The primal element in integral domain

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Abstract

An element $x$ of an integral domain $R$ is called primal if whenever $x$ divides a product $a_1a_2$ with $a_1, a_2 \in R$, $x$ can be written as $x = x_1x_2$ such that $x_i$ divides $a_i$, $i = 1, 2$. We study when in $X^2$ primal in $A + X B[X]$ or $A + X B[[X]]$, when $A \subseteq B$ be an extension of domains. Also we show that if $A$ is an integral domain and $S \subseteq A$ a splitting multiplicative system, then $A + XA_S[X]$ is a semirigid GCD-domain if and only if $A$ is a semirigid GCD-domain and for each two elements of $S$, one of them divides the other.


Introduction:

Let $A \subseteq B$ be an extension of integral domain and $X$ an indeterminate. In this paper, we study some arithmetic properties of the subring $A + X B[X]$ (resp. $A + X B[[X]]$) of $B[X]$ (resp. $B[[X]]$). According to (P. M. Cohn)\(^{(1)}\), an element $x$ of an integral domain $R$ is called primal if whenever $x$ divides a product $a_1a_2$ with $a_1, a_2 \in R$, $x$ can be written as $x = x_1x_2$ such that $x_i$ divides $a_i$, $i = 1, 2$ (an element whose divisors are primal elements is called completely primal). A domain $R$ is called GCD-Domain if every pair of elements of $R$ has a greatest common divisor. Let $A$ be a domain and $S \subseteq A$ a saturated multiplicative system of $A$. A nonzero element $a \in A$ is said to be LCM-prime to $S$\(^{(2)}\), if $aA \cap tA = atA$ (equivalently $tA : a = tA$) for each $t \in S$. $S$ is said to be a splitting multiplicative system \(^{(3)}\), if each nonzero element $x$ of $A$ can be
written as $x = a_s$, where $a$ is LCM-prime to $S$ and $s \in S$. As in $^{(3)}$, an extension of rings $A \subseteq B$ is called inert if whenever $xy \in A$ for nonzero $x, y \in B$, then $xu, yu^{-1} \in A$ for some $u \in U(B)$.

An element $x$ of an integral domain $R$ is called an extractor $^{(4)}$, if $xR \cap yR$ is a principal ideal for each $y \in R$.

In Section 1, we prove that $X$ is primal in $A + X B[X]$ or $A + X B[[X]]$ if and only if $B = A_S$ and $S$ is good, where $S = U(B) \cap A$ (we say that $S$ is good if whenever $s \in S$, $a, b \in A \setminus \{0\}$ and $s \mid_A ab$, there exists $t \in S$ such that $t \mid_A a$ and $s \mid_A tb$). If $n \geq 2$ and $S = U(B) \cap A$, we prove that $X^n$ is primal in $A + X B[X]$ or $A + X B[[X]]$ if and only if $S$ is good, $A_S = B \cap Q(A)$ and for each $b \in B$ there exists $c \in U(B)$ such that $bc \in A$. We also include some remarks about the goodness of a multiplicative system.

In Section 2 we study when $A + X B[X]$ is a semirigid GCD-domain. We recall that, according to (M. Zafrullah $^{1975,1987,1988}$)$^{(5,6,7)}$, an element $x$ of integral domain $R$ is called rigid if whenever $r, s \in R$ and $r, s \mid x$, we have $s \mid r$ or $r \mid s$. Also $R$ is called semirigid if every nonzero element of $R$ can be expressed as a product of a finite number of rigid elements. We show that if $A$ is an integral domain and $S \subseteq A$ a splitting (saturated) multiplicative system, then $A + X A_S[X]$ is a semirigid GCD-domain if and only if $A$ is a semirigid GCD-domain and for each two elements of $S$, one of them divides the other.

Throughout, all rings are commutative with unit element and subrings have the same unit element. If $A$ is a domain, then $U(A)$ denotes the set of invertible elements of $A$ and $A_S$ denotes the quotient ring of $A$ with respect to the multiplicative $S$. Any unexplained notation or terminology is standard as in (R. Gilmer)$^{(8,9)}$.

1. Primal elements
In this section we study the primality of \(X^n\) in domain of type \(A + X B[X]\) or \(A + X B[[X]]\). When \(n = 1\), this primality forces \(B\) to be a fraction ring of \(A\), that is:

**Remark 1.1.** If \(A \subseteq B\) is an extension of domains and \(X\) is primal in \(A + X B[X]\) or \(A + X B[[X]]\), then \(B = A_S\) where \(S = U(B) \cap A\). Indeed, if \(R\) denotes \(A + X B[X]\) or \(A + X B[[X]]\) and \(0 \neq b \in B\), then \(X\) divides \((bX)^2\) in \(R\). So, there exist \(f, g, u, v \in R\), \(f(0) \neq 0\), such that \(X = f g\) and \(bX = fu = gv\). If \(g'\) denotes the (formal) derivative of \(g\), then \(1 = f(0)g'(0)\), so \(f(0) \in S\) and \(b = g'(0)v(0) = v(0)/f(0) \in A_S\).

The next result describes the primality of \(X\) in \(A + X B[X]\) or \(A + X B[[X]]\).

When \(f\) is a nonzero power series (polynomial), the order of \(f\) is denoted by \(\text{ord}(f)\).

**Theorem 1.2:** Let \(A\) be a domain and \(S \subseteq A\) a saturated multiplicative system. The following assertions are equivalent:

(a) \(X\) is primal in \(A + X A_S [X]\),

(b) \(X\) is primal in \(A + X A_S[[X]]\),

(c) If \(s \in S\), \(a, b \in A\) and \(s \mid ab\), there exists \(t \in S\) such that \(t \mid a\) and \(s \mid tb\).

(let us agree to say that \(S\) is good if it satisfies property (c)).

Proof: Set \(R = A + X A_S[X]\) or \(A + X A_S[[X]]\). First, we prove that ((a) or (b)) implies (c). Let \(s \in S\) and \(a, b \in A\) such that \(s \mid ab\). Then \(X \mid_R ab\). Then \(X \mid_R a(bX/s)\), so there exists \(t \in S\) such that \(X = t(X/t)\), \(t \mid_R a\) and \((X/t) \mid_R (bX/s)\).

So \(t \mid_A a\) and \(s \mid_A bt\). Conversely, we prove, that (c) implies ((a) and (b)). Assume that \(X \mid_R fg\) with \(f, g \in R \setminus \{0\}\) and \(\text{ord}(f) \leq \text{ord}(g)\). If \(\text{ord}(g) \geq 2\), then \(X \mid_R g\). If \(\text{ord}(f) = \text{ord}(g) = 1\), then \(0 \neq g'(0) = b/s\) with \(b \in A, s \in S\), hence \(X = s(X/s)\) and \(s \mid_R f, (X/s) \mid_R g\) (again, \(g'\) denotes the formal derivative of \(g\)). If \(\text{ord}(f) = 0\) and \(\text{ord}(g) = 1\), then \(0 \neq f(0) = a \in A\), \(0 \neq g'(0) = b/s\) with \(b \in A, s \in S\) and \(ab/s \in A\). Since \(S\) is good, there
exists $t \in S$ such that $a/t, bt/s \in A$.
Hence $X = (X/t)$ and $t | f, (X/t) | g$.

**Remark 1.3** : Let $A$ be a domain and $S \subseteq A$ a saturated multiplicative system of $A$.

(a). If $S$ is a splitting multiplicative system, then $S$ is good. Indeed, assume that $s \mid ab$ with $s \in S$ and $a,b \in A$. We can write $a = a't$ with $t \in S$ and $a'$ LCM-prime to $S$, so $s \mid bt$.

(b). If $S$ is consisting of (completely) primal elements, then $S$ is good. Indeed, if $s \mid ab$ with $s \in S$ and $a,b \in A$, then $s = tu$ (so $t,u \in S$), such that $t \mid a$ and $u \mid b$, hence $s \mid bt$.

(c). If $A$ is atomic and $S$ is good, then $S$ is splitting. Indeed, it suffices to show that each atom $a$ not belonging to $S$ is LCM-prime to $S$. If $b \in A$, $s \in S$ and $s \mid ab$, then $s \mid b$, because $S$ is good and $a$ has only unit divisors in $S$. As a specific example, we can consider the atomic domain $A = k[X^2, XY,Y^2]$, where $k$ is a field, $X,Y$ are indeterminate and $S$ is the saturated multiplicative system generated by $X^2$.

Here the atom $XY$ is not LCM-prime to $S$, because both $X^2, XY$ divide $(XY)^2$ but their product does not.

(d). $S$ is good if and only if for each $a \in A$, $aA \cap S \cap A = \bigcup_{t \in S} t \mid a \bigcup_{a \in A} (a/t)A$. Indeed, to see that the condition is necessary, let $x \in aA \cap S \cap A$. Then, there exists $x = ab / s$, so there exists $s \in S$ such that $t \in S$ such that $a/t, tb/s \in A$.

(e). Consequently, $S$ is splitting if and only if $S$ is good and every nonzero $a \in A$ has a divisor $t \in S$, such that any other divisor $w \in S$ of $a$ divides $t$.

(f). Let $T$ be the $m$-complement of $S$ (4). Then, cf. (10) each nonzero prime ideal $P$ disjoint of $S$ contains an LCM-prime
to S element if and only if the saturation of ST is $A \setminus \{0\}$ (that is, for each nonzero $x$, there exists $x'$ in $A$, $s \in S$, $t \in T$ such that $xx' = st$). If $S$ is good and every prime ideal $P$ disjoint of $S$ contains an LCM-prime to $S$ element, then $S$ is splitting. Indeed, if $x$ is a nonzero element of $A$, then $xx' = st$ for some $x' \in A$, $s \in S$, $t \in T$. So $s \mid xx'$ and, since $S$ is good, $w \mid x$ and $s \mid wx'$ for some $w \in S$. Now since $T$ is saturated and $t = (x/w)(wx'/s)$, $x/w \in T$, so $x = w(x/w)$. That is $S$ is splitting.

**Corollary 1.4:** Let $A$ be a domain and $S$ a saturated multiplicative system. Then $X$ is completely primal in $A + X_{A S}[X]$ or $A + X_{A S}[[X]]$ if and only if $S$ is consisting of (completely) primal elements of $A$.

**Proof:** Let $R$ denote $A + X_{A S}[X]$ or $A + X_{A S}[[X]]$. Notice that, if $s \in S$, $s$ is primal in $A$ if and only if $s$ is primal in $R$. Indeed, if $s$ is primal in $A$ and $s \mid_R fg$, $f, g \in R$, then $s \mid_A f(0)g(0)$, hence $s$ can be written as $s = tu$ with $t, u \in S$ such that $t \mid_A f(0)$, $u \mid_A g(0)$, thus $t \mid_R f$ and $u \mid_R g$. Since the divisors of $X$ in $R$ are of type $s$ or $X/s$ with $s \in S$ and since $R$ has an automorphism sending $X/s$, the assertion of Corollary 1.4 follows from Remark 1.3 (b).

The next result describes the primality of $X^n$ in $A + X B[X]$ or $A + X B[[X]]$, when $n \geq 2$. Here, if $B$ is a domain and $b \in B$, $b U(B) = \{ bw; w \in U(B)\}$.

**Theorem 1.5:** Let $A \subseteq B$ be an extension of domains, $S = U(B) \cap A$ and $K$ the quotient field of $A$. Let $R$ denote $A + X B[X]$ or $A + X B[[X]]$. The following statements are equivalent:

(a). $X^2$ is primal in $R$,
(b). $X^n$ is primal in $R$ for some $n \geq 2$,
(c). $X^n$ is primal in $R$ for each $n \geq 2$,
(d). $b U(B) \cap A \neq \emptyset$ for each $b \in B$ is good and $A_S = B \cap K$,
(e). $b U(B) \cap A \neq \emptyset$ for each $b \in B$ and whenever $a \in A$, $b \in B$ are nonzero elements such that $ab \in A$, there exists $t \in S$ such that $at^1, bt \in A$. 

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Remark 1.6: Let $A \subseteq B$ be an extension of domain. If condition (d) of Theorem 1.5 holds, then $A \subseteq B$ is an inert extension. Indeed, let $b_1, b_2 \in B \setminus \{0\}$ such that $b_1b_2 = a \in A$. Since $b_1 U(B) \cap A = \emptyset$, there exists $u \in U(B)$ such that $a' = b_1u^{-1} \in A$. Therefore $a = b_1b_2 = (b_1u^{-1})(b_2u) = a'(b_2u)$. Since (d) holds, there exists $t \in S$ such that $b_1u^{-1}t^{-1} = b_1(ut)^{-1} \in A$ and $b_2(ut) \in A$.

Proof of Theorem 1.5: For a nonzero $f \in R$, let $\alpha(f)$ denote the first nonzero coefficient of $f$. In order to show that (e) implies (d), let $a, b \in A$ and $s \in S$, such that $s \mid Aab$. By the second condition of (e) applied for $b/s \in B$, there exists $t \in S$ such that $at^{-1} \in A$, $bt/s \in A$. Let $b = c/a \in B$, where $a, c \in A$ are nonzero. So $ba = c \in A$, hence, by (e), there exists $t \in S$ such that $bt \in A$, thus $b \in A_S$. The inclusion $A_S \subseteq B$ is obvious.

Conversely, let $a \in A$, $b \in B$ be nonzero elements such that $c = ab \in A$. Then $b = c/a \in K \cap B = A_S$. So there exist $s \in S$, $d \in A$ such that $b = d/s$. Since $S$ is good, there exists $t \in S$ such that $a' \in A$ and $bt = (d/s)t \in A$.

(b) $\Rightarrow$ (e). In order to see that the second part of (e) holds, let $a \in A$, $b \in B$ nonzero elements such that $ab = c \in A$. Then $X^n \mid_R a(X^n)$, so $X^n = fg$, $a = ff'$, $bX^n = gg'$ for some $f, f', g' \in R$ of order 0 and $g \in R$ of order $n$. Then $t = \alpha(f) \in S$, because $\alpha(f)\alpha(g) = 1$. Also $a = \alpha(f)\alpha(g')$, so $t \mid_{Aa}$, and $b = \alpha(g')\alpha(g)$, hence $bt = \alpha(g') \in A$. To show the first part of (e), let $0 \neq b \in B$. Then $X^n \mid_R (bX^n)$, so $X^n = fg$, $bX^n = ff'$, $bX = gg'$, for some $f, f', g, g' \in R$. Then $1 = \alpha(f)\alpha(g)$ and $b = \alpha(f)\alpha(g') = \alpha(g\alpha(g'))$ with $\alpha(g') \in A$ or $\alpha(g') \in A$.

Hence $\alpha(f)\alpha(g) \in U(B)$ and of the elements $b\alpha(f)$, $b\alpha(g)$, belongs to $A$. (a) $\Rightarrow$ (e) can be proved similarly. (c) $\Rightarrow$ (b) and (c) $\Rightarrow$ (a) are trivial. (e) $\Rightarrow$ (c).

Assume that $f, g \in R$ are nonzero, $X^n \mid_R fg$ and $\text{ord}(g) = j \geq \text{ord}(f) = i$. Obviously, if $j \geq n + 1$, then $X^n \mid_R g$.

We consider the following three cases.
If $i = 0$ and $j = n$, then $\alpha(f) \in A$, $\alpha(g) \in B$ and $\alpha(f)\alpha(g) \in A$. By (e), there exists $t \in S$ such that $t^{-1}\alpha(f)$, $t\alpha(g) \in A$. Hence $X^n = t( t^{-1}X^n)$ with $t \mid_R f$ and $t^{-1}X^n \mid_R g$, because $t$ divides each nonconstant monomial in $f$. If $i + j = n$ and $i \geq 1$, then $\alpha(f), \alpha(g) \in B$ and $\alpha(f), \alpha(g) \in A$. By Remark 1.6 there exists $w \in U(B)$ such that $w\alpha(f) \in A$ and $w^{-1}\alpha(g) \in A$. Hence $X^n = (w^{-1}X^i)(wX^i)$ with $w^{-1}X^i \mid_R f$ and $wX^i \mid_R g$. If $i + j \geq n + 1$ and $i \geq 1$, then $J \geq 2$ and $\alpha(f), \alpha(g) \in B$. By (e), there exists $c \in U(B)$ such that $\alpha(f)c \in A$. Hence $X^n = (c^{-1}X^i)(cX^{i-1})$ with $c^{-1}X^i \mid_R f$ and $cX^{i-1} \mid_R g$.

**Example 1.7:**

(i). Let $A$ be a domain and $S \subseteq A$ a multiplicative system. In $A + XA[X]$, $X$ is primal if and only if $X^2$ is primal.

(ii). Let $K$ be a field and $A$ a subring of $K$. Then $X^2$ is primal in $A + XK[X]$, but $X$ is primal in $A + XK[X]$ if and only if $K = Q(A)$. For instance, in the ring $Z + X R[X]$, $X^2$ is primal, but $X$ is not primal.

(iii). $X^2$ is not primal in $Z + X Z[\sqrt{2}][X]$.

(iv). If $A' \subseteq B$ is an extension of domains such that $bU(B) \cap A' \neq \emptyset$ for each $b \in B$, then $X^2$ is primal in $A + X B[X]$, where $A = Q(A') \cap B$. Indeed, $Q(A) \cap B = A$, whence $U(B) \cap A = U(A)$.

We recall that an element $x$ of integral domain in $R$ is said to be an extractor if $Rx \cap Ry$ is principal for each $y \in R \setminus \{0\}$. Obviously each extractor is primal. By (D. D. Anderson et al 1995, Theorem 4.1) if $x$ is an extractor, then so is every divisor of $x$.

By the proof of (T. Dumitrescu, Proposition 2.11), a completely primal element $x$ is an extractor if and only if for each $y$, the elements $x.y$ have a maximal common divisor (abbrev. MCD), that is, a common
divisor \( z \) such that \( x / z \) and \( y / z \) are relatively prime.

**Proposition 1.8:** Let \( A \) be a domain and \( S \subseteq A \) a saturated multiplicative system of \( A \). Then \( X \) is an extractor in \( A + X A_S[X] \) or \( A + X A_S[[X]] \) if and only if \( S \) is splitting and consists of extractors.

Proof: Let \( R \) denote \( A + X A_S[X] \) or \( A + X A_S[[X]] \). Assume that \( X \) is an extractor in \( R \). By the above remark, the elements of \( S \) are extractors in \( R \), hence extractors in \( A \). By Remark 1.3 (b), \( S \) is good. Let \( a \in A \) be nonzero and \( f = \gcd_R(X,a) \). Then \( f(0) = t \in S \) and if \( w \mid A a \) with \( w \in S \), then \( w \mid A a , X \), so \( w \mid R t \), hence \( w \mid A t \). By Remark 1.3(e), \( S \) is splitting.

Conversely, assume that \( S \) is splitting and consists of extractors. By Corollary 1.4, \( X \) is completely primal in \( R \). So, by the remarks made before Proposition 1.8, it suffices to see that \( X, f \) have an MCD, for each nonzero \( f \in R \). If \( a = f(0) \neq 0 \), \( a \) can be written \( a = bs \) with \( s \in S \) and \( b \) LCM-prime to \( S \). Thus \( s \) is an MCD of \( X \) and \( f \) in \( R \). If \( \operatorname{ord}(f) = 1 \), say \( f = (a/t)X + c_2X^2 + c_3X^3 + \ldots \) with \( a \in A \), \( t \in S \), we write again \( a = bs \) with \( s \in S \) and \( b \) LCM-prime to \( S \). Since \( s,t \) are extractors, there exists \( w \in S \) such that \( (s,t) = Aw \). Then \( (w/t)X \) is an MCD of \( X \) and \( f \) in \( R \).

**Corollary 1.9:** Let \( A \) be a domain and \( S \subseteq A \) a saturated multiplicative system of \( A \). Then \( A + X A_S[X] \) is GCD-domain if and only if \( X \) is an extractor in \( A + X A_S[X] \) and \( A_S \) is a GCD-domain.

Proof: Apply proposition 1.8.

2. **Semirigid GCD-Domains**

Let \( R \) be an integral domain. We recall that a element \( x \in R \) is said to be rigid, if whenever \( r,s \in R \) and \( r,s \mid x \), we have \( r \mid s \) or \( s \mid r \). So an irreducible element is obviously rigid (we recall that a nonzero nonunit \( x \in R \) is called irreducible if whenever \( y \in R \) and \( y \mid x \), we have \( y \mid 1 \) or \( x \mid y \). Then \( R \) is called
semirigid if every nonzero element of $R$ can be expressed as a product of a finite number of rigid elements. So, each atomic domain, that is a domain in which every nonzero nonunit element is a product of irreducible elements, is semirigid.

**Lemma 2.1:** Let $A$ be a domain, $S \subseteq A$ a saturated multiplicative system and $R = A + x A S[X]$.

(a). $X$ is rigid in $R$ if and only if for every $r,s \in S$, $r | A s$ or $s | A r$. If $X$ is rigid in $R$ and $s \in S$, then $X/s$ is rigid in $R$.

(b). A nonzero element $a \in A$ is rigid in $A$ if and only if $a$ is rigid in $R$.

Proof: Is a consequence of the following remarks. The set of all divisors of $X$ in $R$ is $S \cup \{ X/s ; s \in S \}$ and any nonzero $a \in A$ has the same divisors in $A$ and $R$. Also if $b,c \in A$, then $b | A c$ if and only if $b | A c$. When $s \in S$, $R$ has an $A$-algebra automorphism sending $X$ into $X/s$.

Let $A$ be a domain and $S \subseteq A$ a saturated multiplicative system. In [D. Costa et al., Theorem 1.1]^{12}, is show that $A + X A S[X]$ is a GCD-domain if and only if $A$ is a GCD-domain and $S$ is splitting. Our next result characterizes the semirigid GCD-domain of type $A + X A S[X]$.

**Theorem 2.2:** Let $A$ be a domain and $S \subseteq A$ a splitting (saturated) multiplicative system. Then $A + X A S[X]$ is a semirigid GCD-domain if and only if $A$ is a semirigid GCD-domain and for every $s,t \in S$, $s | A t$ or $t | A s$.

Proof: Set $R = A + X A S[X]$. Using Lemma 2.1, and the comments above, it suffices to show that $R$ is semirigid provided $A$ is a semirigid GCD-domain and $X/s$ is rigid in $R$ for each $s \in S$. We use freely Lemma 2.1. Let $f \in R$ be a nonzero polynomial of minimal degree among all nonzero polynomials of $R$ which cannot be written as product of
rigid elements of $R$. Then any divisor of $f$ in $R$ is constant or have the same degree with $f$. Since a rigid element of $A$ is still rigid in $R$, it follows that $f \notin A$.

If $f(0) = 0$, then $f = (X/s)a$ for some $s \in S$ and $a \in A$. This contradicts the choice of $f$ because $X/s$ is rigid and $A$ semirigid. If $f(0) \neq 0$, then factoring out from $f$ an appropriate element of $A$, we may assume that

$$f = a_0 + (a_1/s)X + \ldots + (a_n/s)X^n$$

with $a_i \in A$, $s \in S$ and $\gcd(a_0, \ldots, a_n) = 1$. Moreover, since $S$ is splitting, we may assume that $a_0$ is LCM-prime to $S$. Then $f$ has no nonunit factor in $A$, thus $f$ is irreducible, a contradiction.

There is a certain resemblance between the proofs of Theorem 2.2 and [T. Dumitrescu, Corollary 1.8]^{(11)}.

Corollary 2.3: If $A$ is a semirigid GCD-domain, so is the polynomial ring $A [X]$.

**Corollary 2.4:** [M. Zafrullah, Example 4]^{(6)} If $V$ is a valuation domain with quotient field $K$, then $V + X K[X]$ is a semirigid GCD-domain.

**Corollary 2.5:** If $A$ is a semirigid GCD-domain and $p \in A$ a prime element such that $\cap_{n \geq 1} p^n A = 0$ (e.g. if $A$ is a factorial domain and $p \in A$ a prime), then $A + X A[1/p][X]$ is a semirigid GCD-domain.

Proof: The saturated multiplicative system generated by $p$ is splitting cf^{(3)}.

**Example 2.6:** Iterating the preceding corollary, we obtain successively that the following rings are semirigid GCD-domains

$$A_1 = Z + X Z[1/2][X_1], \ A_2 = A_1 + X_2 A[1/3][X_2], \ldots,$$

$$A_n = A_{n-1} + X_n A_{n-1}[1/p_{n-1}][X_n],$$

Where $p_n$ is the $n$th prime number.

Indeed, if $q$ is a prime distinct from $p_1$, $p_2$, $\ldots$, $p_n$, then

$$A_n/q A_n \cong (A_{n-1}/q A_{n-1})[X_n] \cong \cdots \cong (Z/q Z)[X_1, X_2, \ldots, X_n]$$

and
∩_{k≥1}q^kA_0 \subseteq \cap_{k≥1}q^kZ/\langle \langle \langle v/p_1,v/p_2,\ldots,v/p_n \rangle \rangle \rangle [X_1,X_2,\ldots,X_n] = 0 \).

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الملخص

يُدعى العنصر في الساحة العددية R عنصرًا أساسيًا عندما x يقسم حاصل ضرب الأعداد R حيث a1, a2∈R، بحيث أن x يقسم a1 في كل i=1,2. يمكن كتابة x كشكل x1=x2x3 عندما يكون عنصرًا أساسيًا. يتناول البحث x التي يقسمها عنصرًا أساسيًا X في A+B[X] أو في [A][X]. حيث أن A⊆B. حيث أن A+B[X] هو توسعة للساحات وكذلك تبين الدراسة أنه إذا كان A ساحة عددية وA+X[x] هو نظام ضربي متخلق، فإن S⊆A عددية وGCD A+X[x] حيث S صارم إذا وفقط إذا كان A ساحة عددية عددية. GCD A+X[x] S صارم وكل عنصر في S أحدهما يقسم الآخر.