Study the properties of the low-lying energy states for $^{184,186}$Os isotopes using IBM-1

Yousra Abd AL-Hassan Kadhim and Assist. Prof. Dr. Nawras Mohammed Shaheed AL-Dahan

Department of Physics - College of Science - University of Kerbala
The field: Nuclear Physics

Abstract:

The energy levels, B(E2) values, the square of rotational energy and the moment of inertia, staggering in $\gamma$-band energies and the potential energy surface for $^{184,186}$Os isotopes have been investigated using IBM-1. The calculated results show good agreement with experimental data.

Keywords: IBM; Energy levels; B(E2) values; Staggering in $\gamma$-band energies; Potential energy surface

1. Introduction

The interacting boson model (IBM-1) was proposed by Arima and Iachello in (1974)[1, 2], it has become widely accepted to describe and predict low-energy collective properties of complex nuclei. This model adopted in terms of the group U(6)[2], it has three dynamical symmetries corresponding to different nuclear shapes: U(5) a spherical nucleus that can vibrate, SU(3) an ellipsoidal deformed axially symmetric rotor, and O(6) an axially asymmetric rotor. Even-even osmium isotopes are of interest because they lie in a complex region and hard to populate experimentally. Many studies have been conducted on the structure of Osmium nucleus in recent years. P. Sarriquin et. al. were studied the evolution of shapes with the number of nucleons in various chains of Yb, Hf, W, Os, and Pt isotopes from neutron number N = 110 up to N = 122, in (2008)[3]. K. Nomura et al. in (2011)[4], used interacting boson model (IBM) to calculate the transitions from prolate to oblate ground-state shapes. The energy levels (positive parity), the reduced probability of E2 transitions, the intrinsic quadrupole moment $Q_0$, and the potential energy surface for $^{184}$W and $^{184}$Os nuclei were calculated using IBM-1 by F. I. Sharrad et al. in (2013)[5]. In (2015) [6], the energy levels for the ground-state band, the properties of the ground and excited-state bands, negative parity band, the $\gamma$-band and $\beta$-band states for $^{178-186}$Os isotopes, have been calculated using Interacting Boson model by I. Mamdouh and M. Al-Jubbori.

In this present work, we investigated the energy levels, probability of electromagnetic transitions B(E2), and potential energy surface for $^{184-194}$Os isotopes using IBM-1.
2. Theory

2.1. The Interacting Boson Model (IBM-1)

The Hamiltonian operator according to IBM-1 describes the system of s (L= 0) and d (L= 2) boson can be written as follows [1, 7]

\[ \hat{H} = \varepsilon \hat{n}_d + a_0 \hat{p}^\dagger \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \]  

(1)

where:-

\( (\varepsilon, a_0, a_1, a_2, a_3, \text{and} a_4) \) are parameter used in IBM-1 ,
\( \varepsilon = (\varepsilon_d - \varepsilon_s) \) is the boson energy
\( \hat{n}_d = (\hat{d}^\dagger \cdot \hat{d}) \) Boson number type of (d-boson)
\( \hat{p} = 1/2 \left[ (\hat{d}^\dagger \cdot \hat{d}) - (\hat{s} \cdot \hat{s}) \right] \) Pairing bosons operator
\( \hat{L} = \sqrt{10} \left[ \hat{d}^\dagger \times \hat{d} \right]^3 \) Angular momentum operator
\( \hat{Q} = \left[ \hat{d}^\dagger \times \hat{s}^\dagger + \hat{s} \times \hat{d} \right]^{(2)} + \text{CHI} \left[ \hat{d}^\dagger \times \hat{d} \right]^{(2)} \) Quadrupole operator
\( \hat{T}_3 = \left[ \hat{d}^\dagger \times \hat{d} \right]^{(3)} \) Octupole operator
\( \hat{T}_4 = \left[ \hat{d}^\dagger \times \hat{d} \right]^{(4)} \) Hexadecapole operator

A form of the IBM Hamiltonian suitable for the study of shape phase transitions from SU(3) to O(6) as follows [7, 8]:

\[ \hat{H} = a_0 \hat{p}^\dagger \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \]  

(2)

The ratio \( a_0/4a_2 \) is closed to (-1), the equation above is used, but if it’s larger the appropriate Hamiltonian is [2]:

\[ \hat{H} = a_0 \hat{p}^\dagger \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 \]  

(3)

The parameters \( a_2 \) and \( a_1 \) determine only features of the eigenvalue spectrum. Furthermore, \( \chi \) (CHI) can be used as a single parameter describing the O(6)-SU(3) transition, since \( \chi = 0 \) corresponds to O(6) eigenfunctions and \( \chi = -\frac{\sqrt{7}}{2} \) corresponds to SU(3) eigenfunctions [9].

The \( n_d \varepsilon_d \) term were added to the equation (2) so the Hamiltonian Writes as follows [10]:

\[ \hat{H} = \varepsilon \hat{n}_d + a_0 \hat{p}^\dagger \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \]  

(4)

The terms in \( n_d \varepsilon_d \) or \( \hat{T}_3 \) will to reduce the ratio \( E4^+_f/E2^+_i \) and thus must be kept small for well deformed nuclei [8].

The general form of B(E2) operator in IBM-1 is given by reduce electric quadrupole transition probability B(E2) operator as follows [11,12,13]:

\[ \hat{T}_\mu^{E2} = \alpha_2 \left[ d^\dagger \times \hat{s} + s^\dagger \times d \right]^{(2)}_{\mu} + \beta_2 \left[ d^\dagger \times d \right]^{(2)}_{\mu} \]  

(5)
The reduced $E2$ transition probability ($E2$) is given by [11,12] :

$$B(E2; I_i \rightarrow I_f) = \frac{1}{(2I_i+1)} |\langle I_f | \mathcal{T}^{(E2)} | I_i \rangle|^2$$  \hspace{1cm} (6)

### 2.2. Staggering in $\gamma$- band Energies

The staggering in $\gamma$ - band energies $S(J)$ and the transition between different structural symmetries in nuclei can be written as follows [7, 14] :

$$S(J) = \frac{\{E(J^+_{2}) - E[(J - 1)^+_{2}]\} - \{E[(J - 1)^+_{2}] - [J - 2]^+_2]\}}{E(2^+_1)}$$  \hspace{1cm} (7)

The odd–even staggering is quite strongly pronounced in nuclear regions characterized by SU(5) and O(6) and relatively weaker in nuclei near the SU(3) region[15].

### 2.3. The square of rotational energy and the moment of inertia

The moment of inertia and the square of rotational energy are [7,16,17]

$$\frac{2\hbar^2}{\hbar^2} = \frac{4L-2}{E(L)-E(L-2)} = \frac{4L-2}{E_\gamma} \text{ (MeV)}^{-1}$$  \hspace{1cm} (8)

where:

- $\hbar$ is moment of inertia, $E_\gamma$ is transition energy and $L$ is angular momentum. Also,

$$\langle \hbar \omega \rangle^2 = \frac{E(L)-E(L-2)}{\sqrt{L(L+1) - \sqrt{(L-2)(L-1)}}} (\text{MeV})^2$$  \hspace{1cm} (9)

### 2.4. Potential energy surface

The calculation the potential energy surface is one of methods to knowledge the deformation of nuclear structure. The general formula for the potential energy surface as a function of geometrical variables $\beta$ and $\gamma$ is given by[1, 8] :

$$E(N, \beta, \gamma) = \frac{N_e \beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)} \left( \alpha_1 \beta^4 + \alpha_2 \beta^3 \cos 3\gamma + \alpha_3 \beta^2 + \alpha_4 \right)$$  \hspace{1cm} (10)

Where:

- $N$ is the total boson number, $\beta$ is the quadrupole deformation parameter and $\gamma$ is a symmetry angle.

### 3. Results and discussion

The energy levels, reduced electric transition probabilities $B(E2)$, relative $B(E2)$ values, the square of rotational energy and the moment of inertia, the staggering in $\gamma$ - band energies, and potential energy surface, for $^{184,186}$Os isotopes have been studied and compared with the experimental data using IBM-1 code PHINT [18].
3.1. Energy levels

The $^{184,186}$Os isotopes, with Z=76 and N=108, 110, have ratio R4/2 equals 3.21, 3.16, and the $\beta$-band above the $\gamma$-band which is incompatible with the SU(3) limit. So the equations 1, 2, and 3 are suitable used to get better results and comparing with experimental data. The parameters of $^{184,186}$Os isotopes are shown in Tables (1, 2, 3). The calculated of the energy levels are shown in Fig. (1), can see when we added EPS and OCT terms better than without them, generally the calculations are in agreement compare with experimental data.

3.2. Reduced probability of electric quadrupole transition B(E2)

Electromagnetic properties were described by IBM-1[2, 8], to calculated the absolute B(E2) values of $^{184,186}$Os isotopes the E2SD and E2DD were estimated correspond to selection rules[2, 8], and it’s shown in Table 4. The calculated and experimental values of absolute B(E2) values given in Table 4, shows the values of BE(2; 2$^+_1$ $\rightarrow$ 0$^+_1$), BE(2; 4$^+_1$ $\rightarrow$ 2$^+_1$) decreases with increasing atomic mass number, from this Table can see the results are in agreement with experimental values. The relative B(E2) values of the $^{184,186}$Os isotopes were calculated, the intraband and interband transitions from 2$^+_1$, 4$^+_1$, 5$^+_1$, and 6$^+_1$ states were compare the calculated values with experimental data [19], as see in Fig. (3, 4), shows the results are in agreement with experimental data[19].

3.3. Staggering in $\gamma$-band energies

Can observed the effect odd-even staggering in the $\gamma$-bands it is among the most sensitive phenomena carrying information about the symmetry changes[15], as see in Fig.(5, 6 and 7), in Fig. (5) the $^{184,186}$Os isotopes S(J) not appear but in Fig.(6 and 7) the S(J) is weak because it's close to SU(3) limit. This results agreement with previous study in Ref.[14].

3.4. The square of rotational energy and the moment of inertia

Fig.(8, and 9) shows the square of rotational energy and the moment of inertia for $^{184,186}$Os isotopes. As see the $^{184}$Os appears a gradual increase in moment of inertia between the lower angular momentum states, then change in behavior and then again extends gradually, this effect is known as back bending[2], that is occurs in some heavy nuclei because the rotational energy increases the energy required to break a pair of coupled nucleons. When this effect occurs, the unpaired nucleons go into different orbits and change the nuclear moment of inertia [16]. But in $^{186}$Os dose not appear any back bending, namely the properties for $^{186}$Os isotope no change.

3.5. Potential energy surface

The potential energy surface were calculated using equation (10) to determined the laste shape for $^{184,186}$Os isotopes. Fig. (10 and 11), shows the isotopes under study have prolate deformed deeper than oblate shape where the $\beta_{min}$ =1.4 for $^{184,186}$Os isotopes as for SU(3) limit.

4. Conclusions

In this work, the IBM-I model was applied for $^{184,186}$Os isotopes, this isotopes lie in transition region SU(3)-O(6) limits. The energy levels have been calculated in three different procedures. The results of added the OCT or EPS term to the eq. 2 are in agreement with experimental data and better than without them. B(E2) values, relative B(E2) values are in agreement with experimental data. The square of rotational energy and the moment of inertia were produced reasonably with experimental values, where the $^{184}$Os isotope has the backbending, odd-even staggering in the $\gamma$-bands has been studied, the S(J) is weak because it's close to SU(3) limit. The final shape for $^{184,186}$Os isotopes were found by calculating potential energy surfaces. They have prolate shape more than oblate.
Reference

  Evaluated Nuclear Structure Data File (ENSDF), online database, National Nuclear Data Center,
  (Site Visited: 3 January, 2015).
Table 1. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV except CHI. using eq. 2

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>N</th>
<th>EPS</th>
<th>PAIR</th>
<th>ELL</th>
<th>Q Q</th>
<th>OCT</th>
<th>HEXA</th>
<th>CHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{184}\text{Os}$</td>
<td>12</td>
<td>0.0</td>
<td>0.0110</td>
<td>0.0312</td>
<td>-0.0238</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.958</td>
</tr>
<tr>
<td>$^{186}\text{Os}$</td>
<td>11</td>
<td>0.0</td>
<td>0.300</td>
<td>0.0389</td>
<td>-0.0200</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.898</td>
</tr>
</tbody>
</table>

Table 2. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV except CHI. using eq. 2 + OCT term.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>N</th>
<th>EPS</th>
<th>PAIR</th>
<th>ELL</th>
<th>Q Q</th>
<th>OCT</th>
<th>HEXA</th>
<th>CHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{184}\text{Os}$</td>
<td>12</td>
<td>0.0</td>
<td>0.0139</td>
<td>0.0300</td>
<td>-0.0219</td>
<td>0.0020</td>
<td>0.0</td>
<td>-2.9580</td>
</tr>
<tr>
<td>$^{186}\text{Os}$</td>
<td>11</td>
<td>0.0</td>
<td>0.242</td>
<td>0.0328</td>
<td>-0.0178</td>
<td>0.0046</td>
<td>0.0</td>
<td>-2.7325</td>
</tr>
</tbody>
</table>

Table 3. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV except CHI. using eq. 2 + EPS term.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>N</th>
<th>EPS</th>
<th>PAIR</th>
<th>ELL</th>
<th>Q Q</th>
<th>OCT</th>
<th>HEXA</th>
<th>CHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{184}\text{Os}$</td>
<td>12</td>
<td>0.383</td>
<td>0.0223</td>
<td>0.0172</td>
<td>-0.0239</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.7680</td>
</tr>
<tr>
<td>$^{186}\text{Os}$</td>
<td>11</td>
<td>0.383</td>
<td>0.300</td>
<td>0.0245</td>
<td>-0.0173</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.7485</td>
</tr>
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</table>

Table 4. The values of parameters (E2SD, E2DD in $(e^2 b^2)$) of B(E2) for $^{184-186}\text{Os}$ isotopes.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>E2SD</th>
<th>E2DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{184}\text{Os}$</td>
<td>1</td>
<td>-1.088</td>
</tr>
<tr>
<td>$^{186}\text{Os}$</td>
<td>1</td>
<td>-0.686</td>
</tr>
</tbody>
</table>
Figure 1. (color online) The calculated low-lying energy levels (Th. 1: using eq. 2, Th. 2: using eq. 2+α_3 term, Th. 3: using eq. 2+εn_d term) and the experimental data of ^{184}Os, taken from Ref. [19].

Figure 2. (color online) The calculated low-lying energy levels (Th. 1: using eq. 2, Th. 2: using eq. 2+α_3 term, Th. 3: using eq. 2+εn_d term) and the experimental data of ^{186}Os, taken from Ref. [19].
### Table 5: B(E2) values for $^{184,186}$Osisotopes in ($e^2b^2$)

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>$J_i \rightarrow J_f$</th>
<th>EXP B(E2) [19]</th>
<th>Th. 1 B(E2)</th>
<th>Th. 2 B(E2)</th>
<th>Th. 3 B(E2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{184}$Os</td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>0.610</td>
<td>0.610</td>
<td>0.610</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>$4^+_1 \rightarrow 2^+_1$</td>
<td>0.870</td>
<td>0.850</td>
<td>0.870</td>
<td>0.870</td>
</tr>
<tr>
<td></td>
<td>$6^+_1 \rightarrow 4^+_1$</td>
<td>-----</td>
<td>0.910</td>
<td>0.930</td>
<td>0.940</td>
</tr>
<tr>
<td></td>
<td>$2^+_2 \rightarrow 0^+_1$</td>
<td>-----</td>
<td>0.011</td>
<td>0.013</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>$2^+_2 \rightarrow 2^+_1$</td>
<td>-----</td>
<td>0.018</td>
<td>0.020</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>$2^+_2 \rightarrow 4^+_1$</td>
<td>-----</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>$3^+_1 \rightarrow 2^+_1$</td>
<td>-----</td>
<td>0.019</td>
<td>0.020</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>$3^+_1 \rightarrow 4^+_1$</td>
<td>-----</td>
<td>0.008</td>
<td>0.010</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>$4^+_2 \rightarrow 2^+_1$</td>
<td>-----</td>
<td>0.010</td>
<td>0.005</td>
<td>0.007</td>
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<tr>
<td></td>
<td>$4^+_2 \rightarrow 4^+_1$</td>
<td>-----</td>
<td>0.021</td>
<td>0.020</td>
<td>0.050</td>
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<tr>
<td></td>
<td>$4^+_2 \rightarrow 2^+_2$</td>
<td>-----</td>
<td>0.290</td>
<td>0.040</td>
<td>0.320</td>
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<tr>
<td>$^{186}$Os</td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>0.580</td>
<td>0.580</td>
<td>0.580</td>
<td>0.580</td>
</tr>
<tr>
<td></td>
<td>$4^+_1 \rightarrow 2^+_1$</td>
<td>0.840</td>
<td>0.820</td>
<td>0.820</td>
<td>0.820</td>
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<tr>
<td></td>
<td>$6^+_1 \rightarrow 4^+_1$</td>
<td>1.160</td>
<td>0.860</td>
<td>0.880</td>
<td>0.880</td>
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<tr>
<td></td>
<td>$2^+_2 \rightarrow 0^+_1$</td>
<td>0.063</td>
<td>0.024</td>
<td>0.028</td>
<td>0.031</td>
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<td></td>
<td>$2^+_2 \rightarrow 2^+_1$</td>
<td>0.140</td>
<td>0.035</td>
<td>0.057</td>
<td>0.073</td>
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<tr>
<td></td>
<td>$2^+_2 \rightarrow 4^+_1$</td>
<td>0.0075</td>
<td>0.002</td>
<td>0.005</td>
<td>0.006</td>
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<td>0.040</td>
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<td></td>
<td>$3^+_1 \rightarrow 4^+_1$</td>
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<td>0.020</td>
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<td>$4^+_2 \rightarrow 2^+_1$</td>
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<td>0.010</td>
<td>0.010</td>
<td>0.010</td>
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<tr>
<td></td>
<td>$4^+_2 \rightarrow 4^+_1$</td>
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<td>0.040</td>
<td>0.064</td>
<td>0.080</td>
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<td></td>
<td>$4^+_2 \rightarrow 2^+_2$</td>
<td>0.450</td>
<td>0.279</td>
<td>0.298</td>
<td>0.320</td>
</tr>
<tr>
<td></td>
<td>$4^+_2 \rightarrow 3^+_1$</td>
<td>-----</td>
<td>0.610</td>
<td>0.587</td>
<td>0.556</td>
</tr>
</tbody>
</table>
Figure 3. (color online) Calculated (Th.1 is using eq.2, Th. 2 is using eq. 2+$\alpha_3$ term, Th. 3 is using eq. 2+ $\varepsilon \hat{n}_d$ term) and experimental values of relative B(E2) for $^{184}$Os, experimental values taken from Ref. [19].
Figure 4. (color online) Calculated (Th. 1: using eq. 2. Th. 2: using eq. 2+$\alpha_3$ term. Th. 3: using eq. 2+ $\varepsilon\tilde{n}_d$ term) and experimental values of relative $B(E2)$ for $^{186}$Os, experimental values taken from Ref. [19].

Figure 5. Staggering $S(J)$ in $\gamma$-band calculations for $^{184-186}$Os isotopes, using eq. 2.

Figure 6. Staggering $S(J)$ in $\gamma$-band calculations for $^{184-186}$Os isotopes, using eq. 2 + OCT term.
Figure 7. Staggering $S(J)$ in $\gamma$-band calculations for $^{184-186}$Os isotopes, using eq. 2 + EPS term.

Figure 8. (color online) The calculated moment of inertia $\frac{2\hbar\omega}{g}$ versus the square of rotational energy $(\hbar\omega)^2$ (Th. 1: using eq. 2, Th. 2: using eq. 2+$a_3$ term, Th. 3: using eq. 2+ $\varepsilon\hat{n}_d$ term) for $^{184}$Os isotope, the experimental data taken from Ref.[19].
Figure 9. (color online) The calculated moment of inertia \( \frac{2\gamma}{\hbar^2} \) versus the square of rotational energy \((\hbar\omega)^2\) (Th.1: using eq.2, Th. 2: using eq. 2+\(a_3\) term, Th. 3: using eq. 2+ \(\epsilon\hat{n}_d\) term) for \(^{186}\text{Os}\) isotope, the experimental data taken from Ref.[19].
Figure. 10. (A) The potential energy surface for $^{184}$Os as a function of $\beta$ at $\gamma=0$ and $60^\circ$. (B) The potential energy surface in $\beta$-$\gamma$ plane for $^{184}$Os Nucleus.

Figure. 11. : (A) The potential energy surface for $^{186}$Os as a function of $\beta$ at $\gamma=0$ and $60^\circ$. (B) The potential energy surface in $\beta$-$\gamma$ plane for $^{186}$Os Nucleus.