027483	0.0000006575
Pr	0.000319	0.0000101	0.0000024973	0.0000006253
Best	Sq	Sq	Sq	Sq

Table 3. (MSE) for parameter θ by using Exponential prior information at $\theta = 0.5$.

Estimator \ **N**		**10**	**50**	**100**	**200**
Sq	$\lambda = 0.5$	0.0051	0.0001143	0.000026952	0.000006612
	$\lambda = 1$	0.0043	0.0001103	0.000026474	0.000006552
MS r=1	$\lambda = 0.5$	0.0073	0.0001278	0.000028621	0.000006822
	$\lambda = 1$	0.0061	0.0001226	0.000028004	0.000006746
JMS r=2	$\lambda = 0.5$	0.0100	0.0001455	0.000030803	0.000007096
	$\lambda = 1$	0.0085	0.0001391	0.000030045	0.000007003
MS r=3	$\lambda = 0.5$	0.0135	0.0001674	0.000033498	0.000007433
	$\lambda = 1$	0.0115	0.0001597	0.000032596	0.000007323
Pr	$\lambda = 0.5$	0.0061	0.0001205	0.00002772	0.000006709
	$\lambda = 1$	0.0051	0.0001159	0.000027173	0.000006641
Best		Sq	Sq	Sq	Sq

Table 4. (MSE) values of the Reliability function estimators by using Exponential prior information at $\theta = 0.5$.

Estimator \ n		10	50	100	200
Sq	$\lambda = 0.5$	0.000332	0.0000102	0.0000025179	0.0000006281
	$\lambda = 1$	0.000290	0.0000099	0.0000024824	0.0000006235
MS r=1	$\lambda = 0.5$	0.0031	0.0000364	0.0000058603	0.000001052
	$\lambda = 1$	0.0030	0.0000354	0.0000057318	0.000001035
MS r=2	$\lambda = 0.5$	0.000508	0.0000119	0.0000027277	0.0000006549
	$\lambda = 1$	0.000443	0.0000114	0.0000026716	0.0000006479
MS r=3	$\lambda = 0.5$	0.000625	0.0000130	0.0000028797	0.0000006745
	$\lambda = 1$	0.000548	0.0000125	0.0000028134	0.0000006661
Pr	$\lambda = 0.5$	0.2320	0.0470	0.0235	0.0118
	$\lambda = 1$	0.2328	0.0470	0.0235	0.0118
Best		Sq	Sq	Sq	Sq

Table 5. (MSE) for parameter θ by using Jeffrey's prior information at $\theta = 1.5$.

Estimator \ n	10	50	100	200
Sq	0.0391	0.00097346	0.00023555	0.00005861
MS r=1	0.0553	0.0011	0.00024721	0.000060085
MS r=2	0.778	0.0012	0.00026350	0.000062128
MS r=3	0.1067	0.0014	0.00028442	0.000064741
Pr	0.1444	0.0422	0.0218	0.0111
Best	Sq	Sq	Sq	Sq

Table 6. (MSE) values of the Reliability function estimators by using prior information at $\theta = 1.5$.

Estimator \ n	10	50	100	200
Sq	0.000640	0.000024186	0.000006000951	0.0000015384
MS r=1	0.000699	0.000024643	0.0000061533	0.0000015461
MS r=2	0.000804	0.000025556	0.0000062697	0.0000015611
MS r=3	0.000945	0.0000120	0.0000064431	0.0000015834
Pr	0.000662	0.000024355	0.0000061167	0.0000015413
Best	Sq	Sq	Sq	Sq

Table 7. (MSE) for parameter θ by using Exponential prior information at $\theta = 1.5$.

Estimator \ n		10	50	100	200
Sq	$\lambda = 0.5$	0.0328	0.00096071	0.0002343	0.00005847
	$\lambda = 1$	0.0211	0.0008813	0.00022438	0.00005720
MS r=1	$\lambda = 0.5$	0.0463	0.0011	0.00024684	0.00006007
	$\lambda = 1$	0.0285	0.0009501	0.00023329	0.00005836
MS r=2	$\lambda = 0.5$	0.0650	0.0012	0.00026395	0.00006223
	$\lambda = 1$	0.0403	0.0011	0.0002467	0.00006008
MS r=3	$\lambda = 0.5$	0.0889	0,0014	0.0002856	0.00006496
	$\lambda = 1$	0.0564	0.0012	0.0002646	0.00006237
Pr	$\lambda = 0.5$	0.388	0.0010	0.00023998	0.00005920
	$\lambda = 1$	0.0242	0.0009110	0.00022826	0.00005771
Best		Sq	Sq	Sq	Sq

Table 8. (MSE) values of the Reliability function estimators by using Exponential prior information at $\theta = 1.5$.

Estimator \ N		10	50	100	200
Sq	$\lambda = 0.5$	0.000553	0.000023591	0.0000060208	0.000001529
	$\lambda = 1$	0.000456	0.00002269	0.0000059033	0.000001513
MS r=1	$\lambda = 0.5$	0.0022	0.00003918	0.0000080152	0.000001783
	$\lambda = 1$	0.0019	0.00003548	0.0000075334	0.000001721
MS r=2	$\lambda = 0.5$	0.000717	0.000025211	0.0000062315	0.000001556
	$\lambda = 1$	0.000516	0.00002336	0.0000059928	0.000001525
MS r=3	$\lambda = 0.5$	0.000849	0.000036657	0.0000064211	0.000001581
	$\lambda = 1$	0.000601	0.00002433	0.0000061224	0.000001542
Pr	$\lambda = 0.5$	0.15170	0.0306	0.0153	0.0077
	$\lambda = 1$	0.15430	0.0307	0.0153	0.0077
Best		Sq	Sq	Sq	Sq

15. Simulation Results and Conclusions

From our simulation study, we conclude that the performance of the Bayes estimators under square error loss function are the best compared to other estimators in all cases that are included in our study, followed by the estimators under Precautionary loss function in the in the cases presented in **Table 2, 3,6.** The estimators under modified square error loss function when (r=1) presented in **Table 1, 5, 7**.

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