Stopping power and straggling of H (proton) and He (helium) in solid targets (Au, AL, K, Cs)

Saher Mazher Mutesher

Thi-Qar University, collage of medicine, Department of physiology and medical physics

**Abstract:**

Stopping power and straggling have been calculated by using Random Phase Approximation (RPA) at low velocity at the first and second approximation order, where the influence of damping has been ignored. At high velocities of a projectile, Plasmon Pole Approximation (PPA) has been used to calculate them without damping. In this research, we discuss interaction of protons H and helium He with solid matter Au (rs=1.49 a.u.), AL (rs=2.12 a.u.), K (rs=4.86 a.u.) and Cs (rs=5.88 a.u.) in different adverbs. The results were obtained in all areas of the present work showed a good agreement with the previous works for stopping power and variance in energy loss (straggling). The results also showed detailed behavior of (H and He) of its interactions with four electron gas targets medium. The results have been achieved by using programs of matlab language, which performed for the numerical calculation.

**Introduction**

A heavy ion passing through a target of convinced thickness will suffer a number of collisions with the atoms and electrons of the target. There is amount of energy will be transferred to the target atom and electron in each collision. Because the collisions are random and discrete, statistical fluctuation is expected in the number of collisions. [1, 2] variance of stopping power will be studied because of the statistical nature for the stopping power quality. The threshold effect describes the energy loss and straggling in a single crystal in channeling they found a mass effect between channeled protons and neutrons in the relative straggling values. [3, 4].

The decreasing soft ions in solid matter due to interaction with the electrons of the atoms have been of interest since the early days of atomic physics [5]. Specific energy loss is also a major amount when the length of the road traveled by the ion at the center of importance. Thus, the exchange of energy between the ion intrusion and mail order to target not only in the fundamental interests but of the utmost importance to the depth profiling techniques employing ionic rays and applied to the thin-film as well as ion beam based materials modification. The decelerating force dE/dx, i.e. the mean energy loss per path length, an ion experiences in a
material is commonly denoted as the stopping power \( S \) of the material [6]. Theoretical model

Stopping power and straggling. The stopping power of matter for charged particles is a main physical quantity in various applications. It is a subject of great importance in numerous areas of fundamental and applied physics [5].

**Theoretical model**

**Stopping power and straggling**

The stopping power of matter for charged particles is a main physical quantity in various applications. It is a subject of great importance in numerous areas of fundamental and applied physics [5]. The interaction of charged particles with matter has been a subject of great interest both for the advance of the knowledge of the basic interaction processes, as well as for a multitude of practical applications [6].

The problem of energy losses suffered by charged particles moving in matter is of continuous interest in physics. When the ion velocity is greater than the average velocity of valence electrons in solids, a good description of the loss can be achieved using a linear response theory in which the screened potential is treated to the lowest order to calculate the loss due to the valence – band electrons with atomic type calculations due to core – electron excitations. However, at low energies, the importance of screening nonlinearities was demonstrated by using a scattering theory approach to the stopping power and density – functional theory [7]. When a charged particle is traversing matter, it is loss energy due to interaction with the target atoms. The energy loss of the projectile per unit distance in the target material is called the stopping power of the material \( \frac{dE}{dx} \). It depends on the charge, velocity of the projectile and, of course, the target material. Stopping power has been evaluated from equation [8]:

\[
\frac{dE}{dx} = \frac{2Z_1^2 e^2}{\pi \nu^2} \int_0^\infty \frac{dk}{k} \int_0^\omega \text{Im} \left[ \frac{-1}{\nu(k, \omega)} \right] d\omega
\]

(1)

Where \( \text{Im} \left[ \frac{-1}{\nu(k, \omega)} \right] \) is energy loss function as a function of \( k \) and \( \omega \).

Where \( Z_1 \) is the projectile atomic number (\( Z_1 = 1 \) for H and \( Z_1 = 2 \) for He) and \( e \) is the elementary charge.

If a beam of monoenergetic charged particles, with initial energy \( E \), passes through an absorber of thickness \( \Delta x \), due to the statistical fluctuation in the energy loss, the variance can be defined as follows [9]

\[
\Omega^2 = \sqrt{\langle (\Delta E - \langle \Delta E \rangle)^2 \rangle}
\]

(2)

Where \( \Delta E \) : the energy loss of the charged particle when it passes through the absorber and \( \langle \Delta E \rangle \) is the average energy loss of the same particle in passing through the same
absorber whose thickness is \( \Delta x \). The square root in Eq. (2) is called standard deviation \( \Omega \) in energy loss \( \Delta E \) from its mean \( \langle \Delta E \rangle \) value.

The energy-loss straggling \( \Omega^2 / dx \) represents the fluctuation in the energy-loss spectrum and it is given by [10]:

\[
\Omega^2 / dx = \frac{2Z_i^2 e^2 h^2}{\pi \nu^2} \int_0^\infty dk \int_0^{k_v} d\omega \omega^2 \text{Im} \left[ -\frac{1}{\epsilon(k, \omega)} \right]
\]  

(3)

Straggling is a complex issue in general. Fluctuations in energy loss are governed by the statistics of energy-loss processes and charge-changing events. For penetrating atomic ions, the former dominates far light ions and hinge on close collision. The processes giving rise to the enhanced stopping power of light molecular ions less efficient with regard to straggling. [10]. On the other hand, charge-exchange straggling goes as the square of the stopping power, therefore, become relatively more important for molecular than for atomic ions [11].

**Dielectric Formalism for Stopping Power and straggling:**

1. **Low velocities**

The Lindhard function [7] gives in a self-consistent way an exact expression of the dielectric constant for a non-relativistic free electron gas of high density at zero temperature. In the low energy limit, within the Random Phase Approximation (RPA) for the dielectric constant, the loss function can be written as follows:

\[
\epsilon(k, \omega) \equiv \epsilon_1(k) + i \epsilon_2(k, \omega)
\]

(4)

or

\[
\frac{1}{\epsilon(k, \omega)} = \frac{1}{\epsilon_1(k) + i \epsilon_2(k, \omega)}
\]

(5)

The imaginary part of \( \frac{-1}{\epsilon(k, \omega)} \) can be obtain by multiplying and dividing Eq. (2) by its conjugate.

Therefore the \( \text{Im} \left[ -\frac{1}{\epsilon(k, \omega)} \right] \) can be written as:

\[
\text{Im} \left[ \frac{1}{\epsilon(k, \omega)} \right] = \frac{\epsilon_2(k, \omega)}{\epsilon_1^2(k) + i \epsilon_2^2(k, \omega)}
\]

(6)

From Nagg et al. [8] we have that:
\[ \varepsilon_1(\vec{k}) = C(\vec{k})f_1(\vec{k}) + 1 \quad \text{for arbitrary } \vec{k} \]
\[ \varepsilon_2(\vec{k}) = C(\vec{k}) \frac{\pi \omega}{2k k_F} \quad \text{for } k \leq 2k_F \]  

(7)

Where \( f_1(\vec{k}) = \frac{1}{2} \left[ 1 + \frac{4k^2 - k^2}{4k k_F} \ln \left| \frac{k + 2k_F}{k - 2k_F} \right| \right] \)

\[ C(\vec{k}) = \frac{4k_F}{\pi k^2} \]

(8)

The approximation of the dielectric constant at low velocities \((v<v_f)\)

i. The first approximation \( f_1(\vec{k}) \):

For \( \in (\vec{k}, \omega) \) an approximation is made to Eq. (3), if \( \varepsilon_1(\vec{k}) >> \varepsilon_2(\vec{k}, \omega) \), therefore,

\[ \text{Im} \left[ \frac{-1}{\varepsilon_1(\vec{k}, \omega)} \right] \approx \frac{\varepsilon_2(\vec{k}, \omega)}{\varepsilon_1(\vec{k})} \]

(9)

Using the first approximation method to \( f_1(\vec{k}) = 1 \) then, equation (6) becomes:

\[ \text{Im} \left[ \frac{-1}{\varepsilon_1(\vec{k}, \omega)} \right] = \frac{C(\vec{k}) \left[ \frac{\pi \omega}{2k k_F} \right]}{[C(\vec{k}) + 1]^2} = \frac{2\omega}{k^3 \left[ \frac{4k_F}{\pi k^2} + 1 \right]^2} \]

\[ = \frac{2k \omega}{(k^2 + k_D^2)^2} \quad \text{for } k \leq 2k_F \]

(10)

Where \( k_D^2 = \frac{4k_F}{\pi} \)

(11)

ii. The Second Approximation to \( f_1(\vec{k}) \):

A good approximation to straggling of energy loss values obtained numerically by using the full (RPA) dielectric response function has been proposed by Lindhard and Winther [9]. Expanding the function \( f_1(\vec{k}) \) and then, \( f_1(\vec{k}) \) up to the second order in \( \vec{k} \) and then, \( f_1(\vec{k}) \) becomes [10].

\[ f_1(\vec{k}) = 1 - \frac{1}{3} \left( \frac{\vec{k}}{2k_F} \right)^2 \]

(12)

The imaginary part of the (RPA) dielectric loss function is given by inserting Eq. (12) and into Eq. (6) as follows:
\[
\text{Im}\left(\frac{-1}{\varepsilon(k, \omega)}\right) \approx \frac{4k_F \pi \omega}{\pi k_F^2 2k_F^2} \left\{ \frac{1}{1 + \frac{1}{3} \left( \frac{\vec{k}}{2k_F} \right)^2} \times \left( \frac{4k_F}{\pi k_F^2} \right)^2 \right\} \\
\approx \frac{2\omega}{k^3 \left(1 - \frac{1}{3\pi k_F^2} + \frac{4k_F^2}{\pi k_F^2}\right)^2}
\]

(13)

Let the constants \[ \Pi^2 = 1 - \frac{1}{3\pi k_F^2} \text{ and } k_D^2 = \frac{4k_F}{\pi} \] then Eq. (13) becomes

\[
\text{Im}\left(\frac{-1}{\varepsilon(k, \omega)}\right) \approx \frac{2\omega}{k^3 \left(\Pi^2 + k_D^2/k^2\right)^2} \approx \frac{2\omega}{k^3 \left(\Pi^2 k^2 + k_D^2/k^2\right)^2}
\]

(14)

2- High velocities:

The key ingredient in the calculation of stopping power is the linear-response function \(\varepsilon(\vec{k}, \omega)\) of the target material, \(\varepsilon(\vec{k}, \omega)\) for a dense, and hence a degenerate electron gas, has been calculated in various approximations in the literature [12]. At high velocities where the projectile can excite plasmons in the medium, Echenique et al., [13] and Basbas et al., [14] have used the plasmon-pole approximation (PPA) of the dielectric function \(\varepsilon(\vec{k}, \omega)\):

\[
\varepsilon(\vec{k}, \omega) = 1 + \frac{\omega_p^2}{\omega^2 + \beta^2 k^2 + \frac{k^4}{4} - \omega(\omega + i\gamma)}
\]

(15)

The constant \(\beta = (3/5)^{1/2} k_F\) is the propagation of density disturbances in an electron gas, \(\omega_p\) is the plasmon frequency \(\omega_p = 3^{1/2} / r_s^{1/2}\) and the effective band gap energy \(\omega_g\) in semiconductors and insulators give a collective resonance frequency \(\Omega_o = (\omega_p^2 + \omega_g^2)^{1/2}\) [15].

Plasmon dispersion is included through the term containing \(\beta^2\). Single-particle effects are accounted for by the term equal to the square of kinetic energy \(k^2/2\), of a free electron with momentum \(\vec{k}\). The small constant \(\gamma\) represents damping processes. It follows that in the limit \(\gamma \to 0\)[10]:

\[
\text{Im}\left[\frac{1}{\varepsilon(k, \omega)}\right] = \frac{\pi \omega_p^2}{2A} \delta(\omega - A)
\]

(16)

Where \(A^2 = \Omega_o^2 + \beta^2 k^2 + k^4 / 4\)
The upper and lower integration limits in "k" are the maximum and minimum momentum transfers \( k_+ \) and \( k_- \) to the target electrons [16].

\[ k_\pm = \left\{ 2(\nu^2 - \beta^2) \pm 2[(\nu^2 - \beta^2)^2 - \Omega_v^2] \right\}^{1/2} \]  

(17)

This gives a threshold for \( \nu \); 

\[ \nu_{th} = (\beta^2 - \Omega_v^2)^{1/2} \]

The results

We have made extensive calculation of stopping power and straggling and the numerical results for four solid targets, Au (\( r_s = 1.49 \) a.u), Al (\( r_s = 2.12 \) a.u), K (\( r_s = 4.86 \) a.u) and Cs (5.88 a.u.) have been presented [17]. These four targets have been chosen because of their frequent use in experiments [18] and also of their different electron densities, where \( (r_s) \) is a measure of electron density. In atomic unit \((m = e = \hbar = 1)\), and 1 a.u. = 0.529 Å, and 1 a.u. ≈ 27.2 eV.

I- First approximation

Different approximations to the function \( f_1(\tilde{k}) \) in Eq. (10) lead to different expressions for the stopping power and straggling. By substituting dielectric function in first approximation \( f_1(\tilde{k}) = 1 \) at low velocities from eq.(10) for determine both stopping power and straggling:

a- Stopping power: Substituting eq. (10) in eq. (1)

\[ \frac{dE}{dx} = \frac{2}{\pi \nu^2} \int_0^\nu \frac{dk}{k} \int_0^{k/k_F} d\omega \omega \frac{2k \omega}{(k^2 + k_F^2)^2} \left( z_i^2 e^2 \right) \]  

(18)

\[ \frac{dE}{dx} = \frac{2}{3\pi \nu^3} \int_0^\nu \frac{dk}{k} \left[ \frac{2k (k^2 - 1)}{(k^2 + k_F^2)^2} \right] \]  

(19)

The maximum momentum transfer between the collision electrons of projectile and medium is \( 2\tilde{k}_F \). Then the average rate of stopping power due to single charges at low velocity with no damping \( (\gamma \rightarrow 0) \) is given by the following Eq.

\[ \frac{dE}{dx} = \frac{2}{3\pi \nu^3} \left( z_i^2 e^2 \right) \int_0^{k_F} \frac{dk}{k} \frac{k^3}{(k^2 + k_F^2)^2} \]  

(20)

b- The straggling

When we substituting eq(10 ) in eq( 2), One can get

\[ \frac{\Omega_v^2}{e^2} (E) = \frac{2\hbar e^2}{\nu^2 \gamma} \int_0^\nu \frac{dk}{k} \int_0^{\nu \omega} d\omega \omega \frac{2k \omega}{(k^2 + k_F^2)^2} \left( z_i^2 \right) \]
The Eqs. (20,21) are solved by the program written by matlab language and the figures have been gotten

Fig.(1): stopping power in the first approximation of low velocities for:

(a) proton

(b) Helium

Fig.(2): straggling in the first approximation of low velocities for:

(a) proton

(b) Helium
2- Second approximation

a- Stopping power

By substituting Eq. (14) into the stopping power Eq. (1), one can get:

\[
\frac{dE}{dx} = \int \frac{dk}{k} \int_0^{\beta v} d\omega \omega \frac{2k\omega}{\Pi^4 [k^2 + (k_d/\Pi)^2]} \left( z_i^2 \right)
\]

By using the standard integral solution [19], one can get the final solution to Eq. (22) as follows: The stopping power for the second approximation at low velocity is given by the following Eq.

\[
\frac{dE}{dx} = \frac{2e^2 \bar{\nu}(z_i^2)}{3\pi \Pi} \int_0^{2\beta v} k^3 \frac{d\bar{k}}{k^2 + (k_d/\Pi)^2} \left( z_i^2 \right)
\]

b- Straggling

Now we discuss the variance (straggling) at low velocity ions with no damping by using the second approximation by using Eq. (14) into Eq. (2) then the variance (straggling) is given as follows:

\[
\frac{\Omega^2}{dx}(E) = \frac{2he^2}{\pi \nu^2} \int_0^{2\beta v} \frac{d\bar{k}}{k} \int_0^{\beta v} d\omega \omega^2 \left[ \frac{2\bar{k}\omega}{\Pi^4 [k^2 + (k_d/\Pi)^2]} \right] \left( z_i^2 \right)
\]

\[
= \frac{e^2 h(z_i^2)}{\pi \Pi^4} \int_0^{2\beta v} \frac{k^3 d\bar{k}}{k^2 + (k_d/\Pi)^2} \left( z_i^2 \right)
\]

The Eqs. (23, 25) are solved by the program written by matlab language and the figures have been gotten
Fig. (3): stopping power in the second approximation of low velocities for:

(a) proton

(b) Helium

\[ f_i(\tilde{k}) = 1 - \frac{1}{3} \left( \frac{\tilde{k}}{2k_f} \right)^2 \] for He

\[ f_i(\tilde{k}) = 1 - \frac{1}{3} \left( \frac{\tilde{k}}{2k_f} \right)^2 \] for H

Fig. (4): straggling in the second approximation at low velocities for:

(a) proton

(b) Helium

\[ f_i(\tilde{k}) = 1 - \frac{1}{3} \left( \frac{\tilde{k}}{2k_f} \right)^2 \] for He

\[ f_i(\tilde{k}) = 1 - \frac{1}{3} \left( \frac{\tilde{k}}{2k_f} \right)^2 \] for H
3- high velocities

By substituting the imaginary part of \( \frac{1}{\varepsilon(k, \omega)} \) for a suitable approximation of the dielectric response \( \varepsilon(k, \omega) \) in the limit of damping process in Eq. (1) then we can find stopping power for high velocities. [20] At high velocity imaginary part (energy loss function) given by eq. (16):

\[
\text{Let us take two special cases: (a) If } A^2 = \Omega_p^2 = \omega_p^2, \text{ then }
\]

\[
\text{Below which plasmon contributions subside. Then by substituting Eq. (26) in Eq. (1) one can get,}
\]

\[
\text{If a- } A^2 = \Omega_p^2 = \omega_p^2, \text{ then From the property of the Dirac-} \delta \text{ function}
\]

\[
f(x') = \int \delta(x - x') f(x) dx
\]

From comparison Eq. (27, 28) we get \( \int_0^\infty d\omega \omega A \delta(\omega - A) = 1 \)

Therefore, the stopping power for high velocity of ions with no damping being as follows [10]:

\[
\text{(29) } \frac{dE}{dx} = \frac{2e^2}{\pi} \left( \frac{\omega}{v} \right)^3 \ln \left( \frac{k}{k_-} \right)
\]

By substituting Eq. (16) into Eq. (2) and therefore the variable in energy loss (straggling) becomes,

\[
\text{(30) } \frac{\Omega^2}{dx} = \frac{2e^2}{\pi^2} \left( \frac{\omega}{v} \right)^3 \int \frac{dk}{k} \int d\omega \omega \Omega_p^2 \left[ \int \frac{\pi \delta(\omega - \omega_p)}{2} \right] \times \left( \frac{1}{\varepsilon_i} \right)
\]

By useful from eq. (28) in eq. (29), one can get:

\[
\text{(31) } \frac{\Omega^2}{dx} = \frac{e^2}{v^3} \left( \frac{\omega}{v} \right)^3 \ln \left( \frac{k}{k_-} \right)
\]

(b) \text{ If } A^2 = \Omega_p^2 + \beta^2 K^2 + K^4 / 4 \text{ Then Eq. (1) becomes}

\[
\text{(32) } \frac{dE}{dx} = \frac{2e^2}{\pi^2} \left( \frac{\omega}{v} \right)^3 \int \frac{dk}{k} \int d\omega \omega \Omega_p^2 \delta(\omega - A) \times \left( \frac{1}{\varepsilon_i} \right)
\]

\[
\text{(33) } = \frac{e^2}{v^3} \left( \frac{\omega}{v} \right)^3 \int \frac{d\omega}{\omega} \Omega_p^2 \left( \frac{1}{\varepsilon_i} \right)
\]

\[
\text{(34) } \frac{dE}{dx} = \frac{e^2}{v^3} \left( \frac{\omega}{v} \right)^3 \left[ \frac{\Omega_p^2}{k^2} + \beta^2 + K^4 / 4 \right]
\]

By take the same last steps we can find the straggling equation written it as the following:

\[
\text{(35) } \frac{\Omega^2}{dx} = \frac{e^2}{v^3} \left( \frac{\omega}{v} \right)^3 \left[ \frac{\Omega_p^2}{k^2} + \beta^2 + K^4 / 4 \right]
\]

Eq. (29,31,34,35) has been solved numerically by using the program matlab (2012) and figures have been obtained the following:
Fig.(5): stopping power in the first case $A^i = \Omega^j = \alpha^j$ at high velocities for:

(a) proton  
(b) Helium

\[ A^i = \Omega^j = \alpha^j \quad \text{for He} \]

Fig.(6): straggling in the first case $A^i - \Omega^j - \alpha^j$ at high velocities for:

(a) Helium

\[ A^i = \Omega^j = \alpha^j \quad \text{for H} \]
Discussion and conclusions

The dielectric formalism have been applied to calculate the main significant magnitudes in the energy loss of hydrogen- and helium-ion beams in a target, namely, stopping power and straggling. The calculations have been done taking into account velocities of projectile.

A good relationship has been obtained from calculating the total stopping power, straggling stopping power by using Lindhard function of Random Phase Approximation with
no damping for first and second order of approximation at low velocities as used by Nagy and Echenique [21] and Plasmon Pole Approximation at high velocities. This study is comparable to the result of Arista [22].

fig.(1) shows stopping power of proton (H) and helium(He) at low velocities in first approximation \( f_1(\vec{k}) = 1 \) for four different targets (Au,Al,K,Cs) and fig.(2) which shows straggling of proton (H) and helium(He) at low velocities in first approximation for four different targets (Au,Al,K,Cs). while fig.(3) shows stopping power of proton (H) and helium(He) at low velocities in second approximation \( f_2(\vec{k}) = 1 - \frac{1}{3} \left( \frac{\vec{k}}{2k_F} \right)^2 \) for four different targets (Au,Al,K,Cs) and fig.(4) which shows straggling of proton (H) and helium(He) at low velocities in second approximation for four different targets (Au,Al,K,Cs). in spite of the difference between them (first and second approximation respectively) both stopping power and straggling remain on the same conduct. This means that stopping power and the straggling have the same behavior in both approximations formula. Stopping power and straggling have large value for Au \((r_s=1.49 \text{ a.u.})\) which mean that stopping power and straggling decreasing with density parameter or wigner-sietz radius \((r_s)\). The increasing of Winger Seitz radius \((r_s)\) lead to decreasing with stopping power and straggling of energy loss at low velocities with no damping. This means that relation is inverse proportional.

Fig.(5) shows stopping power of proton (H) and helium(He) at high velocities in first case \( A^2 = \Omega_p^2 = \omega_p^2 \) for four different targets (Au,Al,K,Cs), Fig.(6) shows straggling of proton (H) and helium(He) at high velocities in first case \( A^2 = \Omega_p^2 = \omega_p^2 \) for four different targets (Au,Al,K,Cs), while fig.(7) shows stopping power of proton (H) and helium(He) at high velocities in second case \( A^2 = \Omega_p^2 + \beta^2 K^2 + K^4 / 4 \) for four different targets (Au,Al,K,Cs) and fig.(8) shows stopping power of proton (H) and helium(He) at high velocities in second case \( A^2 = \Omega_p^2 + \beta^2 K^2 + K^4 / 4 \) for four different targets (Au,Al,K,Cs). figures (5,6,7,8) show dependant of energy loss on the Wigner Seitz radius \((r_s)\) for different values of its at high velocities. The increasing values of \( r_s \) leads to decrease \( \vec{v}_F \) (Fermi Velocity) of a target and also the density of electrons according to the relation as \( r_s = \left( \frac{3}{4\pi n_e} \right)^{1/3} \), where \( r_s \) is the radius of a sphere contains one electron [21] and \( n \) is the density of electrons. The collision of ions in a target of small \( r_s \) means that there will be high dense of electrons to screen the projectile and delay it. In addition, one may expect a short interaction time, therefore each target medium exhibits prevention against the
projectile dependent on its density where 
\[ Au(r_s = 1.49) \] represents the highest screening and then, 
\[ Al(r_s = 2.12) , \quad K = (r_s^4 = 4.86) \quad \text{and} \quad C_i(r_s^4 = 5.88). \]

Reference

البلازمون لحساب قدرة الارتقاف والتطور للأهداف عند وجود الاضحلال. في هذا البحث، ناقشنا تفاعل البروتونات والهيليوم مع المواد الصلبة (Au (rs=1.49 a.u.)، AL (rs=2.12 a.u.)، K (rs=4.86 a.u.) وCs (rs=5.88 a.u.) في ظروف مختلفة. أن النتائج التي تم حصول عليها خلال هذه الدراسة أظهرت أن هناك توافق جيد بين قدرة الارتقاف والتبان في الطاقة المفقودة (التطور)، أن النتائج المستحصلة بين السلوك التفصيلي للأيونات (البروتون والهيليوم) لتفاعله مع أربعة أوساط لأهداف ذات كثافة كترونية مختلفة. حُصلت على النتائج باستخدام برامج بلغة الماتلاب والتي اعدت لإجراء الحسابات العددي.