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1. Introduction

In general, a wheeled mobile robot system considered a multi-input multi-output nonlinear dynamic and time variant system and the one of fundamental problems in the control engineering is the motion control design to track the desired path [1] because the mobile robot has many applications in various life directions such as: science; education; industry; entertainment; security and military therefore the mobile robot is still active region of research [2].

There are different types of control algorithms based on kinematics and dynamics mathematical model of the wheeled mobile robot are proposed to solve the motion control of the mobile robot in order to follow the desired trajectory with high performance of the controller in terms of generating optimal control action that lead to minimizing the pose error during tracking reference path, such as nonlinear neural PID controllers [3], fuzzy logic and PID controllers [4], neural networks controllers [5], adaptive fuzzy with back-stepping controllers [6], adaptive sliding mode controllers [7], neural predictive controllers [8] and nonlinear fractional order PID Neural Controller [9].

The motivation for this research is taken from [2, 3, 4, 8, 9] in order to track and stabilize the mobile robot on the desired path and to generate best torque control action without saturation state and no spike action. The main core of the contribution novelty of this research is described as follows.

- High computational accuracy that has been derived of the control law in order to find the best torque action that lead to minimizing tracking pose error of the wheeled mobile robot.
- Using different types of cognitive optimization algorithms to show the ability in the fast search in local and global region in order to find and tune on-line the best parameters of the neural controller based on the proposed hybrid algorithm.
- Adding a dynamic disturbance to investigate the robustness performance of the proposed controller.
- Changing the initial pose state to verify the adaptation performance of the proposed controller.
- Tracking a variable radius continuous trajectory to validate the capability of the proposed controller.

The organization of this paper can be described as follows: Section two is a description of the dynamic wheeled mobile robot model. Section three is deriving the proposed nonlinear neural controller. In section four, the different types of the cognitive optimization algorithms is explained especially the proposed hybrid Bees-PSO algorithm. Section five presents the simulation results of the proposed hybrid Bees-PSO algorithm.

2. Model of dynamic wheeled mobile robot

Figure 1 shows the schematic diagram of the cart wheeled mobile robot. It consists of a two DC motors, which is driving the two wheels with one
or two and omni-directional caster wheels that will stabilize the platform of the mobile robot [7]. The motion and orientation of the mobile robot depends on two independent actuators (DC motors) for left and right wheels, \( r \) is the radius of each wheel and the distance between the two wheels is \( L \) and \( c \) is the centre of gravity of the mobile robot.

In general, the global coordinate frame is defined as \([0, x, y]\) while the pose vector of the mobile robot in the surface is defined as:

\[
q = (x, y, \theta)^T
\]  
(1)

The coordinates of \( x \) and \( y \) are at point \( e \) while the orientation angle \( \theta \) is measured with respect to global frame in the \( X \)-axis therefore the configuration of mobile robot can be described by these three generalized coordinates. To investigate the motion and orientation of the wheeled mobile robot, two conditions should be achieved; the first is pure rolling and the second is non-slipping in order to make the mobile robot’s lateral velocity is equal to zero in equation (2) [3].

\[
-x(t)\sin \theta(t) + y(t)\cos \theta(t) = 0
\]  
(2)

Then, the equations of the kinematics wheeled mobile robot in the world frame can be represented as follows [10]:

\[
\dot{x}(t) = \frac{r(w(t) + w(t))\cos \theta(t)}{2}
\]  
(3)

\[
\dot{y}(t) = \frac{r(w(t) - w(t))\sin \theta(t)}{2}
\]  
(4)

\[
\dot{\theta}(t) = \frac{r(w(t) - w(t))}{L}
\]  
(5)

Based on Euler Lagrange formulation [6], the dynamic model of the mobile robot can be described as follows:

\[
M \ddot{q} + C(q, \dot{q}) + G(q) = \tau \]

(6)

In the computer simulation, the pose equations of the mobile robot can be described as follows:

\[
F(k) = \frac{\tau_L(k) + \tau_R(k)}{r} 
\]  
(7)

\[
\tau_a(k) = 0.5r[\tau_L(k) - \tau_R(k)]
\]  
(8)

\[
Acc_{Lin}(k) = \frac{F(k)}{M}
\]  
(9)

\[
Acc_{ang}(k) = \frac{\tau_a(k)}{r}
\]  
(10)

\[
V_{Lin}(k) = Acc_{Lin}(k) \ast \Delta t
\]  
(11)

\[
V_{ang}(k) = Acc_{ang}(k) \ast \Delta t
\]  
(12)

\[
wR(k) = \frac{0.5[2V_{Lin}(k)+V_{ang}(k)L]}{r}
\]  
(13)

\[
wL(k) = \frac{0.5[2V_{Lin}(k)-V_{ang}(k)L]}{r}
\]  
(14)

\[
x(k) = 0.5r[wR(k) + wL(k)]\cos \theta(k)\Delta t + x(k-1)
\]  
(15)

\[
y(k) = 0.5r[wR(k) + wL(k)]\sin \theta(k)\Delta t + y(k-1)
\]  
(16)

\[
\theta(k) = \frac{r}{L}[wR(k) - wL(k)]\Delta t + \theta(k-1)
\]  
(17)

Where \( \tau_L \) is the left wheel torque, \( \tau_R \) is the right wheel torque; \( M \) is the mass of the mobile robot; \( I \) is the inertia of the mobile robot; \( \lambda \) is the constraint forces; \( \tau_d \) is bounded dynamics disturbances; \( F(k) \) is linear total force; \( \tau_a(k) \) is total angular torque; \( Acc_{Lin}(k) \) is linear acceleration; \( Acc_{ang}(k) \) is angular acceleration; \( V_{Lin}(k) \) is linear velocity; \( V_{ang}(k) \) is angular velocity; \( wr(t) \) and \( wL(t) \) are the right and left angular velocities respectively; \( x(k), y(k), \theta(k) \) are the pose of the mobile robot at each step \( k \) of the movement and \( \Delta t \) is the sampling time.

3. Nonlinear Neural Controller Methodology

In general, the feedback control signal based on neural controller is very important in the structure of the proposed controller in terms of stabilizing and minimizing the tracking pose error of the wheeled mobile robot during the mobile robot’s pose is drifted from the reference path. Block diagram of the proposed structure of the nonlinear neural trajectory-tracking controller for wheeled mobile robot is shown in Figure 2.

Figure (3) shows the structure of the proposed nonlinear neural controller.
The proposed nonlinear neural control structure for the right and left torques is as follows:

\[
\tau_r(k) = B_1[e_1(k)] + B_2[e_1(k) + e_1(k-1)] + B_3[e_1(k) - e_1(k-1)]
\]

\[
\tau_l(k) = B_4[e_2(k)] + B_5[e_2(k) + e_2(k-1)] + B_6[e_2(k) - e_2(k-1)]
\]

For the outputs \( O_\alpha, O_\gamma, \) and \( O_\theta \) of the neural networks, sigmoid function is used as nonlinear relationship as in equation (20) [3]:

\[
\sigma(r) = \frac{2}{1 + e^{-net_r}} - 1
\]

Where \( \gamma = \alpha, \gamma, \theta \).

\( net_r \) is calculated from this equation:

\[
net_r(k) = B_1[e_1(k)] + B_2[e_1(k) + e_1(k-1)] + B_3[e_1(k) - e_1(k-1)]
\]

(21)

The control parameters B1, B2, … and B15 are online updated by using different types of cognitive optimization algorithms.

4. Cognitive Optimization Algorithms

The purpose of the on-line cognitive optimization algorithms are to find and tune the optimal gains control for the proposed controller to generate the best and smooth torque signal that will lead to minimizing the tracking pose error of the wheeled mobile robot during dynamic disturbance has been added. In this research, three types of the optimization algorithms are used, there are Bees, PSO and the proposed hybrid Bees-PSO algorithms.

Bees Algorithm

In this research, to find and tune the parameters of the proposed controller of the trajectory tracking for wheeled mobile robot, Bees algorithm is used. In general, this algorithm mimics the food foraging behavior of swarms of honey bees and this algorithm carries out by using local (neighborhood) search that has two types of Bees (Selected and Recruit Bees) and global (random) search that has also two types of Bees (Scout and Fittest Bees) [11-12]. The Bees algorithm’s steps can be described as follows:

- Scout Bees (n) are generated in the global (random) search as the initial population with randomly values of the controller’s parameters
- Calculated the fitness of the (n) Scout Bees by using the fitness equation as [10]:

\[
\text{fitness}(e(k)) = \sum_{i=1}^{n} e_i(k)^2
\]
\[
\text{fitness} = \frac{1}{\mu + \text{Cost Function}} \tag{24}
\]

Where:
\( \mu > 0 \) to avoid division by zero.

- The proposed cost function is a mean square error as in equation (25).
- \( \text{MSE} = \frac{1}{N} \sum_{i=1}^{N} ((x_{\text{ref}} - x)^2 + (y_{\text{ref}} - y)^2 + (\theta_{\text{ref}} - \theta)^2 + (V_{\text{lin ref}} - V_{\text{lin}})^2 + (W_{\text{ref}} - W)^2) \) \tag{25}

\( N \): is the number of iteration.

- Selected Bees (m) are chosen as the highest fitness for the Scout Bees (n) to use in a local search.
- The size of (patch size) the local search is determined by applying the proposed equations (26) as follows:
- \( \Delta \beta = 0.025 \times \beta_{\text{old}} \times \text{random}(0,1) \) \tag{26}
- \( \beta_{\text{new}} = \beta_{\text{old}} + \Delta \beta \) \tag{27}

Where
- \( \beta \) is the vector of the fifteen parameters of the proposed controller.
- Recruit Bees are generated by using equations (26 and 27) to search and find the best controller’s parameters.
- Fittest Bees are chosen the highest fitness for the Recruit Bees by using equation (24).
- Return to global search by assign the (n-m) remaining Bees to random search and generating a new population of Scout Bees.

These steps of the on-line optimization algorithm for the controller’s parameters are repeated for each sample.

**PSO Algorithm**

In general, the particle swarm optimization (PSO) is considered one of the algorithms type that has a capability to search for the optimal solution by simulating the movement and flocking of birds. In PSO algorithm, the particles that have a position and velocity are a population of individuals in order to move around the search space to find best or optimal solution through evaluation of each particle by using a fitness function [3].

In PSO algorithm, \( pbest \) is the previous best value and it is related only to a particular particle and in the swarm, the best value of all the particles \( pbest \) is \( gbest \). Fifteen weights parameters for the proposed nonlinear neural controller and random initialized all particles and updated the position and velocity of all particles using equations (28 and 29) to find and tune on-line the control gains of proposed controller:

\[
\Delta B_{m}^i = \Omega B_{m} + c_{1}r_{1}(pbest_{i}-B_{m}) + c_{2}r_{2}(gbest-B_{m}) \tag{28}
\]

\[
\bar{B}_{m} = B_{m} + \Delta B_{m} \tag{29}
\]

where \( m = 1, 2, 3, ..., \text{pop} ; \text{pop} \) is particles number; \( B_{m}^i \) is particle’s weight \( m \) at \( k \) iteration; \( \Omega \); is the inertia weight factor; \( c_1 \) and \( c_2 \) are the positive values where \( (c_1+c_2<4) \); \( r_1 \) and \( r_2 \) are random values between 0 and 1.

The PSO algorithm’s steps can be described as follows:

- Particles (n) are generated in the local search as the initial population with randomly values of the controller’s parameters.
- Evaluating the proposed cost function of each particle by using the mean square error equation (25).
- Set to \( pbest \) for each particle in the current searching point. The best-evaluated value of \( pbest \) is set to \( gbest \) and the particle number with the best value is stored.
- The \( pbest \) value is replaced by the current value if the value is better than the current \( pbest \) of the particle and if the best value of \( pbest \) is better than the current \( gbest, gbest \) is replaced by the best value and the particle number with the best value is stored.
- Each particle is update by using Eqs. (28 and 29).
- Return to step two if the current iteration number did not reach to the predefined maximum iteration number, otherwise exit.

These steps of the on-line optimization algorithm for the controller’ parameters are repeated for each sample.

**Hybrid Bee-PSO (HBPSO) Algorithm**

In this paper, the proposed on-line optimization algorithm is a hybrid algorithm, which consists of Bees algorithm [11-12] and PSO algorithm [3]. The main advantages for this proposed optimization algorithm are to solve the problem of local search of the Bees algorithm by using PSO algorithm which has the high ability for local search through generating population (Recruit Bees) as particles and this will speed up the process of optimization while the problem of the global search in the PSO algorithm is solved by using Bees algorithm which has the capability of combining the (Fittest Bees) and a new population of (Scout Bees) generating in the global search. The proposed on-line HBPSO algorithm is applied as a powerful optimization algorithm to find and tune the best stable the nonlinear neural controller’s parameters to enhance the performance characteristics of the system by reducing the processing time as well as improve the response accuracy through minimizing the tracking error for mobile robot.

The proposed HBPSO algorithm’s steps can be described as follows:
Scout Bees (n) are generated in the global (random) search as the initial population with randomly fifteen values of the controller’s parameters.

Calculated the fitness of the (n) Scout Bees by using the fitness equation (24) based cost function.

In the local search, chosen the number of particles depends on the number of the highest fitness for the Scout Bees (population) as (m) Selected Bees.

The size of (patch size) of the local search is determined by applying the proposed equations (26).

Particles are generated by using equations (27) to search and find the best controller’s parameters.

Evaluating the proposed cost function of each particle by using the mean square error equation (25).

Set to \( p_{best} \) for each particle in the current searching point. The best-evaluated value of \( p_{best} \) is set to \( g_{best} \) and the particle number with the best value is stored.

The \( p_{best} \) value is replaced by the current value if the value is better than the current \( p_{best} \) of the particle and if the best value of \( p_{best} \). \( g_{best} \) is replaced by the best value and the particle number with the best value is stored.

Each particle is updated by using Eqs. (28 and 29).

Return to step six if the current iteration number did not reach to the predefined maximum iteration number otherwise pick out the highest fitness for the particles as Fittest Bees (m).

Return to global search by assigning the (n-m) remaining Bees to random search and generating a new population of Scout Bees.

In general, these steps of the on-line optimization algorithm for finding and tuning control parameters are repeated at 0.1 second (sampling time) for each sample. The proposed flow chart of the on-line HBPSO tuning control algorithm is shown in Figure 4.

5. Simulation Results

MATLAB package used to verify the proposed nonlinear neural controller of the trajectory tracking for the dynamic model of the wheeled mobile robot. The specifications of the Eddie mobile robot platform are picked from [13]: \( M=12\text{kg}; I=1.536\text{kg.m}^2; r=0.075\text{m} \) and \( L=0.39\text{m} \). The MATLAB simulation is carried out on-line cognitive optimization algorithm with proposed controller as shown in Figure 2 to track reference pose with continuous various radius path and 0.1 sec is sampling time.

In this paper, the three types of the optimization algorithms is used (Bees; PSO and HBPSO) to show which of them is better in term of finding and tuning the parameters of the controller that lead to generating optimal and smooth torque control action and minimizing the tracking error.
The first optimization method is Bees algorithm that will define the following parameters as follows: The Scout Bees (n) is equal to 10; the Selected Bees (m) is equal to 5; the Recruit Bees is equal to 40; the Fittest Bees is equal to 5; the weights in each Bee is equal to 15 and the best iteration number (N) is equal to 25.

The second optimization method is PSO algorithm that will define the following parameters as follows: The particle is equal to 10; the weights in each particle is 15; the inertia weight factor is equal to 0.65; the acceleration constants c1 and c2 are the positive values equal to 1.27; the random number r1 and r2 are random numbers between 0 and 1 and the best iteration number is equal to 25.

The third optimization method is hybrid Bees-PSO algorithm that will define the following parameters as follows: The Scout Bees (n) is equal to 10 in the global search; the Selected Bees is equal to 5; the particle is equal to 40 in the local search; the Fittest Bees is equal to 5; the best iteration number (N) in the global search is equal to 3 while the best iteration number (P) in the local search is equal to 3.

Case Study

The reference wheeled mobile robot pose trajectory is described in these equations (30, 31 and 32):

\[ x_i(t) = \sin\left(\frac{2\pi}{20}\right) \]  
\[ y_i(t) = \sin\left(\frac{2\pi}{40}\right) \]  
\[ \theta_i(t) = 2\tan^{-1}\left(\frac{\Delta y_i(t)}{\sqrt{(\Delta x_i(t))^2 - (\Delta y_i(t))^2 + \Delta x_i(t)}}\right) \]

The mobile robot has initial pose as \( q(0) = [-0.1, 0, \pi/4] \). After applying the structure of the proposed controller with three different types of the cognitive optimization algorithms with adding small values of the dynamic disturbances as the term that taken from [8] \( \bar{\omega} = [0.01\sin(2t) \ 0.01\sin(2t)]^T \) to show the controller ability of robustness through the on-line adapting the parameters of the controller which could eliminated these effect.

The trajectory tracking for mobile robot model can be shown in Figure 5 which it is clearly, the excellent tracking performance when the optimization algorithm was hybrid Bees-PSO algorithm because this proposed algorithm has a capability to perfect search in two search space (local and global) with minimum number of iteration in both search space in spite of adding disturbances to the system.

Figure 6 demonstrates the orientation tracking performance of the mobile robot with three types of optimization algorithms.

In these on-line control algorithms, the proposed Mean Square Error (MSE) clearly improved the performance of the controller by showing the pose error convergence for the mobile robot motion at 200 steps, as shown in Figure 7.

Investigating the on-line tuning control algorithm based hybrid Bees-PSO it appears to be better than other algorithms in this paper in terms of the effects of iteration number in the local and global search and summing all the values of on-line mean square error as shown in Figure 8.

It is observed that the best value of the iteration in the global search equal to 3 and the best value of the iteration in the local search equal to 3, these values lead to minimum mean square error which it is equal to 0.0847 therefore the total number of the iteration in proposed HBPSO algorithm is equal to \((3 \times 3 = 9)\) while the best number of the iteration in the Bees algorithm and PSO algorithm is equal to 25.

Figure 9 shows the effectiveness of the proposed nonlinear neural controller response through generating smooth torque control action without saturation control action state to track the desired path in minimum time.

To verify the platform Eddie wheeled mobile robot needs to minimum power that will drive
the actuators through the response of angular velocity of the left and right wheels were smoothness without sharp spikes, as shown in Figure 10.

Figures 11 (a, b, c) show the robustness and adaptation performance of the proposed controller in terms of keeping on minimum tracking pose error for the wheeled mobile robot and stabilizing the pose of the mobile robot when the mobile robot tries to drift from the desired path because of the effect of the bounded dynamic disturbances to the system.

6. Conclusions

The simulation results on the on-line cognitive optimization algorithms of nonlinear neural controller are presented in this paper for the dynamic wheeled mobile robot model, which shows preciousness, and that the proposed hybrid Bees-PSO tuning control algorithm has the following capabilities of:

- Fast and stable on-line finding and tuning the parameters of the controller with minimum number of iteration in the local and global search.
- Obtaining a smooth and best torque control action, without spikes as well as no saturation torque action state.
• Minimizing the pose error and no output oscillation for the wheeled mobile robot during path-tracking.
• Strong adaptability and robustness performance when dynamic disturbances have been added to the mobile robot.

References

Author biography
I have finished my B.Sc. degree in Control and Systems Eng. from the University of Technology in 1997 and M.Sc. degree in Mechatronics Engineering for the University of Technology in 2001. I have finished my PhD degree in Electronics and Computers in the School of Engineering and Design, Brunel University, London, UK in 2012. Currently I am a head of Mechatronics Eng. Dept. at University of Technology, Baghdad - Iraq. My current project is Cognitive Methodologies.