Mathematical Model For Estimating Particle Deposition Velocity Over Baghdad City
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ABSTRACT
This paper provides mathematical model to calculate the speed of the dry deposition particles. Measurements taken over the centre of Baghdad city (Bab Al-Mhadham area) at level 15m for wind speed and temperature, by this model, we can find deposition velocity for any particle by interring the diameter after installing friction velocity, temperature, wind speed and over parameter values for neutral condition, and draw a geometric shapes of the relationship between deposition velocity and diameter. We can notice that the deposition velocity is high when the particle is small and then decreases when the particle size increases.

INTRODUCTION
The atmospheric aerosol consists of particles that range widely in size from nanometers to tens of micrometers. When studying the atmospheric aerosol particles, it is important to know the dynamics of the particles: formation, growth and removal processes [1]. In this paper, the description of mathematical model for dry deposition particle is over land terrain under natural condition, in addition to evaluating the model outputs and analysis of these outputs. Deposition of air pollutants is an important loss of aerosol particles from the atmosphere. At the same time, deposition processes of different air pollutants can cause various harmful effects both on ecosystems and built environment. Deposition of an air pollutant affects its atmospheric concentration as well as the state of the environment and human health. Therefore; it is an important factor in different types of atmospheric chemical-transport models, and surface exchange models. The remove of particulates from the atmosphere can occur by dry or wet forms. Dry deposition is a continuous process, while wet removal can be realized only in the presence of precipitation. Therefore, despite of the dry process is slower than wet deposition, the accumulated removal quantity of a pollutant could be more important in case of dry deposition. Both the dry and wet depositions depend on the properties of the gases or particles and removal processes are governed by several environmental factors [2]. On this research, focusing on the dry deposition velocity of particles, they have been parameterized as a function of particle size and density surface properties, and micrometeorological conditions near the surface [3]. In the natural condition, variation of deposition velocity described by some authors and works, described deposition velocity over different conditions such as study of dry deposition velocity for global modeling, measure dry deposition by flux of particles measure over land, sea and global [3], and discussing effective dry deposition velocities of gas and particles over heterogeneous terrain [4].

MATERIALS & METHODS
Dry deposition is commonly measured by the deposition velocity vd, which depends on many characteristics of the underlying surface, surface layer and parameters of deposition particles including their size. Therefore, it is reasonable to define the dry deposition velocity as the gravitational settling velocity Vg, and the inverse of sum of resistances, in the following relation [5],

\[ V_d = \frac{(R_a + R_d + R_a R_d V_g)}{V_g} \]

Where Ra is the aerodynamic resistance of the atmosphere surface layer, Rd is the resistance to transport through the quasi-laminar layer, and Vg the gravitational settling [5] [6] [7].

\[ R_a = \frac{U}{(U_r)^2} \]

Where U is the wind speed and U_r is the friction velocity, which is by the following logarithmic equation in the natural air calculated by,

\[ U_r = \frac{\kappa}{\ln(Z/Z_0)} \]

\[ \kappa \text{ is Von Kármán constant } = 0.4, Z_0 \text{ aerodynamic roughness length, } Z \text{ height above the surface } [9]. \]

\[ R_d = \left( \frac{1}{U_r} \right) \left( \frac{S_c^{1/3}}{V/Db} \right) \]

Where Sc the Schmidt number can give:

\[ S_c = \frac{V}{Db} \]

Where V viscosity of air (taking as 0.15 cm²s⁻¹), Db the Brownian diffusivity of the pollutant (in cm²s⁻¹) which is computed by the following relation:

\[ Db = \left( 8.09 \times 10^{-10} \times Ta \times Scf \right) / D \]

Where Ta temperature in (Kelvin), D diameter of particle, Scf Cunningham slip correction factor which is computed as:
\[ S_{cf} = 1.1 \left( \frac{2x_2(a_1 + a_2 e^{-(a_3 D/x_2)})}{D \times 10^{-4}} \right) \]  

Where, \( x_2, a_1, a_2, a_3 \) are constants with values of 6.5 \times 10^{-6}, 1.257, 0.4, 0.55 \times 10^{-4} respectively.

\[ S_t = (\nu g/g) \times \left( \frac{U^2}{V} \right) \]  

Where \( S_t \) Stokes number (dimensionless), \( \nu \) acceleration due to the gravity (981 cm/s^2), \( V \) The gravitational settling velocity in (cm/s) which is calculated by:

\[ V_g = \rho (\rho - \rho_{air}) \times \frac{\rho_{air} \times \mu \times C_2}{D} \times S_{cf} \]  

Where \( \rho \) density of particle, \( \rho_{air} \) the air density, \( \mu \) the absolute viscosity of air, \( C_2 \) air units conversion constant (1 \times 10^{-6} cm^2 μm^-2) [7][8].

**THE SITE AND DATA**

The model that calculates dry deposition velocity of any particle in the air of urban city near the surface under natural condition requires the observational data such as wind speed and temperature. These data were obtained over adistrid called (Bab Al-Mhadham) located at the centre of Baghdad city (320 14" N, 440 14" E and 31.7 m above mean sea level. These data were measured by two sets of classical slow-response instruments such as three-cup anemometer, wind vane and thermometer. These instruments were mounted on two masts with 15 and 20 m heights which were set up on one of the roofs belonging to the engineering college buildings in Al-Mustansiriyah University, the field around the measuring area consists of mostly government offices with different heights including very big public hospital, scientific institutes, ministries, public schools…etc. Many low houses with 4 m height are located to the east of measuring mast. Lastly many old and tall trees are scattered in whole area around the mast. The data used at each level were observed at the same time at afternoon from 4:00 to 7:00 Baghdad local standard time from 4-20 April, 2006. These data were recorded every minute for interval of 20 min, and then 20-minute averages were computed. Particle diameter applied in this model is [0.01, 0.1, 0.3, 0.5, 1, 1.5, 5, 7.5, 10, 15, 20, and 30 μm]. The mean value of surface roughness length of Baghdad city across all directions within the circle of 1km was 1.2m calculated using the logarithmic equation under neutral conditions [9].

**DRY DEPOSITION MODEL**

A mathematical model is an abstract model that uses mathematical language to describe the behavior of a system. Mathematical models are used particularly in the natural sciences and engineering disciplines. Mathematical models can take many forms, usually it is the pest for using as much a priori information as possible to make the model more accurate. The earth’s surface is the interface between the atmosphere and earth – and as such it is the surface through which all deposition processes. The development of models for the deposition of material from the air to the earth’s surface has relied heavily on historical studies of gravity, heat, wind speed, friction velocity, and diameter. The Model, which I’m working on to calculate the speed of the particles over the city of Baghdad, through the use of mathematic programs, the first program Math Lab we enter the equations for the deposition velocity, which is in turn extracting results. And the second program is a program origin pro, by which we introduce the data extracted from the first program, and the creation of empirical relation give the deposition velocity for any diameter, and also how it affects each coefficient atmosphere on the deposition velocity. The model show in the figure (1).
RESULTS AND DISCUSSION

1. Effect of Friction Velocity \( u^* \) on Deposition Velocity

Friction velocity can reflect the rough status of surface. In order to recalculate wind speed at any level by use of logarithmic equation, it’s required to determine \( U^* \) first. According to the values of Ri, the \( U^* \) results are plotted against atmospheric stability in figure 1.

In near neutral conditions, \( u^* \) values are nearly constant around 0.43 ms\(^{-1}\) with significant scatter, which is expected because surface roughness features around the experimental site are largely changed, as shown in figure 2.

\[
V_d = B_0 + (B_1 \times U^*) + (B_2 \times U^*^2) \ldots \ldots \ldots \quad (10)
\]

Where \( B_0, B_1, B_2 \) are constants and their values are (0.75, -0.01, and 0.002) respectively.

Table 1: Shows the amount of the increase in deposition velocity with increase friction velocity at a number of different diameters at neutral condition.

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( 0.36 ) ms(^{-1})</th>
<th>( 0.41 ) ms(^{-1})</th>
<th>( 0.48 ) ms(^{-1})</th>
<th>( 0.55 ) ms(^{-1})</th>
<th>( 0.59 ) ms(^{-1})</th>
<th>( 0.68 ) ms(^{-1})</th>
<th>( 0.76 ) ms(^{-1})</th>
<th>( 0.82 ) ms(^{-1})</th>
<th>( 0.86 ) ms(^{-1})</th>
<th>( 1.02 ) ms(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01 ( \mu ) m</td>
<td>0.106</td>
<td>0.121</td>
<td>0.142</td>
<td>0.163</td>
<td>0.175</td>
<td>0.201</td>
<td>0.225</td>
<td>0.243</td>
<td>0.255</td>
<td>0.285</td>
</tr>
<tr>
<td>0.1 ( \mu ) m</td>
<td>0.023</td>
<td>0.026</td>
<td>0.030</td>
<td>0.035</td>
<td>0.037</td>
<td>0.043</td>
<td>0.048</td>
<td>0.052</td>
<td>0.055</td>
<td>0.061</td>
</tr>
<tr>
<td>1 ( \mu ) m</td>
<td>0.01</td>
<td>0.011</td>
<td>0.012</td>
<td>0.013</td>
<td>0.01</td>
<td>0.01</td>
<td>0.016</td>
<td>0.01</td>
<td>0.017</td>
<td>0.019</td>
</tr>
<tr>
<td>10 ( \mu ) m</td>
<td>10.0</td>
<td>14.96</td>
<td>22.5</td>
<td>30.5</td>
<td>35.12</td>
<td>45.52</td>
<td>54.70</td>
<td>61.53</td>
<td>66.05</td>
<td>77.24</td>
</tr>
<tr>
<td>20 ( \mu ) m</td>
<td>25.87</td>
<td>31.4</td>
<td>39.0</td>
<td>46.64</td>
<td>50.93</td>
<td>60.50</td>
<td>68.92</td>
<td>75.20</td>
<td>79.37</td>
<td>89.74</td>
</tr>
<tr>
<td>30 ( \mu ) m</td>
<td>32.78</td>
<td>38.07</td>
<td>45.43</td>
<td>52.7</td>
<td>56.87</td>
<td>66.16</td>
<td>74.36</td>
<td>80.50</td>
<td>84.58</td>
<td>94.75</td>
</tr>
</tbody>
</table>

Deposition velocity increases near linearly with friction velocity for small particles (diameter smaller than 1 \( \mu \)m). However, this dependency does not exist for larger particles where deposition velocity does not change with variations of friction velocity until the latter reaches a certain threshold; beyond this threshold, deposition velocity rises again near linearly with friction velocity. According to the values of Ri used to obtain value of the \( U^* \) and used to classify atmosphere stability Given that the Ra +Rd term decreases when the friction velocity increases, deposition velocity of particles smaller than 1 mm is, in all cases of friction velocity, controlled by the 1/Ra +Rd term, while for particles larger than 10 mm velocity is controlled by the Vg term. For particles in the 1–10 mm range, the Ra +Rd term is larger than the 1/Vs term at small values of the friction velocity; in this, case the deposition velocity is controlled by settling velocity and is thus independent on the friction velocity. This remains true until the friction velocity reaches a threshold when Ra+Rd becomes smaller than 1/Vg; and then 1/Ra+Rd controls the deposition velocity, which becomes linearly dependent on the friction velocity [2].

2. Effect of Surface Roughness Length on Deposition Velocity

Because there are several values of \( z_0 \) determine, their effects on \( v_d \) can be studied, as shown in figure 3.
The relation between roughness and deposition velocity is a near linear correlation because whenever it found with high roughness will catch a larger number of particles, and the piece will be large deposition. The data points can be fitted by the Empirical relationship between $z_0$ with $v_d$ as follows:

$$v_d = C_0 + (C_1 \times z_0) + (C_2 \times z_0^2) \ldots \ldots \ldots \ (11)$$

Where $C_0$, $C_1$ and $C_2$ are constants which their values are 1.1, 1.5 and 0.1 respectively.

The surface roughness length is a measure of the amount of mechanical mixing introduced by the surface roughness elements over a region of transport. It is measured directly from meteorological data [11]. Roughness length appears to be one of the determining factors in particle deposition velocity. On the one hand, it is directly related to the aerodynamic resistance by ruling the drag coefficient, and on the other hand it controls the quasi-laminar layer resistance through the friction velocity term. Thus, as the surface gets rougher the two resistances become smaller and deposition velocity decreases [2].

![Figure 5: Heterogeneity Means Values of $z_0$ with Diameter](image)

We notice through the figure that the effect of roughness in the case of natural condition, the increase in deposition velocity with increasing roughness and this effect is small and equal increase for the particles that have a diameter less than 1.5 μm, and at this diameter the influence will be equal but after this point, also shows an increase in the speed of deposition for large particles and the increase is not completely equal and will be bigger than larger particles.

### 3. Deposition Velocity

This section describes and discusses deposition velocities and relation with diameter under constant condition $u=4.3 \text{ ms}^{-1}$, $z_r=1.2 \text{ ms}^{-1}$, $m=2.81 \text{ ms}^{-1}$ and $T=301.5 \text{ K}$. Dry deposition velocities have been calculated from experimental data of particles. In general, the deposition velocities observed for any particular material have a wide range of values. The dry deposition velocity may be modeled using an analogy to electrical resistance. Resistances are associated with atmospheric conditions, physical and chemical characteristics of the material, and the physical, chemical and biological properties of the surface [12]. Content the data of both diameter of particle and velocity are presented in figure 5, relation when input any diameter in this equation can create velocity of this diameter, can followed by the empirical relation as follows:

$$v_d = B_0 + (B_1 \times D) + (B_2 \times D^2) \ldots \ldots \ldots \ (12)$$

Where $B_0$, $B_1$ and $B_2$ are constants which their values are -1.44, 1.22, 0.75 respectively.

![Figure 6: The Fitting of Velocity and Equation of Relation between Diameter and Velocity](image)

### Table 2: Shows the amount of increase in deposition velocity with increase roughness length at a number of different diameters at neutral condition.

<table>
<thead>
<tr>
<th>$D$ (μm)</th>
<th>0.2m</th>
<th>0.54m</th>
<th>0.6m</th>
<th>0.7m</th>
<th>0.8m</th>
<th>0.9m</th>
<th>1m</th>
<th>1.1m</th>
<th>1.8m</th>
<th>1.9m</th>
<th>2.2m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01μm</td>
<td>0.020</td>
<td>0.038</td>
<td>0.056</td>
<td>0.079</td>
<td>0.103</td>
<td>0.124</td>
<td>0.144</td>
<td>0.162</td>
<td>0.200</td>
<td>0.200</td>
<td>0.038</td>
</tr>
<tr>
<td>0.1μm</td>
<td>0.004</td>
<td>0.008</td>
<td>0.012</td>
<td>0.017</td>
<td>0.022</td>
<td>0.026</td>
<td>0.031</td>
<td>0.035</td>
<td>0.040</td>
<td>0.060</td>
<td>0.008</td>
</tr>
<tr>
<td>1μm</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
<td>0.013</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>10μm</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
<td>0.013</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>20μm</td>
<td>0.010</td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
<td>0.013</td>
<td>0.014</td>
<td>0.015</td>
<td>0.015</td>
<td>0.010</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>30μm</td>
<td>0.014</td>
<td>0.014</td>
<td>0.015</td>
<td>0.016</td>
<td>0.017</td>
<td>0.017</td>
<td>0.018</td>
<td>0.019</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
</tr>
</tbody>
</table>

### Table 2: Shows the amount of the deposition velocity with different diameters at neutral condition.

<table>
<thead>
<tr>
<th>$D$ (μm)</th>
<th>0.01</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_0$</td>
<td>0.1300</td>
<td>0.028</td>
<td>0.014</td>
<td>0.011</td>
<td>0.0120</td>
<td>0.018</td>
<td>1.347</td>
<td>7.235</td>
<td>11.785</td>
<td>16.032</td>
<td>18.161</td>
<td>21.44</td>
</tr>
</tbody>
</table>
Two noticeable features can be found from this figure: first, deposition velocity strongly depends on particular size, and varies from 0.01 to several tens of cm s$^{-1}$; with a minimum in the 0.5-1.0 mm diameter range. Thus, for these sizes, dry deposition does not appear to be an efficient removal process, and this could lead to longer atmospheric residence times. Second, for particles in the range of 0.1-1.5 μm diameter, which have low deposition velocities and particle in range more than 5 μm have large deposition velocities. In order to study the effect of wind speed on deposition velocity of particles, it is illustrated as follows:

Through the drawing, wind speed does not have any effect on the deposition of particles with a diameter less than 5 micrometers to 1 for diameters bigger than 5 micrometers note the effect begins speed increases and be clear and this means that the large particles are largely affected by wind speed and this leads to increased settling velocity.

CONCLUSIONS

This paper attempts to estimate deposition velocity in near neutral stratification over Baghdad city. Particular dry deposition velocity as a function of surface properties and micro-meteorological conditions near the surface develops within our model; this dry deposition velocity derived from change resistance near the surface, before deriving deposition velocity, the valid of estimated values by those laws at the surface was compared to these observed in this paper and the result was in good agreement. The used data of wind and temperature in this paper were measured at levels 15 by three-cup anemometer and thermometer, respectively. The atmospheric stability parameter is indicated by calculating the Richardson number. The main results can be summarized as:

1. Within urban canopy, $U_1$ values are small at low level constant value around 4.3 ms$^{-1}$ near the natural condition.
2. The particle deposition velocity was very sensitive to surface type and meteorological conditions, as well as particle size.
3. Increasing the friction velocity will tend to increase deposition velocity.
4. The relation between roughness and deposition velocity is a direct correlation because whenever it was high deposition will catch a larger number of particles and the piece will be large deposition.
5. Particles in the range of 0.1-1.5 μm diameter have low deposition velocities and particle in range more than 5 μm have large deposition velocities.

REFERENCES