Parameters estimation for modified weibull distribution  
based on singly type one censored samples

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Abstract

The three parameters distribution called modified Weibull distribution (MWD) introduced by sarhan and zaindin (2009). In this paper, we deal with point estimation to estimate the parameters of modified Weibull distribution based on complete data, using maximum likelihood method Ordinary least squares estimator method and rank set sampling estimator method to obtain the estimate parameters for modified Weibull distribution, then estimate the death function , survival function and hazard function obtained these estimate functions by applied on set of real data which taken for Leukemia disease in the Baghdad general hospital.

Key Words:
Modified Weibull distribution(MWD), Maximum likelihood estimation method(MLEM), Ordinary least squares estimator method(OLSEM), Rank set sampling estimator method(RSSEM), Newton-Raphson method, Survival function, Hazard function.

1-
Introduction

The modified Weibull distribution was first introduced by Sarhan and Zaindin (2009)(1). Which is a very important distribution that it can be used to describe several reliability models. This distribution contain three parameters one scale parameter α and two shape parameters are λ,γ respectively. Sarhan and Zaindin ;(2009)(1, 2) introduced MWD( α,λ,γ ) and prove some basic properties , Sarhan and Zaindin ;(2009)(1) estimate parameters by MLE based on type two censored data, Said in(2010)(3) considered rank sampling to estimate parameters based on MLE, Mazen ; (2010)(4) presented estimators for parameter based on type one censored data by using MLE, Soufiane and Maher ;(2011)(5) presented that the hazard rate function of MWD( α,λ,γ ) is constant if γ = 1, increasing if γ > 1 and decreasing if γ < 1. Our aim of the this
paper is interested in MWD(α,λ,γ) and defines the properties of this distribution, and then estimates the three parameters in this distribution by using three Non-Bayesian methods (Maximum likelihood estimator method, Ordinary least squares estimator method and Rank set sampling estimator method), after that finding and estimating the death density function, survival probability function and hazard probability function based on complete data. At least applying the mentioned probability functions for areal data which are collected for the Leukemia disease in hospital. The rest of paper is organized as follows: In section two definition and some properties of MWD(α,λ,γ). In section three deriving point estimation for the parameters of MWD(α,λ,γ) by using MLEM, OLSEM and RSSM. In section four apply the real set data compute the estimation of death density function, survival function and hazard function. Finally we conclude the paper in section five.

2-definition and properties of MWD:

The cdf of MWD(α,λ,γ) take the follows from:

\[ F(t; \alpha, \lambda, \gamma) = 1 - \exp(-\alpha t - \lambda t^\gamma) \quad t > 0 \quad \ldots \quad (2.1) \]

The pdf of the MWD(α,λ,γ) is:

\[ f(t; \alpha, \lambda, \gamma) = \begin{cases} (\alpha + \lambda t^{\gamma-1}) \exp(-\alpha t - \lambda t^\gamma), & t > 0 \\ 0, & \text{o.w} \end{cases} \quad (2.2) \]

Where the parameter space is

\[ \Omega = \{ (\alpha, \lambda, \gamma) ; \alpha, \lambda \geq 0 ; \gamma > 0 \} \]

The mean of MWD(α,λ,γ) is:

\[
\mu_1 = E(t) = \sum_{i=0}^{\infty} \frac{(-\lambda)^i}{i!} \left[ \frac{\Gamma(iy + 2)}{\alpha^{i+y}} + \frac{\lambda y \Gamma(y(1 + i) + 1)}{\alpha^{1+y(i+1)}} \right] 
\] (2.3)

The variance of the MWD(α,λ,γ) is:

\[
\sigma^2 = \text{var}(t) = \sum_{i=0}^{\infty} \frac{(-\lambda)^i}{i!} \left[ \frac{\Gamma(iy + 3)}{\alpha^{2+i+y}} + \frac{\lambda y \Gamma((1 + i)y + 2)}{\alpha^{2+(1+i)y}} \right] - \sum_{i=0}^{\infty} \frac{(-\lambda)^{2i} (i!)^2}{(i!)^2} \left[ \frac{\Gamma(iy + 2)}{\alpha^{2+i+y}} + \frac{\lambda y \Gamma((1 + i)y + 1)}{\alpha^{1+(1+i)y}} \right]^2 
\] (2.4)

The survival function of the MWD(α,λ,γ) takes following form:

\[ s(t; \alpha, \lambda, \gamma) = e^{(-\alpha t - \lambda t^\gamma)} , \quad t \geq 0 \quad (2.5) \]

The hazard rate function of MWD(α,λ,γ) is:

\[ h(t; \alpha, \lambda, \gamma) = (\alpha + \lambda t^{\gamma-1}) , \quad t \geq 0 \quad (2.6) \]

3-Parameters Estimation:

In this section we shall clarify how to derive and estimate the parameter by using Non-Bayesian methods which are as follows

3.1- The Maximum Likelihood Estimators:
The likelihood function for MWD(\(\alpha, \lambda, \gamma\))

\[
L = \prod_{i=1}^{n} (\alpha + \lambda \gamma t_i^{\gamma-1}) e^{(-\alpha \Sigma_{i=1}^{n} t_i - \lambda \Sigma_{i=1}^{n} t_i^{\gamma})} \quad (3.1)
\]

Taking the logarithm for the likelihood function so we get function:

\[
\ln L = \ln \prod_{i=1}^{n} (\alpha + \lambda \gamma t_i^{\gamma-1}) - \alpha \Sigma_{i=1}^{n} t_i - \lambda \Sigma_{i=1}^{n} t_i^{\gamma} \quad (3.2)
\]

We take the first derivative of equation (3.2) with respect to \(\alpha, \lambda, \gamma\) and equating each equation to zero, then we get three nonlinear equation for \(\alpha, \lambda, \gamma\) respectively as follows:

\[
\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{n} \frac{1}{\alpha + \lambda \gamma t_i^{\gamma-1}} - \sum_{i=1}^{n} t_i
\]

\[
\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{n} \frac{\gamma t_i^{\gamma-1}}{(\alpha + \lambda \gamma t_i^{\gamma-1})} - \sum_{i=1}^{n} t_i^{\gamma}
\]

\[
\frac{\partial \ln L}{\partial \gamma} = \sum_{i=1}^{n} \frac{\lambda[\gamma t_i^{\gamma-1} \ln t_i + t_i^{\gamma-1}]}{(\alpha + \lambda \gamma t_i^{\gamma-1})} - \lambda \sum_{i=1}^{n} t_i^{\gamma} \ln t_i
\]

\[
\sum_{i=1}^{n} \frac{1}{\alpha + \lambda \gamma t_i^{\gamma-1}} - \sum_{i=1}^{n} t_i = 0 \quad (3.3)
\]

\[
\sum_{i=1}^{n} \frac{\gamma t_i^{\gamma-1}}{(\alpha + \lambda \gamma t_i^{\gamma-1})} - \sum_{i=1}^{n} t_i^{\gamma} = 0 \quad (3.4)
\]

\[
\sum_{i=1}^{n} \frac{\lambda[\gamma t_i^{\gamma-1} \ln t_i + t_i^{\gamma-1}]}{(\alpha + \lambda \gamma t_i^{\gamma-1})} - \lambda \sum_{i=1}^{n} t_i^{\gamma} \ln t_i = 0 \quad (3.5)
\]

To find the maximum likelihood estimations for \(\alpha, \lambda, \gamma\) we must solve the system of three nonlinear equation (3.2),(3.4),(3.5) by using the iterative method. Such as Newtone-Raphson method to obtain the solution which is as follows:

\[
[z(\gamma)] = \begin{bmatrix} \alpha_{i+1} \\ \lambda_{i+1} \\ \gamma_{i+1} \end{bmatrix} = \begin{bmatrix} \alpha_i \\ \lambda_i \\ \gamma_i \end{bmatrix} - J_i^{-1} \begin{bmatrix} f(\alpha) \\ g(\lambda) \\ z(\gamma) \end{bmatrix} \quad (3.6)
\]

\[
f(\alpha) = \sum_{i=1}^{n} \frac{1}{(\alpha + \lambda \gamma t_i^{\gamma-1})} - \sum_{i=1}^{n} t_i
\]

\[
g(\lambda) = \sum_{i=1}^{n} \frac{\gamma t_i^{\gamma-1}}{(\alpha + \lambda \gamma t_i^{\gamma-1})} - \sum_{i=1}^{n} t_i^{\gamma}
\]

\[
z(\gamma) = \sum_{i=1}^{n} \frac{\lambda[\gamma t_i^{\gamma-1} \ln t_i + t_i^{\gamma-1}]}{(\alpha + \lambda \gamma t_i^{\gamma-1})} - \lambda \sum_{i=1}^{n} t_i^{\gamma} \ln t_i
\]

Thus,\(J_i^{-1}\) is Jacobean matrix which is defined as follows:

\[
J_i^{-1} = \begin{bmatrix} \frac{\partial f(\alpha)}{\partial \alpha} & \frac{\partial f(\alpha)}{\partial \lambda} & \frac{\partial f(\alpha)}{\partial \gamma} \\ \frac{\partial g(\lambda)}{\partial \alpha} & \frac{\partial g(\lambda)}{\partial \lambda} & \frac{\partial g(\lambda)}{\partial \gamma} \\ \frac{\partial z(\gamma)}{\partial \alpha} & \frac{\partial z(\gamma)}{\partial \lambda} & \frac{\partial z(\gamma)}{\partial \gamma} \end{bmatrix}
\]

Now, we find the formulas of Jacobean matrix as follows:

\[
\frac{\partial f(\alpha)}{\partial \alpha} = \sum_{i=1}^{n} \frac{-1}{(\alpha + \lambda \gamma t_i^{\gamma-1})^2}
\]

\[
\frac{\partial f(\alpha)}{\partial \lambda} = \sum_{i=1}^{n} \frac{-\gamma t_i^{\gamma-1}}{(\alpha + \lambda \gamma t_i^{\gamma-1})^2}
\]

\[
\frac{\partial f(\alpha)}{\partial \gamma} = \sum_{i=1}^{n} \frac{-\lambda[\gamma t_i^{\gamma-1} \ln t_i + t_i^{\gamma-1}]}{(\alpha + \lambda \gamma t_i^{\gamma-1})^2}
\]
\[
\frac{\partial g(\lambda)}{\partial \alpha} = \sum_{i=1}^{n} -\gamma y_i^{y-1} (\alpha + \lambda y_i^{y-1})^{-2}
\]

\[
\frac{\partial g(\lambda)}{\partial \lambda} = \sum_{i=1}^{n} -\gamma^2 y_i^{2(y-1)} (\alpha + \lambda y_i^{y-1})^{-2}
\]

\[
\frac{\partial g(\lambda)}{\partial \gamma} = \sum_{i=1}^{n} \frac{[y_i^{y-1} \ln t_i + t_i^{y-1}]}{(\alpha + \lambda y_i^{y-1})} - \sum_{i=1}^{n} \lambda y_i^{2(y-1)}(\alpha + \lambda y_i^{y-1})^{-2} - \sum_{i=1}^{n} t_i^{y} \ln t_i
\]

\[
\frac{\partial z(\gamma)}{\partial \alpha} = \sum_{i=1}^{n} -\lambda [y_i^{y-1} \ln t_i + t_i^{y-1}] (\alpha + \lambda y_i^{y-1})^{-2}
\]

\[
\frac{\partial z(\gamma)}{\partial \lambda} = \sum_{i=1}^{n} [y_i^{y-1} \ln t_i + t_i^{y-1}] (\alpha + \lambda y_i^{y-1})^{-2} - \sum_{i=1}^{n} t_i^{y} \ln t_i
\]

\[
\frac{\partial z(\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \frac{\lambda [y_i^{y-1} (\ln t_i)^2 + 2t_i^{y-1} \ln t_i]}{\alpha + \lambda y_i^{y-1}} - \sum_{i=1}^{n} \lambda^2 [y_i^{y-1} \ln t_i + t_i^{y-1}] (\alpha + \lambda y_i^{y-1})^{-2} - \lambda \sum_{i=1}^{n} t_i^{y} (\ln t_i)^2
\]

The absolute values of the difference between the new founded values of parameters and initial values are error terms, where the error are very small\(\epsilon\) and calculate as follows:

\[
\epsilon = \frac{\ln[1 - F(t_i)]}{t_i} + \alpha + \lambda y_i^{y-1}
\]
By applying the formula (3.11), we get:

\[
\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[ \frac{\ln[1 - F(t_i)]}{t_i} + \alpha + \lambda t_i^{\gamma - 1} \right]^2
\]  
(3.12)

Now, deriving the equation (3.12) with respect to the unknown parameters \((\alpha, \lambda, \gamma)\), we get three functions which respectively as follows:

\[
f(\alpha) = \frac{\partial \varepsilon_i^2}{\partial \alpha} = 2 \sum_{i=1}^{n} \frac{\ln[1 - F(t_i)]}{t_i} + 2n \alpha + 2\lambda \sum_{i=1}^{n} t_i^{\gamma - 1}
\]

\[
g(\lambda) = \frac{\partial \varepsilon_i^2}{\partial \lambda} = 2 \sum_{i=1}^{n} t_i^{\gamma - 2} \ln[1 - F(t_i)] + 2\alpha \sum_{i=1}^{n} t_i^{\gamma - 1} + 2\lambda \sum_{i=1}^{n} t_i^{2(\gamma - 1)}
\]

\[
z(\gamma) = \frac{\partial \varepsilon_i^2}{\partial \gamma} = 2\lambda \sum_{i=1}^{n} t_i^{\gamma - 2} \ln t_i \ln[1 - F(t_i)] + 2\alpha \sum_{i=1}^{n} t_i^{\gamma - 1} \ln t_i + 2\lambda^2 \sum_{i=1}^{n} t_i^{2(\gamma - 1)} \ln^2 t_i
\]

These three functions are the system of nonlinear equations and cannot solve it simulateaneasily, so we can solve it by employing Newton-Raphson method such as in maximum likelihood method by applying the Jacobean matrix as in equation (3.6), then:

\[
\frac{\partial f(\alpha)}{\partial \alpha} = 2n
\]

\[
\frac{\partial f(\alpha)}{\partial \lambda} = 2 \sum_{i=1}^{n} t_i^{\gamma - 1}
\]

\[
\frac{\partial f(\alpha)}{\partial \gamma} = 2\lambda \sum_{i=1}^{n} t_i^{\gamma - 1} \ln t_i
\]

\[
\frac{\partial g(\lambda)}{\partial \alpha} = 2 \sum_{i=1}^{n} t_i^{\gamma - 1}
\]

\[
\frac{\partial g(\lambda)}{\partial \lambda} = 2 \sum_{i=1}^{n} t_i^{2(\gamma - 1)}
\]

\[
\frac{\partial g(\lambda)}{\partial \gamma} = 2 \sum_{i=1}^{n} t_i^{\gamma - 2} \ln t_i \ln[1 - F(t_i)] + 2\alpha \sum_{i=1}^{n} t_i^{\gamma - 1} \ln t_i + 4\lambda \sum_{i=1}^{n} t_i^{2(\gamma - 1)} \ln t_i
\]

Where \(F(t_i)\) is empirical cumulative distribution function, then we can find it by using the following formula:
\[
F(t_i) = \frac{i - 0.5}{n}
\]

Also, the Jacobean matrix here are non-singular and symmetric matrix. Now applying the equation (3.6) to get the estimators of the three parameters modified Weibull distribution by ordinary least squares method also the absolute distance values between the next founded values with the last found values are the error term where\(e\) is very small value then we compute error term by equation (3.7).

3.3 Rank Set Sampling Estimator Method:
Let \(t_1, t_2, \ldots, t_n\) be random samples from \(MWD(\alpha, \lambda, \gamma)\), and assume that order statistics obtained by arranging the sample in increasing order where \(t(1), t(2), \ldots, t(n)\) form the theory of order statistics \([3]\). The probability density function (pdf) of \(y_i\) which is an order statistic formulated as follows:

\[
g(y_i; n) = \frac{n!}{(i-1)! (n-i)!} [F(y_i)]^{i-1}[1 - F(y_i)]^{n-i} f(y_i)
\]

\[
g(y_i; n) = \frac{n!}{(i-1)! (n-i)!} [\sum_{i=1}^{n} (\alpha + \lambda y_i^{\gamma-1})]^{i-1} [1 - \sum_{i=1}^{n} (\alpha + \lambda y_i^{\gamma-1})]^{n-i} (\alpha
\]

\[
+ \lambda y_i^{\gamma-1}) e^{(-\alpha t_i - \lambda t_i^\gamma)} \ldots (3.13)
\]

The likelihood function of sample \(t(1), t(2), \ldots, t(n)\) are:

\[
L(t(1), t(2), \ldots, t(n); \alpha, \lambda, \gamma)
\]

\[
= k^n \prod_{i=1}^{n} (\alpha
\]

\[
+ \lambda y_i^{\gamma-1}) e^{(-\alpha t_i - \lambda t_i^\gamma)} \ldots (3.15)
\]

where: \(k = \frac{n!}{(i-1)! (n-i)!}\)

Taking the logarithm for likelihood function we get the following function:

\[
\ln L = n \ln k + \sum_{i=1}^{n} \ln (\alpha + \lambda y_i^{\gamma-1})
\]

\[
- \sum_{i=1}^{n} (n - i + 1) \left(\alpha t_i + \lambda y_i^{\gamma-1}\right)
\]

\[
+ \sum_{i=1}^{n} (i - 1) \ln \left(1 - e^{(-\alpha t_i - \lambda y_i^{\gamma})}\right)
\]

The partial derivatives for the log-likelihood function with respect to unknown parameters \(\alpha, \lambda, \gamma\) setting them to zero, the following equations are found:

\[
\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{n} \frac{1}{(\alpha + \lambda y_i^{\gamma-1})}
\]

\[
- \sum_{i=1}^{n} (n - i + 1) t_i
\]

\[
+ \sum_{i=1}^{n} (i - 1) t_i (\frac{y_i^{\gamma-1}}{1 - e^{(-\alpha t_i - \lambda y_i^{\gamma})}})
\]

\[
\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{n} \frac{y_i^{\gamma-1}}{(\alpha + \lambda y_i^{\gamma-1})} - \sum_{i=1}^{n} (n - i + 1) t_i^{\gamma}
\]

\[
+ \sum_{i=1}^{n} (i - 1) t_i^{\gamma} (\frac{y_i^{\gamma-1}}{1 - e^{(-\alpha t_i - \lambda y_i^{\gamma})}})
\]
\[
\frac{\partial \ln L}{\partial \gamma} = \sum_{i=1}^{n} \frac{\lambda [\gamma t_i^{\gamma-1} \ln t_i + t_i^{\gamma-1}]}{\alpha + \lambda t_i^{\gamma-1}} + \sum_{i=1}^{n} \lambda(n - i + 1) t_i^{\gamma} \ln t_i + \sum_{i=1}^{n} \frac{(i - 1) \lambda t_i^{\gamma} \ln t_i e^{(-\alpha t_i - \lambda t_i^{\gamma})}}{1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})}}
\]

\[
\frac{1}{\tilde{\alpha} + \lambda \gamma t_i^{\gamma-1}} - \sum_{i=1}^{n} (n - i + 1) t_i^{\gamma-1} - \sum_{i=1}^{n} (i - 1) t_i^{\gamma} e^{(-\alpha t_i - \lambda t_i^{\gamma})}
\]

\[
\frac{\partial f(\alpha)}{\partial \alpha} = \sum_{i=1}^{n} \frac{-1}{(\alpha + \lambda t_i^{\gamma-1})^2} - \sum_{i=1}^{n} \frac{(i - 1) t_i^{\gamma} e^{(-\alpha t_i - \lambda t_i^{\gamma})}}{(1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})})^2}
\]

These functions are system of three non-linear equations and cannot solve it simultaneously, so we can solve it by Newton-Raphson. Thus, we must found the Jacobean matrix \( J_i \) as in equation (3.6).

\[
\frac{\partial f(\alpha)}{\partial \lambda} = \sum_{i=1}^{n} \frac{\gamma t_i^{\gamma-1}}{(\alpha + \lambda t_i^{\gamma-1})^2} - \sum_{i=1}^{n} \frac{(i - 1) t_i^{\gamma+1} e^{(-\alpha t_i - \lambda t_i^{\gamma})}}{(1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})})^2}
\]

The three-function \( f(\alpha), g(\lambda), z(\gamma) \) are the first derivative of log-likelihood function with respectively to unknown parameters \( \alpha, \lambda, \gamma \) respectively.

\[
f(\alpha) = \frac{1}{\tilde{\alpha} + \lambda \gamma t_i^{\gamma-1}} - \sum_{i=1}^{n} (n - i + 1) t_i^{\gamma-1} - \sum_{i=1}^{n} (i - 1) t_i^{\gamma} e^{(-\alpha t_i - \lambda t_i^{\gamma})}
\]

\[
g(\lambda) = \frac{\gamma t_i^{\gamma-1}}{\tilde{\alpha} + \lambda \gamma t_i^{\gamma-1}} - \sum_{i=1}^{n} (n - i + 1) t_i^{\gamma-1} - \sum_{i=1}^{n} (i - 1) t_i^{\gamma} e^{(-\alpha t_i - \lambda t_i^{\gamma})}
\]

\[
\frac{\partial g(\lambda)}{\partial \alpha} = \sum_{i=1}^{n} \frac{-\gamma t_i^{\gamma-1}}{(\alpha + \lambda t_i^{\gamma-1})^2} - \sum_{i=1}^{n} \frac{(i - 1) t_i^{\gamma+1} e^{(-\alpha t_i - \lambda t_i^{\gamma})}}{(1 - e^{(-\alpha t_i - \lambda t_i^{\gamma})})^2}
\]
\[ \frac{\partial g(\lambda)}{\partial \lambda} = \sum_{i=1}^{n} \frac{-\gamma^2 t_i^{2(y-1)}}{(\alpha + \lambda t_i^{y-1})^2} \]
\[ - \sum_{i=1}^{n} \frac{(i - 1)t_i^y e^{-\alpha t_i - \lambda t_i^y}}{\left(1 - e^{-\alpha t_i - \lambda t_i^y}\right)^2} \]
\[ \frac{\partial g(\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \frac{\alpha t_i^{y-1} \ln t_i + \alpha t_i^{y-1}}{(\alpha + \lambda t_i^{y-1})^2} - \sum_{i=1}^{n} \frac{(n - i + 1)t_i^y \ln t_i}{\left(1 - e^{-\alpha t_i - \lambda t_i^y}\right)^2} \]
\[ + \sum_{i=1}^{n} \frac{(i - 1)t_i^y \ln t_i \left[-\lambda e^{-\alpha t_i - \lambda t_i^y} + e^{-(\alpha t_i - \lambda t_i^y)} - e^{2(\alpha t_i - \lambda t_i^y)}\right]}{\left(1 - e^{-\alpha t_i - \lambda t_i^y}\right)^2} \]
\[ \frac{\partial g(\gamma)}{\partial \alpha} = \sum_{i=1}^{n} \frac{-\lambda \left[y t_i^{y-1} \ln t_i + t_i^{y-1}\right]}{(\alpha + \lambda t_i^{y-1})^2} \]
\[ - \sum_{i=1}^{n} \frac{\lambda(i - 1)t_i^{y+1} \ln t_i e^{-(\alpha t_i - \lambda t_i^y)}}{\left(1 - e^{-\alpha t_i - \lambda t_i^y}\right)^2} \]
\[ \frac{\partial g(\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \frac{\lambda \left[\alpha + \lambda t_i^{y-1}\right] \left[y t_i^{y-1} \ln t_i^2 + 2t_i^{y-1} \ln t_i\right]}{(\alpha + \lambda t_i^{y-1})^2} \]
\[ - \sum_{i=1}^{n} \frac{\lambda \left[y t_i^{y-1} \ln t_i + t_i^{y-1}\right]}{(\alpha + \lambda t_i^{y-1})^2} \]
\[ - \sum_{i=1}^{n} \frac{\lambda(i - 1)t_i^y \left(\ln t_i^2\right)^2}{\left(1 - e^{-\alpha t_i - \lambda t_i^y}\right)^2} \]
\[ + \sum_{i=1}^{n} \frac{\lambda(i - 1)t_i^y \ln t_i e^{-(\alpha t_i - \lambda t_i^y)} - t_i^y \ln t_i e^{2(\alpha t_i - \lambda t_i^y)}}{\left(1 - e^{-\alpha t_i - \lambda t_i^y}\right)^2} \]

Also, the Jacobean matrix is non-singular and symmetric matrix because finding it depending upon the first derivative. Now, by applying the equation (3.6), we get the estimators of three-parameter MWD(\(a, \lambda, \gamma\)) by using Rank set sampling method. The absolute value of the difference between the next founded value with the last founded value is the error term, then we can find the error term by the equation (3.7)

4-Result and discussion:

In this paper, depending on real data for the Leukemia disease, choosing this type of disease because it is widespread and deadly in time in Iraq and this type of diseases has failure time (death time) occurs which is interesting phenomenon in this paper. To collect data for the Leukemia disease, returning the Baghdad general hospital. The time of study point in this paper determined form (1-4-2012) until (31-12-2012), that means the duration time of study is constant and fixed for (9) months or (275) days. The number of patients in the experiment for the above duration time is (50) patients. (22) Patients left the hospital and any follow-up could not be done for them, but all (28) patients were dead during the time of study. When applying the test statistic (chi-square) in order to fit MWD(\(a, \lambda, \gamma\)) data, it is discovered that the calculated value is (9.331645), when comparing
this value with tabulated value (14.07) we find out that the calculated value is less than the tabulated value at level of significant (0.05) with (7) degree of freedom that means the data is distributed according to MWD( \( \alpha, \lambda, \gamma \) ).

4.1 - Estimation the parameters:
In this section, we shall use MLEM, OLSEM and RSSEM to estimate the three parameters in MWD( \( \alpha, \lambda, \gamma \) ) for complete data. Applying the Newton-Raphson method to estimate the parameters which requires the initial values through assuming them. Trying and considering many initial values of the three parameters in MWD( \( \alpha, \lambda, \gamma \) ), which gives us best results with smallest value of error term and smallest number of iterations. The assumed initial value for three parameters are follows:

| Table (3-1) |
| Initial values of parameters |

<table>
<thead>
<tr>
<th>Initial value of MLE</th>
<th>Initial value of OLS</th>
<th>Initial value of RSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 = 0.046 )</td>
<td>( \alpha_0 = 0.122 )</td>
<td>( \alpha_0 = 0.002 )</td>
</tr>
<tr>
<td>( \lambda_0 = 0.578 )</td>
<td>( \lambda_0 = 0.109 )</td>
<td>( \lambda_0 = 0.015 )</td>
</tr>
</tbody>
</table>

By using MATLAB program, we've got the following estimated parameters values for MLEM, OLSEM and RSSEM

| Table (3-2) |
| Estimated values for the parameters |

<table>
<thead>
<tr>
<th>Estimate values of MLEM</th>
<th>Estimate values of OLSEM</th>
<th>Estimate values of RSSEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\alpha} = 0.0781 )</td>
<td>( \hat{\alpha} = 0.0233 )</td>
<td>( \hat{\alpha} = 0.0065 )</td>
</tr>
<tr>
<td>( \hat{\lambda} = 0.2462 )</td>
<td>( \hat{\lambda} = 0.0756 )</td>
<td>( \hat{\lambda} = 0.008 )</td>
</tr>
<tr>
<td>( \hat{\gamma} = 0.9442 )</td>
<td>( \hat{\gamma} = 1.3764 )</td>
<td>( \hat{\gamma} = 1.994 )</td>
</tr>
</tbody>
</table>

Then computing the numerical values for probability death density function \( f(t) \), survival function \( s(t) \) and hazard function \( h(t) \) for MLEM, OLEM and RSSEM

| Table (4-3) |
| Estimated Values for Functions \( f(t) \), \( s(t) \), \( h(t) \) for MLE Method |

\[
\begin{array}{ccc}
\gamma_0 &=& 0.523 \\
\gamma_0 &=& 1.959 \\
\gamma_0 &=& 1.809 \\
\end{array}
\]
<table>
<thead>
<tr>
<th>Failure Time /days</th>
<th>$\hat{f}(t)$</th>
<th>$\hat{g}(t)$</th>
<th>$\hat{h}(t)$</th>
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<tbody>
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Table (4-4)
Estimated values for functions $f(t)$, $s(t)$, $h(t)$ for OLSE method
<table>
<thead>
<tr>
<th>Failure Time /days</th>
<th>( \hat{f}(t) )</th>
<th>( \bar{s}(t) )</th>
<th>( \hat{h}(t) )</th>
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Table (3-5)  
Estimated values for function \( f(t) \), \( s(t) \), \( h(t) \) for RSSE method
<table>
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<tr>
<th>Fuiler Time /days</th>
<th>( \hat{f}(t) )</th>
<th>( \hat{s}(t) )</th>
<th>( \hat{h}(t) )</th>
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5-conclusions

Note we can make the following mention about the results above tables:

1. The values of death density function $f(t)$ of maximum likelihood method are decreasing progressively with the increasing of the failure time for the Leukemia patients in the hospital, that means there is an opposite relationship between failure times and death density function. While the estimate values of death functions for ordinary least squares method are increasing until ($t = 60$) then it became decreasing to the end of failure times, but the estimated values of death density function for rank set sampling are increasing until failure time ($t = 228$) then the values become decreasing until the end of failure time.

2. For all estimation methods, observing that the estimate values of probability survival functions are decreasing with the increasing of failure times, which means that there are all estimation methods an opposite relationship between failure times and probability survival functions.

3. For maximum likelihood estimation methods, noting that the estimate values of probability hazard functions are decreasing with increasing of failure times, that means there is an opposite relationship between failure times and probability hazard functions, it is known that the value of $h(t)$ depends on the shape parameter values such that ($\hat{\gamma} = 0.9442$) thus, the hazard function is decreasing function as ($t$) increasing when ($\hat{\gamma} < 1$), while ordinary least squares method and the rank set sampling method are increasing with increasing failure times, that means there is a direct relationship between failure times and probability hazard functions it is known that the value of $h(t)$ depends on the shape parameter values such that ($\hat{\gamma} = 1.3764$) of OLSEM ($\hat{\gamma} = 1.994$) of RSSEM, thus, the hazard function is increasing function as ($t$) increasing when ($\hat{\gamma} > 1$).

References


(3) S. A . Al-Hadhrami ,(2010)," Parameters Estimation of the Modified Weibull Distribution"
Based on Ranked Set Sampling". *European Journal of Scientific Research* ISSN 1450-216X Vol.44 No.1.pp.73-78.


**Parameters estimation for…**

**Based on Ranked Set Sampling**

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