The Nuclear Structure for Exotic Neutron-Rich of $^{42, 43, 45, 47}$K Nuclei

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Abstract:
In this paper the proton, neutron and matter density distributions and the corresponding root mean square (rms) radii of the ground states and the elastic magnetic electron scattering form factors and the magnetic dipole moments have been calculated for exotic nucleus of potassium isotopes K ($A= 42, 43, 45, 47$) based on the shell model using effective W0 interaction. The single-particle wave functions of harmonic-oscillator (HO) potential are used with the oscillator parameters b. According to this interaction, the valence nucleons are assumed to move in the $d3f7$ model space. The elastic magnetic electron scattering of the exotic nuclei $^{42}$K ($J^πT= 2^- 2$), $^{43}$K ($J^πT=3/2^+ 5/2$), $^{45}$K ($J^πT= 3/2^+ 7/2$) and $^{47}$K ($J^πT= 1/2^+ 9/2$) investigated through Plane Wave Born Approximation (PWBA). The inclusion of core polarization effect through the effective g-factors is adequate to obtain a good agreement between the predicted and the measured magnetic dipole moments.

Key words: Exotic nucleus, proton, neutron and matter density, magnetic dipole moments, elastic magnetic form factors

Introduction:
One of the key issues in current nuclear physics research is to investigate the properties of so-called 'exotic nuclei' and of 'exotic nuclear structures'. Exotic nuclei are nuclei with a proton-to-neutron ratio that is very different from the proton-to-neutron ratio in stable nuclei (a technical term related to this ratio is the 'isospin'). The exotic nuclear structures can be defined as excitation modes of nuclei that have a very different structure than the structure (or shape) of the nuclear ground state [1]. Because of the rapid decay of exotic nuclei, it is rather difficult to make targets with them, therefore, experiments have been done in inverse kinematics with a beam of exotic nuclei incident on a target.

The electron scattering from exotic nuclei is not presently available; the technical proposal for an electron-ion collider has been incorporated in the GSI/Germany physics program at FAIR [2]. A similar program exists for RIKEN/Japan facility [3]. In both cases the main purpose is to study the structure of nuclei far from the stability line. Such facilities in future will explore the charge density distributions for nuclei far from the valley of
stability line, having skins or halos. Therefore, the halo nuclei are an extreme case of exotic nuclei with almost zero binding energy.

The electric and magnetic properties of a nuclear state, namely on what the static magnetic dipole and electric quadrupole moments can teach us about the nucleus as a system of independently moving particles in a central potential or as a system of collectively moving nucleons. The magnetic moment is very sensitive to the single particle orbits occupied by the unpaired nucleons, while the quadrupole moment is a unique instrument to study the deformation and collective behavior of nuclei both at low and high excitation energy. Both quantities can be directly compared with the predicted values in different nuclear models. Consequently, the magnetic moment may also give a means to distinguish between spherical and deformed states [4, 5].

Karataglidis and Amos [6] presented elastic and inelastic electron scattering form factors for several neutron-rich exotic nuclei. The results have been obtained using large space no-core shell models. While the elastic scattering form factors are insensitive to the details of the neutron density, it is found that inelastic scattering may be influenced by extensive neutron distributions.

Sieja and Nowacki [7] investigated the neutron rich nuclei which can be described by shell model calculations in the p–sd and sd–pf model spaces. They quantified the effects of the core polarization on the multipole part (pairing and quadrupole) of the effective Hamiltonians. They showed that proton core polarization contributions are responsible for the reduction of the neutron–neutron nuclear matrix elements which, in the recent shell model studies, appeared necessary between p–sd carbon and oxygen and sd–pf silicon and calcium nuclei.

Neyens [8] measured the ground state magnetic moments and spins of the exotic isotopes $^{49,51}$K at the ISOLDE facility at CERN using bunched-beam high-resolution collinear laser spectroscopy. The re-inversion of the ground state spin from $I = 1/2$ in $^{47,49}$K back to the normal $I = 3/2$ in $^{51}$K. At GANIL (Caen, France) the quadrupole moment of the $^{33}$Al ground state has been measured using the continuous-beam-nuclear magnetic resonance method applied to a spin-polarized beam produced at the LISE fragment separator. The large value establishes a very mixed wave function with about equal amounts of normal and neutron particle-hole excited configurations contributing to its ground state wave function.

Kreim et al., [9] reported and deduced on the measurement of optical isotope shifts for $^{38,39,42,44,46-51}$K relative to $^{47}$K from which changes in the nuclear mean square charge radii across the N=28 shell closure. The investigation was carried out by bunched-beam collinear laser spectroscopy at the CERN-ISOLDE radioactive ion-beam facility. Mean square charge radii are now known from $^{37}$K to $^{51}$K, covering all $\nu_{f7/2}$-shell as well as all $\nu_{p3/2}$-shell nuclei. These measurements, in conjunction with those of Ca, Cr, Mn and Fe, provide a first insight into the Z dependence of the evolution of nuclear size above the shell closure at N=28.

The aim of the present work is to study the magnetic elastic electron scattering form factors and to calculate the magnetic dipole moments of exotic nucleus $^{42,43,44,45}$K (neutron-rich) using W0 interaction [10] in d3f7 -model space. The elastic magnetic electron scattering of the exotic $^{42,43,45,47}$K nuclei are investigated through Plane Wave Born Approximation (PWBA).
Also the proton, neutron and matter density distributions and the corresponding root mean square (rms) radii of the ground states are calculated. Calculations are presented with model space and with core-polarization (CP) effects by using effective g-factors.

Theory:
The interaction of the electron with the spin and currents distributions of nuclei can be considered as an exchange of a virtual photon with angular momentum $\pm 1$ along

$$F_{j}^{M}(q) = \frac{4\pi}{Z^2(2J_{j}+1)} \sum_{T=0,1} (-1)^{T_{f}-T_{g}} \left\{ \begin{array}{ccc} T_{f} & T & T_{i} \\ -T_{g} & M_{T} & T_{2i} \end{array} \right\} \left\{ \begin{array}{ccc} \Gamma_{f} & \Gamma_{f, \tau} & \Gamma_{i} \end{array} \right\}_{T_{f}}^{2} \ldots (1)$$

where $\hat{T}_{\tau}(q)$ is the magnetic electron scattering multipole operator. For a magnetic operator $T_{\tau}$ the reduced matrix elements are written as the sum of the product of the one-body density matrix elements (OBDM) times the single-particle transition matrix elements [14]:

$$\langle \Gamma_{\tau}, \Gamma_{\tau}^{\prime} | F_{\tau, \tau}^{a,b} \rangle = \sum_{a,b} \text{OBDM} \left| \langle \Gamma_{\tau}, \Gamma_{\tau}^{\prime}, a, b \rangle \hat{T}_{\tau} \right|^{2} \ldots (2)$$

where $\Lambda = JT$ is the multipolarity and the states $\Gamma_{\tau} = \Gamma_{\tau}$ and $\Gamma_{\tau}^{\prime} = \Gamma_{\tau}$ are initial and final states of the nucleus. While $\alpha$ and $\beta$ denote the final and initial single-particle states, respectively (isospin is included). The momentum transfer $\vec{q}$ direction. This is called transverse scattering [11]. From the parity and angular momentum selection rules, only electric multipoles can have longitudinal components, while both electric and magnetic multipoles can have transverse components [11, 12].

The squared magnetic form factors for electron scattering between nuclear states $J_{i}$ and $J_{f}$ involving angular momentum transfer $J$ are given by [13]:

$$\left| F_{j}^{M}(q) \right|^{2} = \frac{4\pi}{Z^2(2J_{j}+1)} \sum_{T=0,1} (-1)^{T_{f}-T_{g}} \left\{ \begin{array}{ccc} T_{f} & T & T_{i} \\ -T_{g} & M_{T} & T_{2i} \end{array} \right\} \left\{ \begin{array}{ccc} \Gamma_{f} & \Gamma_{f, \tau} & \Gamma_{i} \end{array} \right\}_{T_{f}}^{2} \ldots (4)$$

The single particle matrix element in spin- isospin state is given by [14]:

$$\left\langle a \left| O_{\tau}(m) \right| b \right\rangle = \left\langle n_{a} l_{a} \right| F_{\tau, \tau}^{m} \left| n_{b} l_{b} \right\rangle \ldots (5)$$

where
for $J = 1$ equation (5) become:

$$
\left< a \parallel \hat{O}_T (m1) \parallel b \right> = f_{s-m}^{-1} (a , b)
$$

... (7)

The reduce matrix element of the magnetic transition operator $\hat{O}_T (m1)$ is

$$
\left< J \sum_{aT} \hat{O}_T (m1) J \right> = \sum_{aT} (-1)^{j-T} \left< \frac{T_a}{T} \frac{T}{T} \frac{T_i}{i} \right> \text{OBDM}_{(a , b , J = 1 , T)} \left< a \parallel \hat{O}_T (m1) \parallel b \right> \ldots (8)
$$

The magnetic dipole moment $\mu$ of a state of total angular momentum $J$ is given by [14]:

$$
\mu = \sqrt{\frac{4\pi}{3}} \left< J \sum_{aT} \hat{O}_T (m1) J \right> \ldots (9)
$$

The neutron skin occurs as a consequence of the neutron excess in heavy nuclei. The neutron skin in general is defined as the radial difference (root-mean square-radius difference) of the neutron and proton distributions with surface thickness ($t$) [18]:

$$
t = R_s - R_p = \left< r_n^z \right> - \left< r_p^z \right> \ldots (10)
$$

where $R_s = \left< r_n^z \right>$ and $R_p = \left< r_p^z \right>$ are the neutron and proton root mean square radius, respectively.

**Results and Discussion:**

The radial wave functions for the single-particle matrix elements were calculated with harmonic oscillator (HO) potential with size parameters $b$ are adjusted for each isotope of potassium to reproduce the measured root mean square matter radius ($R_m$). The calculations of the proton, neutron, and matter rms radii and magnetic form factors are carried out using $d3f7$-shell model space with effective W0 interaction [10] in OXBASH code [15]. The core polarization (CP) effect is included by using effective $g$-factors.

The values of calculated matter ($R_m$), proton ($R_p$) and neutron ($R_n$) rms radii, magnetic dipole moments ($\mu$), and oscillator size parameter ($b$) of potassium isotopes nuclei are displayed in table 1.

**1. $^{42}$K nucleus ($J^T = 2^- , 2\pi_1/2 = 12.36$ h)**

Calculations are performed with $d3f7$-shell model space including core-polarization effects. The configurations $(1s_{1/2})^4 (1p_{3/2})^3 (1p_{1/2})^3 (1d_{5/2})^2 (2s_{1/2})^4 (1d_{3/2})^7 (1f_{1/2})^3$ are used for $^{42}$K. The oscillator size parameter is taken to be 2.073fm, which gives the rms matter radius equal to 3.4467fm in agreement with the measured value 3.4467(34)fm [19]. The difference between the neutron and proton rms radii is $R_n - R_p = 3.5905 - 3.3166 = 0.2339$ fm, which provides an additional evidence for the exotic structure of this nucleus.
The calculated total magnetic form factors for $^{43}$K ground state ($J^p=2^+$) using W0 interactions in $d3f7$-model space and with free $g_s$-factors are shown in Fig.(1). Unfortunately, there are no experimental data to compare with it. The individual multipoles contributions M1 and M3 are denoted by dashed and dashed-dot curves respectively, while the E2 and E4 multipoles are disappeared because it has negligible contributions. The diffraction minimum for M1 component located at momentum transfer $q=2.1\text{fm}^{-1}$, but for M3 component the diffraction minimum located at $q=1.4$ and $2.35\text{ fm}^{-1}$. The total form factors in $d3f7$-shell model space are included by solid curve.

The calculated magnetic dipole moments ($\mu$) of $^{43}$K isotope are tabulated in table 1 together with the available experimental data. The calculated magnetic dipole moment of $g$(free) is $\mu=-1.3594\ n.m$. The inclusion the effective $g$-factors with $g$(eff)$=0.9\ g$(free) decreased the magnetic moment value by $-1.22352\ n.m$ which is in a good agreement with the measured value $\mu_{exp.}=-1.1425\ (6)\ n.m$ [20].

2. $^{43}$K nucleus ($J^p=3/2^+5/2,\tau_{1/2} = 22.3\ h$)

Calculations are performed with $d3f7$-shell model space including core-polarization effects. The configurations $(1s_{1/2})^4(1p_{3/2})^4(1p_{1/2})^4(1h_{5/2})^2(2s_{1/2})^4(1d_{5/2})^7(1f_{7/2})^4$ are used for $^{43}$K. The oscillator size parameter is taken to be $1.916\text{fm}$, which gives the rms matter radius equal to $3.4489\text{fm}$ in agreement with the measured value $3.4489(42)\text{fm}$ [19]. The difference between the neutron and proton rms radii is $R_n - R_p = 3.5261 - 3.3488 = 0.1773 \text{ fm}$, which provides an additional evidence for the exotic structure of this nucleus.

The calculated total magnetic form factors for $^{43}$K ground state ($J^p=3/2^+5/2$) using W0 interactions in $d3f7$-model space and with free $g_s$-factors are shown in Fig.(2). The individual multipoles contributions M1 and M3 are denoted by dashed and dashed-dot curves respectively, while the E2 is disappeared because it has negligible contribution. The diffraction minimum for M1 component located at momentum transfer $q=1.9\text{fm}^{-1}$, but for M3 component the diffraction minimum located at $q=0.6\text{ fm}^{-1}$. The total form factors in $d3f7$-shell model space are included by solid curve.

The calculated magnetic dipole moments ($\mu$) of $^{43}$K isotope are tabulated in table 1 together with the available experimental data. The calculated magnetic dipole moment of $g_s$(free) and inclusion the effective $g$-factors with $g$(eff)$=0.9\ g$(free) are $\mu=0.17642\ n.m$ and $0.15878\ n.m$, respectively, which is in a good agreement with the measured value $\mu_{exp.} = 0.1633\ (8)\ n.m$ [20].
magnetic dipole moment. The choice for free g-factors gives $\mu = 0.19753 \ n.m$. The inclusion the effective g-factors with g(eff)=0.9 g(free) decreased the magnetic moment value by 0.17778 \ n.m, which is a good agreement with the measured value $\mu_{exp} = 0.1734 \ (8) \ n.m$ [20].

**3. $^{45}$K nucleus ($J^pT = 3/2^+ 7/2$, $\tau_{1/2} = 17.3 \text{ m}$)**

The configurations $(1s_{1/2})^4 (1p_{3/2})^8 (1p_{1/2})^4(1d_{5/2})^{12} (2s_{1/2})^4 (1d_{3/2})^7(1f_{7/2})^6$ are used for $^{45}$K. The single – particle wave functions of harmonic oscillator potential are used with oscillator size parameter b=1.849fm chosen to reproduce the experimental matter rms radius 3.4523(45) fm [19]. The difference between the neutron and proton rms radii is $R_n - R_p = 3.5113 - 3.3698 = 0.1415 \ \text{fm}$, which provides an additional evidence for the exotic structure of this nucleus.

The calculated total form factors for the ground state of unstable nucleus $^{45}$K with ($J^pT= 3/2^+ 7/2$) with free spin g$_\tau$-factors are shown in Fig.(3). The individual multipoles contributions M1 and M3 are denoted by dashed and dashed-dot curves, respectively, while the E2 multipole is disappeared because it has a negligible contribution. The diffraction minimum for M1 component located at momentum transfer $q = 2.0\text{fm}^{-1}$. The different values of g$_\tau$-factors, which give different values for
are shown in Fig.(4). The individual multipole contribution M1 is denoted by solid curve. The diffraction minimum for M1 component located at momentum transfer q=0.7 and 1.9 fm⁻¹.

The calculated magnetic dipole moment of g(free) and of inclusion the effective g-factors are \( \mu = 1.61367 \ n.m \) and 1.4523 \( n.m \), respectively, which are less than the measured value \( \mu_{\text{exp}} = 1.933 \ n.m \) [20].

**Table 1: The calculated matter, proton and neutron rms radii, magnetic dipole moments (\( \mu \)), and oscillator size parameter (b) of 42,43,45,47 \( K \) isotopes compared with experimental results.**

<table>
<thead>
<tr>
<th>A</th>
<th>( J^T )</th>
<th>( T_{1/2} )</th>
<th>b (fm)</th>
<th>( R_m ) (fm)</th>
<th>( R_p ) (fm)</th>
<th>( R_n ) (fm)</th>
<th>( \mu ) (n.m) Calc.</th>
<th>( g(free) )</th>
<th>( g(eff.)=0.9 ) ( g(free) )</th>
<th>( \mu ) (n.m) Exp. [20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>2( ^+ )</td>
<td>12.36 h</td>
<td>2.073</td>
<td>3.4467</td>
<td>3.4467(34)</td>
<td>3.3166</td>
<td>3.5505</td>
<td>-1.13947</td>
<td>-1.22352</td>
<td>-1.1425(6)</td>
</tr>
<tr>
<td>43</td>
<td>3/2( ^+ )</td>
<td>5/2</td>
<td>1.916</td>
<td>3.4489</td>
<td>3.4489(42)</td>
<td>3.3488</td>
<td>3.5261</td>
<td>0.17642</td>
<td>0.15878</td>
<td>0.1633(8)</td>
</tr>
<tr>
<td>45</td>
<td>3/2( ^+ )</td>
<td>7/2</td>
<td>1.849</td>
<td>3.4523</td>
<td>3.4523(45)</td>
<td>3.3698</td>
<td>3.5113</td>
<td>0.19753</td>
<td>0.17778</td>
<td>0.1734(8)</td>
</tr>
<tr>
<td>47</td>
<td>1/2( ^+ )</td>
<td>9/2</td>
<td>1.898</td>
<td>3.4418</td>
<td>3.4418(32)</td>
<td>3.3309</td>
<td>3.5150</td>
<td>0.161367</td>
<td>0.14523</td>
<td>0.1933(9)</td>
</tr>
</tbody>
</table>

**Conclusions:**

Shell-model calculations are performed for K isotopes including core-polarization effects through first-order perturbation theory. The magnetic dipole moments \( \mu \) of the 42,43,45,47 \( K \) nuclei depend clearly on assigned configurations and their experimental data will be useful to determine the deformations of the ground states of nuclei near the neutron drip line. The inclusion of core polarization (the effective g-factors) is adequate to obtain a good agreement between the predicted and measured magnetic dipole moments. The elastic magnetic form factors are influenced by details of nuclear wave function and the center of mass correction which depends on the mass number and the size parameter b.

**References:**


التركيب النووي للنوى الغريبة الغنية بالنيوترونات لـ $^{42,43,45,47}K$

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الخلاصة:

في هذا البحث تم حساب توزيعات الكثافة البروتونية والنيوترونية والمادة النووية بالإضافة إلى أنواع الأقترب للنيوترونات والنيوترونات للحالات الأرضية وعوامل التشكل المغناطيسية للأسطارة الإلكترونية المرنة وعوامل ثانية القطب المغناطيسية للنوى الغريبة لانقار البتاوسوم $(A=42,43,45,47)$ المستندة على أنواع الفيبرة باستخدام تقاعد $W_0$ الفعال. استخدمت الدوال الموجية للسيناريا المفردة لجهد المتذبذب مع قيمة للثابت التواقي $b$. بناءً على هذا التفاعل، تم افتراض نيكلونات التكافؤ تتراوح في $(HO)$ التواقي، مما كتب $b$. تم تحقيق عوامل التشكل المغناطيسية للإسطارة الإلكترونية المرنة للنوى الغريبة $^{43,45}K(J^pT=1/2^+, 9/2^+, 7/2^+)$ بواسطة تقريب بورن $(J^pT=3/2^+, 5/2^+)$. تضمن تأثير استقطاب القلب من خلال عوامل $g$ الفعالة كافية للحصول على تفاوت جيد بين الفيبرة المتوقعة والمقاسة للعوامل ثانية القطب المغناطيسية.

الكلمات المفتاحية: النوى الغريبة، كثافة البروتون والنيوترون والمادة النووية، العوامل المغناطيسية، عوامل التشكل المغناطيسية المرنة.