Dynamical behavior of quantum-dot laser

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Abstract

In this work we have investigated the dynamical properties of InAs/InGaAs laser emitting $1.3\mu m$ pumped by injection current density of fast rise and falling times with a duration of $5ns$. In (capture) and out (escape) scattering rates of electrons and holes are studied against temperature, injection current density, Auger coefficient. Scattering times are studied against carrier reservoir loss coefficient, injection current density and temperature. Temporal behavior of photons density against injection current density is given too.

Introduction

In this work we study overall dynamics of InAs/InGaAs quantum dot laser such as photon density and number of electrons and holes in QD (against injection current density), the in and out scattering rates between the QD and WL (against temperature, injection current density and Auger coefficient) and number of electrons and holes in WL (against temperature and Auger coefficient). The three-dimensional confinement of electrons and holes in a semiconductor quantum dot (QD) profoundly changes the density of state compared to a bulk semiconductor or thin–film quantum well (QW). In ensembles of QDs, the ideal delta- function density of states of a single dot is modified into a nearly Gaussian contour that is determined by the degree of inhomogeneity in the QD sizes and shapes. QD lasers have attracted much attention in recent years due to their superior properties, such as ultra–low and temperature–stable threshold current density, high speed operation, and low frequency chirping [1]. In this paper we investigate the performance of QD...
semiconductor laser by considering a two level system of $\hbar\omega = 0.96 \text{eV}$ as a common for self–organized QDs InAs/InGaAs material system. The carrier-carrier scattering rates for electron and hole capture into the QD levels $S_{e}^{\text{in}}$ and $S_{h}^{\text{in}}$ and those for carrier escape from the QD levels $S_{e}^{\text{out}}$ and $S_{h}^{\text{out}}$ , the scattering times for electrons $\tau_{e}$ and holes $\tau_{h}$ carrier densities in wetting layer $w_{e}$ and $w_{h}$, the Auger coefficient $B_{A}$, the shift of the device temperature inside the laser medium ($T$), a carriers densities in the QD and photon density are studies versus Auger coefficient, current density and time.

**QD Laser Model:**

The numerical investigations of the laser turn-on dynamics of the QD laser presented here are based on the model given by Kathy Ludge et.al [2-5]. In the QD laser system the electrons are first injected into the wetting WL before they are captured by the QDs. The laser dynamics is determined by the rate equations for the photon density $n_{ph}$ of the ground state, GS, transition, and carrier densities in the QD, $n_{e}$ and $n_{h}$ and the carrier densities in the WL, $w_{e}$ and $w_{h}$ ( $e$ and $h$ stand for electrons and holes, respectively) this model reads :

$$n_{p} = -2k n_{ph} + \Gamma R_{\text{ind}}(n_{e}, n_{h}, n_{ph}) - \beta R_{\text{sp}}(n_{e}, n_{h})$$ (1)

$$n_{i} = \frac{1}{\tau_{e}} n_{e} + S_{e}^{\text{in}} N_{e}^{QD} - \frac{1}{\tau_{e}} n_{e} + S_{e}^{\text{out}} - \Gamma R_{\text{ind}}(n_{e}, n_{h}, n_{ph}) - \beta R_{\text{sp}}(n_{e}, n_{h})$$ (2)

$$n_{i} = \frac{1}{\tau_{h}} n_{h} + S_{h}^{\text{in}} N_{h}^{QD} - \frac{1}{\tau_{h}} n_{h} + S_{h}^{\text{out}} - \Gamma R_{\text{ind}}(n_{e}, n_{h}, n_{ph}) - \beta R_{\text{sp}}(n_{e}, n_{h})$$ (3)

$$w_{e} = \eta \frac{j(t)}{e} + \frac{n_{e}}{\tau_{e}} \frac{N_{e}^{\text{sum}}}{N_{e}^{QD}} + S_{h}^{\text{in}} + \frac{\tilde{R}_{\text{sp}}(w_{e}, w_{h})}{\tilde{R}_{\text{sp}}(w_{e}, w_{h})}$$ (4)

$$w_{h} = \eta \frac{j(t)}{e} + \frac{n_{h}}{\tau_{h}} \frac{N_{h}^{\text{sum}}}{N_{h}^{QD}} - S_{h}^{\text{in}} + \frac{\tilde{R}_{\text{sp}}(w_{e}, w_{h})}{\tilde{R}_{\text{sp}}(w_{e}, w_{h})}$$ (5)

$R_{\text{ind}}(n_{e}, n_{h}, n_{ph})=WA(n_{e} + n_{h} - N_{QD}^{QD})n_{ph}$ is the linear gain, $N_{QD}^{QD}$ denotes twice the QD density of the lasing subgroup (the factor of 2 accounts for spin degeneracy), $W$ is the Einstein coefficient, and $A$ is the WL normalization area ($A=4.14 \mu m \times 1 mm$). The density $N_{\text{sum}}$ is twice the total QD density. The spontaneous emission in the QDs is approximated by $R_{\text{sp}}(n_{e}, n_{h})=(W/N_{e}^{QD})n_{e} n_{h}$ [1].

$\tilde{R}_{\text{sp}}(w_{e}, w_{h})=B_{S} w_{e} w_{h}$ expresses the WL spontaneous recombination rate where $B_{S}$ is the band–band recombination coefficient in the WL. $\beta$ is the spontaneous emission coefficient and $\Gamma=\Gamma_{g} N_{QD}^{QD}/N_{\text{sum}}$ is the optical confinement factor. $\Gamma$ is the product of the geometric confinement factor $\Gamma_{g}$ (i.e the ratio of the volume of all QDs and the mode volume) and the ratio $N_{QD}^{QD}/N_{\text{sum}}$. The total cavity loss is expressed by $2\kappa$. The variable $j(t)$ is the injection current density, $e$ is the electronic charge, and $\eta=1-w_{e} / N_{WL}$ is the injection efficiency that accounts for the fact that we cannot inject any more carriers if the WL is already filled ($w_{e} = N_{WL}$). A sketch of the epitaxial structure as well as the energy diagram of the band structure is shown in Fig. (1) [4].

![Fig. 1.](image)

The spectral properties of the laser output are not addressed in the model, as the photon density is an average over all longitudinal modes. Changes in the QD size distribution are taken into account only by changes in the active QD density, which
It can be seen that for the hole scattering time, the dependence on $w_h$ is close to what would be expected with the assumption of a linear scattering rate. In contrast to this, researchers find a more complicated functional relation for the electron scattering rate. It has to be noted that for even higher WL carrier densities, both electron and hole scattering times will finally increase due to Pauli blocking of the scattered Auger electron [1].

**Turn-on Characteristics:**

To depict the measured laser output for different pump currents $j$ (given in units of the laser threshold $j_{th}$, which is determined from the simulated steady-state input-output characteristic $n_{ph}(j)$). The injection current pulse with a duration of 5 ns is switched on at ($t=0$). The results of the simulation is shown in Fig. (2). For the simulation a current pulse $j(t)=j_0\exp[-(t-t_0)^2/2.5ns]$ with $t_0=2.49$ ns and $n=90$ is used, yielding a flat plateau $j=j_0$ with rise and fall times of 100 ps [Fig. (3)].

![Fig. (2) Steady-state input-output characteristic: simulated photon density $n_{ph}$ vs. injection current density $j$. The threshold current density $j_{th}$ is close to what would be expected with the assumption of a linear scattering rate. In contrast to this, researchers find a more complicated functional relation for the electron scattering rate. It has to be noted that for even higher WL carrier densities, both electron and hole scattering times will finally increase due to Pauli blocking of the scattered Auger electron [1].](image-url)
The simulation results of photon density in the QD laser are obtained by solving the set of equations (1-5) using the fourth order Runge–Kutta numerical integration method with Matlab. The results are shown in fig.(4) at constant injection current density are studied for different injection current densities (the results are shown in Fig.(5)).

Fig.(3). Injection current pulse used in the simulations of turn-on characteristics

Fig.(4) Simulation of the temporal variation of (a) photon density $n_{ph}$ and (b) electrons (upper) and holes (lower) densities in the QD ($n_e$, $n_h$), respectively, for injection current density ($j = 2.7j_{th}$).

Fig.(5) Simulation of the temporal variation of (a) photon density $n_{ph}$, (b) electron density $n_e$ and (c) hole density $n_h$ in the QD for different injection current densities $j = (1.6, 2.2, 2.7, 3.2, 3.9)$.
Temperature-Dependent Losses in the Reservoir:

The temperature, $T$, dependence of the in- and out-scattering rates, the carrier losses inside the reservoir can modeled as a function of $T$. The effect of these $T$-dependent losses will be most prominent for the large signal response of the laser while its effect on the turn-on dynamics and modulation response is small. The rate 

$$ R_{loss} = B(w_e) w_e w_h $$

that accounts for these losses is a sum of the spontaneous bimolecular band–band recombination and Auger-related losses inside the quantum well (or wetting layer) QW given by 

$$ B_A w_e w_h. $$

The Auger coefficient $B_A$ has been shown [8] to depend significantly on the temperature $T$, and is therefore implemented such that it leads to a doubling of the rate for a temperature change of 60 K as found in [1]. Thus, $B_A = 305 \text{nm}^4 \text{ps}^{-1} \left( \frac{T}{300K} \right)^4$ is used as given in [4]. Keep in mind that in this work a laser with only GS levels in the QDs is modeled, which results in a different $B_A$ for the remaining Auger processes within the QW. An alternative approach to model temperature characteristics is described by M. Rossetti et. al [9] by assuming nonradiative losses in the reservoir, which are modeled by capture processes from the reservoir to a mid gap defect level. The Auger scattering rates depend on the carrier temperature inside the QW. The following analytic expressions in order to allow for an implementation into the rate equations are used [4]:

$$ S_{e}^{in}(T, w_e, w_h) = (1 + 0.22(T - 300)/100K) S_{e}^{(300, w_e, w_h)} \quad \text{(7a)} $$

$$ S_{e}^{out}(T, w_e, w_h) = (1 + 0.22(T - 300)/100K) S_{e}^{(300, w_e, w_h)} \quad \text{(7b)} $$

The out-scattering rates are related to the in-scattering rates by detailed balance as derived in [2] and [5]:

$$ S_{e}^{out}(T, w_e, w_h) = S_{e}^{in}(T, w_e, w_h) \frac{e^{\frac{\Delta E_e}{kT}}}{e^{\frac{\Delta E_h}{kT}} - 1} \quad \text{(8a)} $$

$$ S_{h}^{in}(T, w_h, w_e) = S_{h}^{out}(T, w_h, w_e) \frac{e^{\frac{-\Delta E_h}{kT}}}{e^{\frac{-\Delta E_e}{kT}} - 1} \quad \text{(8b)} $$

Here, $\Delta E_e$ and $\Delta E_h$ are the energy separations between the QD electron and hole GS and the lowest respective QW state (see fig.(1)) $\rho_e = m_e / \pi \hbar^2$ and $\rho_h = m_h / \pi \hbar^2$ are the respective 2-D densities of state in the QW, $m_e$ and $m_h$ are the electron and hole masses respectively. As the injection current density increased so does the in-scattering rates to then QD while the out-scattering of electrons and holes shows optimum values then decreases, see Fig.(6).

![Fig.(6) Simulation results of variation of (a) scattering rates of electrons $(S_{e}^{in}, S_{e}^{out})$ against injection current density $(j)$ and (b) scattering rates of holes $(S_{h}^{in}, S_{h}^{out})$ against injection current density $(j)$.](image)

As the temperature change between 0 and 325 K the relations between $S_{e,h}^{in}$ and $S_{e,h}^{out}$ and Auger coefficient are shown in Fig.(7) together with the direct relation between the temporal $T$ and scattering rates.
As a consequence of the result shown in Fig.(7), the behavior of scattering times, $\tau_e$ and $\tau_h$ of electrons and holes respectively, are shown in Fig.(8).

**Dynamic Parameters:**

There are dynamical effects that occur with increasing injection current density. The shift of the device temperature inside an electrically pumped optical amplifier (with identical active region) affected by changing the injection current density. The functional relationship between the temperature shift and injection current density is given as $\Delta T(j) \sim j^2$. 

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Fig.(7) (a, b) : Simulation result of variation of holes $(S_{ba}^{in}, S_{ba}^{out})$, against Auger coefficient $B_A$ and (c, d) : simulation result of variation of scattering rates of $(S_{ba}^{in}, S_{ba}^{out})$ against temperature $T(K)$. $b = e, h$. 

Fig (8) Simulation of the temporal variation of electrons and holes life times (microscopically calculated electron and hole scattering times $\tau_e$ and $\tau_h$ of the confined QD level) against (a) carrier reservoir loss coefficient $B$, (b) injection current density $(j)$ and (c) temperature $T(K)$. 

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*Note: The equations and figures are not transcribed into text format as they are not legible in the image.*
Since this temperature change is due to an increasing QW carrier density, and the QW carrier density itself depends via \( w_e \sim \sqrt{j} \) [2] on the pump current, it implemented \( \Delta T(w_e) \sim (w_e)^4 \) as given by (9)

\[
T = 300 + 0.245 \times 10^{12} \text{nm}^8 (w_e)^4 \quad \ldots .(9)
\]

The loss inside the QW can be written as \( R_{\text{loss}} = B w_e w_h \) which can be implemented in the rate equations (4) and (5) [4]. The constant \( B \) can be written as:

\[
B = B_{\text{loss}} + B_A w_e
\]

\[
= 0.03 \text{nm}^2 \text{ps}^{-1} + 305 \text{nm}^4 \text{ps}^{-1} \left( \frac{T}{300} \right)^4 w_e \quad \ldots .(10)
\]

Thus Auger coefficient \( B_A \) has been shown to depend significantly on the temperature \( T \) [8]. The dynamic parameters \( (T(w_e), B(w_e)) \) depended on the pump current under CW operation. This effect of implementing according to equations (9) and (10).

As expected the temperature hence \( B \) should increase with increase of injection current density as shown in Fig.(9).

As the current density increased so does the temperature hence \( B_A \) increased, as shown in Fig.(10.a). \( B_A \) depends on temperature which leads to the increase in electrons and holes in the wetting layer, see(10.b). As the temperature increased the WL electron density increased too (Fig.(10.c)).
**Discussion**

Results given in Fig (5) can be explained as follows: for a certain injection current the number of electrons and holes in QD show fast increase followed by a peak then it settle down after the transient regime. The number of photons, follow the same behavior of the populations e and h. As the temperature increases $w_e$ increased too (see equation (9)). Since $w_e$ and $w_h$ show direct proportionality against injection current, the result of Fig (6 (a, b)) of the $(S_e^{in/out}, S_h^{in/out})$ can be explained as follows: the calculated scattering rates depend in a strongly nonlinear way upon the WL carrier densities [7]. The curves in Fig. 6(a) and (b) show the in- and out-scattering rates for electrons and holes, respectively. Note that the values for electron and hole in-scattering rates differ by about a factor of 2 and the out-scattering rates differ by two orders of magnitude. For very low WL carrier densities, the in-scattering rate shows a quadratic increase as predicted by mass action kinetics, but it deviates from this functional relation for increasing WL carrier densities. The out-scattering is characterized by a sharp increase with increasing density of scattering partners followed by a decrease that is caused by Pauli blocking of the WL states. Since the holes have a larger effective mass, the maximum of the out-scattering rate lies at higher WL carrier densities than for electrons [2]. At the same time as $w_e$ increases scattering times increased too. These behaviors are shown in Fig. 7(c , d). $B_A$ depends strongly on temperature [1] so that scattering times shows behavior given in Fig.7(a, b) which is the same as those shown in Fig.(6). As temperature increases so does ($w_e$); at the same time $B_A$ increased too. The relation between scattering times of electrons and holes ($\tau_e$ and $\tau_h$) the carrier reservoir loss coefficient ($B$) is equivalent to the reciprocal of the relation between scattering times and temperature and $w_e$ [2]. The effects of $B$, current injection density($j$) and temperature are shown in Fig(8). Fig (9) summarizes the effect of injection current on different parameters affecting the dynamics of QD laser via the effect of $j(t)$ on temperature.

**Conclusion**

Based on the work of Kathy Ludge, we have studied the dependence of different variables such photon density, in- and out-scattering rates, scattering times of electrons and holes, temperature and carrier loss coefficient, on number of parameters such injection current density, Auger coefficient. Results obtained agree with experimental result of other researchers [2,7].

**References**