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Effect of Dissipative Forces on the Theory of a Single-Atom Microlaser

A single-atom microlaser involving Poissonian input of atoms with a fixed f light time through an optical resonator is described. The influence of the cavity reservoir during the interactions of successive individual atoms with the cavity field is included in the analysis. Atomic decay is also considered, as it is nonnegligible in the optical regime. During the random intervals of absence of any atom in the cavity, the field evolves under its own dynamics. The steady-state characteristics of the cavity field are discussed. Away from laser threshold, the field can be nonclassical in nature.

***Keywords*: Microlaser, Single-atom laser, Dissipative forces**

The single-atom laser, an optical counterpart of the micromaser, has generated extensive interest after the recent experimental demonstration by An *et al.*[1] In that experiment, two-level ^{138}Ba atoms in their upper states were pumped into an optical cavity in such a way that the average number of atoms in the resonating mode satisfied the condition $\langle N \rangle \leq 1.0$. The average number of photons $\langle n \rangle$ in the mode showed a linear dependence on the pump until $\langle N \rangle = 0.6$. But a further increase in $\langle N \rangle$ resulted in a thresholdlike jump in $\langle n \rangle$. Calculations based on a single-atom theory, such as the one given in Ref. 1, can explain the linear regime only. An and Feld[2] incorporated the cavity-mode structure into the single-atom theory to explain this jump. Kolobov and Haake[3] addressed the problem by use of a Poissonian pumping model with partial overlaps of the travel times of successive atoms in the cavity. Alternative explanations[4,5] have also been proposed to explain the thresholdlike structure.

In this Letter a slightly different pump mechanism in the microlaser setup is addressed: Atoms are streamed into the cavity in such a way that strictly one atom can pass through it at a time. The pumping is Poissonian, with the intervals between successive atomic flights being random. Thus the flight time through the cavity is fixed for each atom. In other words, an exact optical counterpart of a micromaser setup[6] is considered here. The single-atom laser theories available in the literature, for example, in Ref. 7, involve analysis of the steady-state properties of the Jaynes–Cummings interaction[8] in a damped cavity.[9] The single-atom theory of Filipowicz *et al.*[10] needs recasting because it does not consider atomic damping, which is important in the optical regime, and because of the cavity-enhancement factor, the so-called Purcell factor. In addition, the theory disregards leakage of radiation from the cavity during the atom–field interaction. The magnitudes of the cavity dissipation constant and the atom–field coupling constant in the microlaser experiment[1] indicate that radiation dissipation from the optical resonator is nonnegligible during the entire dynamics, irrespective of the presence of any atom in the cavity.

The above factors lead us to the need for a theory capable of handling reservoir-induced interactions and the Jaynes–Cummings interactions[8] simultaneously. The theory that I proposed in Ref. 11 may be suitable for the present purpose. The theory assumes that the atom–field coupling constant is independent of cavity-mode structure, which is all right for the microwave cavity.[6] In fact, such a situation is necessary for the generation of nonclassical fields.[4] In the optical cavity described in Ref. 1, atoms travel through a length of ~40 wavelengths of the interacting mode, thus making the coupling constant dependent on the mode structure. However, a novel and simple technique for streaming the atoms through the cavity, adopted by An *et al.*[12] to improve their earlier setup,[1] provides a uniform atom–field coupling constant in the cavity. This setup with single-atom events would be suitable for generating nonclassical fields in the optical regime. With such a system in mind, we follow the method in Ref. 11.

We assume that atoms arrive individually at the cavity, with an average interval $\bar{t}_c = 1/R$, where R is the flux rate of atoms. We have $t_c = \tau + t_{\text{cav}}$, where τ is the interaction time, fixed for every atom, and t_{cav} is the random time lapse between one atom leaving and a successive atom entering the cavity. $\bar{t}_c$ is the average of t_c taken over a Poissonian distribution in time of incoming atoms. The cavity field evolves from a near vacuum by means of this repetitive dynamics, as thermal photons in the optical cavity are almost nonexistent. Thus, during τ, we have to solve the equation of motion:

$$\dot{\rho} = -i[H,\rho] - \kappa(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) - \gamma(S^+S^-\rho - 2S^-\rho S^+ + \rho S^+S^-), \qquad (1)$$

where H is the Jaynes–Cummings Hamiltonian[8] and κ and γ are the cavity-mode and the atomic-decay constants, respectively. a is the photon annihilation operator, and S^+ and S^- are the Pauli pseudospin operators for the two-level atomic system. During t_{cav}, the cavity field evolves under its own dynamics, represented by Eq. (1), with $H = \gamma = 0$. The method for obtaining a coarse-grained time derivative, which is valid for a Poissonian process,[13,14] is given in detail in Ref. 11 for the photon-number distribution $P_n = \langle n|\rho|n\rangle$. The steady-state photon statistics is then

$$P_n = P_0 \prod_{m=1}^{n} v_m \,, \tag{2}$$

and P_0 is obtained from the normalization $\sum_{n=0}^{\infty} P_n = 1$. The v_n is given by the continued fractions

$$v_n = f_3{}^{(n)}/[f_2{}^{(n)} + f_1{}^{(n)} v_{n+1}], \tag{3}$$

with $f_1{}^{(n)} = (Z_n + A_{n+1})/\kappa, f_2{}^{(n)} = -2N + (Y_n - A_n)/\kappa$ and $f_3{}^{(n)} = -X_n/\kappa$. $N = R/2\kappa$ is the number of atoms passing through the cavity in a photon lifetime. $A_n = 2n\kappa$, and X_n, Y_n, and Z_n are given by

$$X_n = R \sin^2(g\sqrt{n}\tau)\exp\{-[\gamma + (2n-1)\kappa]\tau\},$$

$$Y_n = \frac{1}{2} R\Bigg(\Bigg[2\cos^2(g\sqrt{n+1}\,\tau) - \frac{1}{2}(\gamma/\kappa + 2n + 1) + F_1(n-1)\Bigg]\exp\{-[\gamma + (2n+1)\kappa]\tau\} + \Bigg[\frac{1}{2}(\gamma/\kappa + 2n + 1) - F_2(n-1)\Bigg] \times \exp\{-[\gamma + (2n-1)\kappa]\tau\}\Bigg),$$

$$Z_n = \frac{1}{2} R\Bigg(\Bigg[\frac{1}{2}(\gamma/\kappa + 2n + 3) + F_2(n)\Bigg] \times \exp\{-[\gamma + (2n+1)\kappa]\tau\} - \Bigg[\frac{1}{2}(\gamma/\kappa + 2n + 3) + F_1(n)\Bigg] \times \exp\{-[\gamma + (2n+3)\kappa]\tau\}\Bigg).$$

The functions F_1 and F_2 are

$$\begin{aligned} F_i(n) = {} & \frac{\kappa/4g}{(\sqrt{n+2} - \sqrt{n+1})^2}\Bigg(\frac{\gamma}{\kappa}(\sqrt{n+2} - \sqrt{n+1}) \\ & \times \sin(2g\sqrt{m}\,\tau) - \frac{\gamma}{g}\cos(2g\sqrt{m}\,\tau) \\ & - \{2n + 3 + 2[(n+1)(n+2)]^{1/2}\} \\ & \times (\sqrt{n+2} - \sqrt{n+1})\sin(2g\sqrt{m}\,\tau)\Bigg) \\ & + \frac{\kappa/4g}{(\sqrt{n+2} + \sqrt{n+1})^2} \\ & \times \Bigg(\pm\frac{\gamma}{\kappa}(\sqrt{n+2} + \sqrt{n+1}) \\ & \times \sin(2g\sqrt{m}\,\tau) - \frac{\gamma}{g}\cos(2g\sqrt{m}\,\tau) \\ & \mp \{2n + 3 - 2[(n+1)(n+2)]^{1/2}\} \\ & \times (\sqrt{n+2} + \sqrt{n+1})\sin(2g\sqrt{m}\,\tau)\Bigg), \end{aligned}$$

where $m = n + 2$ and $n + 1$ for $i = 1$, and $i = 2$, respectively, with the upper sign for $i = 1$. Once P_n is obtained, we can describe the characteristics of the cavity field by evaluating its various moments.

We find that the photon statistics of the cavity field given by Eqs. (2) and (3) is a function of the dimensionless parameters N, κ/g, and γ/g. The theory in Ref. 10 does not take into account atomic relaxation. As cavity dissipation during τ is neglected, the photon distribution function derived in Ref. 10 is dependent on κ only through the parameter $N = R/2\kappa$. Hence, in the context of the microlaser, it is difficult to make a proper judgment of the dissipative effects on the photon statistics based on the work of Filipowicz *et al.*[10] Reference 11 discusses in detail the degree of influence of the reservoir-induced interactions on the steady-state photon statistics. That study indicates that the influence of the interactions is nonnegligible for the optical cavities of the type used in the studies reported in Refs. 1 and 12. In fact, it is found that the photon statistics obtained by use of the results from Ref. 10 (dashed–dotted curves in Figs. 1 and 2) mostly differ from the present results. The pump parameter $D = \sqrt{N}\,g\tau$ is introduced here, as I find it useful for the description of microlaser characteristics. The structures in $\langle n\rangle$ as D is varied for fixed N (see Fig. 1) reflect the characteristics of the Jaynes–Cummings interaction.[8] Soon after the threshold is attained at $D = \sim 1.0$, the photon number rises sharply. The reason is as follows: The field is almost in vacuum before the first atom enters the cavity. Thus the atoms that are initially in their upper states contribute varying fractions of their energies to the cavity, and at $n = N$ and $D = \pi/2$ the atoms are completely in their respective lower states. Thus $\langle n\rangle$ peaks at $D = \sim 1.6$, depending on κ, γ, and N. For higher κ and γ, the peak moves slightly toward higher D because the threshold is attained at higher D. This happens even for increasing R, which is due to an increase in the percentage of time $R\tau \times 100\%$ in the duration of 1 s taken by R atoms in interacting with the cavity field, resulting in an increase in dissipation of energy to the atomic reservoir. It is further found that $\langle n\rangle = 0$ near $D = 31.4, 62.8, 94.2, \ldots$, which gives $g\tau = \pi, 2\pi, 3\pi, \ldots$, respectively. At such values of $g\tau$, the atom absorbs

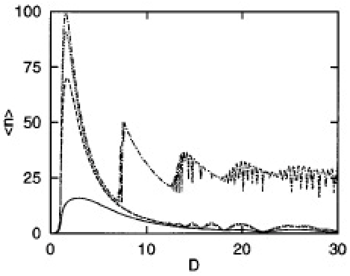

Fig. 1. Cavity field intensity, proportional to $\langle n\rangle$, as a function of the pump parameter D for $N = 100$ and $\gamma/g = 0.1$. Solid curve, $\kappa/g = 0.01$; dashed curve, $\kappa/g = 0.001$; dotted curve, $\kappa/g = 0.0001$; dotted–dashed curve, results from Ref. 10, in which $\kappa = \gamma = 0.0$ during τ.

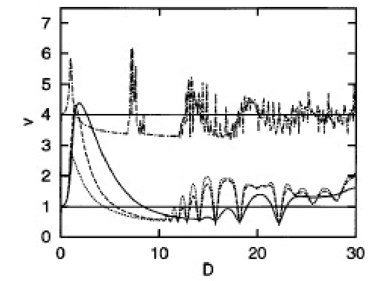

Fig. 2. Variance of the cavity field, v, versus pump parameter D. The other parameters are the same as in Fig. 1. For clarity, the dashed–dotted curve is shifted upward by 3.0. The horizontal lines are $v = 1.0, 4.0$. The sub-Poissonian nature of the radiation field is indicated by $v < 1.0$.

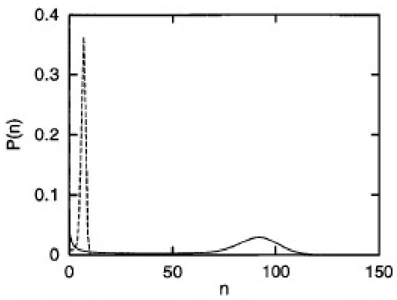

Fig. 3. Photon distribution function $P(n) = P_n$ for $N = 100$, $\gamma/g = 0.1$, and $\kappa/g = 0.001$. Solid curve, $D = 1.7$; dashed curve, $D = 10.0$.

the photon that it has emitted before leaving the cavity.

Figure 2 shows that the variance of the cavity field, $v = [(\langle n^2\rangle - \langle n\rangle^2)/\langle n\rangle]^{1/2}$, increases sharply at $D = \sim 1.6$, where $\langle n\rangle$ also peaks (see Fig. 1). It is found that near this value of D the P_n is doubly peaked at $n = 0$ and $n \approx N$, as shown in Fig. 3, and this increases the variance in photon number. However, for a slightly higher value of D the cavity field is highly sub-Poissonian in nature (Figs. 2 and 3). This is also the case for still-higher values of D. The sub-Poissonian nature of the cavity field appears because the flight time τ is short compared with the atomic lifetime $(2\gamma)^{-1}$ as well as the photon lifetime $(2\kappa)^{-1}$, as was discussed in detail in Ref. 11. Such situations, which are depicted in Fig. 2, do not happen in conventional lasers. Thus it is found that the photon field characteristics in the present microlaser case (Figs. 2 and 3) are different from those of a conventional laser.[15] Further, it may be noted in Fig. 1 that $\langle n\rangle$ gets very small as D is increased, because the increase in interaction time τ increases the influence of atomic as well as cavity reservoirs on the atom–field interaction. The sub-Poissonian nature of the cavity field appears for such values of D since the situation $\tau < (2\tau)^{-1}, (2\kappa)^{-1}$ is met in the range of D depicted in Figs. 1 and 2.

The cavity field intensity, which is proportional to $\langle n\rangle$, saturates as N is increased for fixed τ. The saturation value of $\langle n\rangle$ depends on τ, as dictated by the Jaynes–Cummings interactions.[8] This saturation characteristic is not seen in Fig. 1, in which the increase in D is due to an increase in τ for fixed N.

The characteristics of a single-atom microlaser capable of generating nonclassical optical fields in the realistic regimes of atomic and cavity dissipations (e.g., $\kappa/g = 0.01$ and $\gamma/g = 0.1$) have been analyzed. Results have also been displayed for $\kappa/g = 0.001, 0.0001$ in the figures. This shows the influence of κ on the photon statistics that can be attained if the cavity Q factor in the present experimental setup is enhanced by a factor of 100 or more. In addition, the influences of atomic as well as field reservoirs on the coherent atom–field interaction have been included. Poissonian pumping of atoms into the cavity, which was the case in previous experiments,[1,12] was considered. However, the parameter N in the present study has to be distinguished from $\langle N\rangle$ in those experiments, in which the statistical averaging was taken over $N_0 = 1, 2, 3, \ldots$ atomic events. The arrival of N_0 atoms at the cavity is, however, Poissonian. Yang and An,[4] in their attempt to analyze the experimental results presented in Ref. 1, considered values as great as $N_0 = 15$, whereas here N was just a simple addition of single-atom events in the photon lifetime $(2\kappa)^{-1}$. The technique adopted in Ref. 12 can already be used to select atoms with a particular velocity. Further refinement to restrict the dynamics to only $N_0 = 1$ events should be possible. Then it would be possible to have nonclassical optical fields of the type shown in Fig. 2.

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