Characterization of Soft Semi Separation Axioms in Soft Quad Topological Spaces

Arif Mehmood Khattak¹, Saleem Abdullah², Asad Zaighum¹, Fahad Jamal³, Muhammad Afzal Rana¹, Zaheer Anjum⁴, Muhammad Zamir, Muhammad Ishfaq³
¹Department of Mathematics and Statistics, Riphah International University Islamabad, Pakistan.
²Department of Mathematics, Abdul Wali Khan University Mardan, Pakistan.
³Department of Mathematics, Qurtuba University Peshawar, Pakistan.
⁴Department of Mathematics, University of Sciences and Technology Bannu, Pakistan.

Abstract
Our main interest in this study is to look for soft semi separations axioms in soft quad topological spaces. We talk over and focus our attention on soft semi separation axioms in soft quad topological spaces with respect to ordinary points and soft points. Moreover study the inherited characteristics at different angles with respect to ordinary points and soft points. Some of their central properties in soft quad topological spaces are also brought under examination.

Keywords: soft sets, soft topology, soft semi open set, soft semi closed set, soft quad topological space, soft $qT_0$ structure, soft $qT_1$ structure, soft $qT_2$ structure, soft $qT_3$ structure and soft $qT_4$ structure.

1. Introduction.

In real life condition the complications in economics, engineering, social sciences, medical science etc. we cannot handsomely use the old-fashioned classical methods because of different types of uncertainties existing in these problems. To finish out these complications, some types of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, in which we can safely use a mathematical methods for dealing with uncertainties. But, all these theories have their inherent worries. To overcome these difficulties in the year 1999, Russian scholar Molodtsov [1] introduced the idea of soft set as a new mathematical methods to deal with uncertainties. Which is free from the above difficulties. J.C Kelly [2] studied Bi topological spaces and discussed different results.

Recently, in 2011, M. Shabir and M. Naz [3] initiated the idea of soft topological space and discussed different results with respect to ordinary points. they beautifully defined soft topology as a collection of $\tau$ of soft sets over $X$. they also defined the basic concept of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axioms, soft regular and soft normal spaces and published their several performances. Soft separation axioms are also discussed at detail. Aktas and N. In the recent years, many interesting applications of soft sets theory and soft topology have been discussed at great depth [4 -26] explained Soft results included connectedness via soft ideal developed soft set theory. A. Kandil et al [27] launched Soft regularity and normality based on semi open soft sets and soft ideals.

In[28-33] discussion is launched soft semi hausdorff spaces via soft ideals, semi open and semi closed sets, separation axioms, decomposition of some type supra soft sets and soft continuity are discussed.

In this present paper, concept of soft semi separation axioms in Soft quad topological spaces is announced with respect to ordinary and soft points.

Many mathematicians made discussion over soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open, soft $\alpha$-open set and soft $\beta$-open

*Email: mehdaniyal@gmail.com
set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present article hand is tried and work is encouraged over the gap that exists in soft quad-topology related to soft semi \( qT_0 \), soft semi \( qT_1 \), soft semi \( qT_2 \), soft semi \( qT_3 \) and soft semi \( qT_4 \) structures. Some propositions in soft quad topological spaces are discussed with respect to ordinary points and soft points. When we talk about distances between the points in soft topology then the concept of soft separation axioms will automatically come in force. That is why these structures are catching our attentions. We hope that these results will be valuable for the future study on soft quad topological spaces to accomplish general framework for the practical applications and to solve the most complicated problems containing doubts in economics, engineering, medical, environment and in general mechanic systems of various varieties. In future these beautiful soft topological structures may be extended in to soft n-topological spaces provided \( n \) is even.

2. Preliminaries

The following Definitions which are pre-requisites for present study

**Definition 1:** [4].

Let \( X \) be an initial universe of discourse and \( E \) be a set of parameters. Let \( P(X) \) denotes the power set of \( X \) and \( A \) be a non-empty sub-set of \( E \). a pair \((F, A)\) is called a soft set over \( U \), where \( F \) is a mapping given by \( F: A \rightarrow P(X) \)

In other words, a set over \( X \) is a parameterized family of sub set of universe of discourse \( X \). For any \( e \in A, F(e) \) may be considered as the set of e-approximate family of the soft set \((F, A)\) and if \( e \notin A \) then \( F(e) = \emptyset \) that is \( F_A = \{F(e): e \in A \subseteq E, F: A \rightarrow P(X)\} \) the family of all these soft sets over \( X \) denoted by \( SS(X)_A \)

**Definition 2:** [4].

Let \( F_A, G_B \in SS(X)_E \) then \( F_A \), is a soft subset of \( G_B \) denoted by \( F_A \subseteq G_B \), if

1. \( A \subseteq B \) and
2. \( F(e) \subseteq G(e), \forall e \in A \)

In this case \( F_A \) is said to be a soft subset of \( G_B \) and \( G_B \) is said to be a soft super set \( F_A, G_B \supseteq F_A \)

**Definition 3:** [5].

Two soft subsets \( F_A \) and \( G_B \) over a common universe of discourse set \( X \) are said to be equal if \( F_A \) is a soft subset of \( G_B \) and \( G_B \) is a soft subset of \( F_A \)

**Definition 4:** [6].

The complement of soft subset \((F, A)\) denoted by \((F, A)^C\) is defined by \((F, A)^C = (F^C, A)\)

\( F^C \rightarrow P(X) \) is a mapping given by \( F^C(e) = U - F(e), \forall e \in A \) and \( F^C \) is called the soft complement function of \( F \). Clearly \((F^C)^C\) is the same as \( F \) and \((F, A)^C)^C = (F, A)\)

**Definition 5:** [7].

The difference between two soft subset \((G, E)\) and \((G, E)\) over common of universe discourse \( X \) denoted by \((F, E) - (G, E)\) is the soft set \((H, E)\) where for all \( e \in E, \emptyset \) or \( \emptyset_A \) if \( \forall e \in A, F(e) = \emptyset \)

**Definition 6:** [7].

Let \((G, E)\) be a soft set over \( X \) and \( x \in X \) We say that \( x \in (F, E) \) and read as \( x \) belong to the soft set \((F, E)\) whenever \( x \in F(e) \forall e \in E \) The soft set \((F, E)\) over \( X \) such that \( F(e) = \{x\} \forall e \in E \) is called singleton soft point and denoted by \( x \) or \( (x, E) \)

**Definition 7:** [7].

A soft set \((F, A)\) over \( X \) is said to be Null soft set denoted by \( \emptyset \) or \( \emptyset_A \) if \( \forall e \in A, F(e) = \emptyset \)

**Definition 8:** [7].

A soft set \((F, A)\) over \( X \) is said to be an absolute soft denoted by \( \bar{A} \) or \( X_A \) if \( \forall e \in A, F(e) = X \)

Clearly, we have, \( X_A^C = \emptyset_A \) and \( \emptyset_A^C = X_A \)

**Definition 9:** [8].

The soft set \((F, A) \in SS(X)_A\) is called a soft point in \( X_A \), denoted by \( e_F \), if for the element \( e \in A, F(e) \neq \emptyset \) and \( F(e^t) = \emptyset \) if for all \( e^t \in A - \{e\} \)

**Definition 10:** [8].
The soft point \( e_F \) is said to be in the soft set \((G,A)\), denoted by \( e_F \in (G,A) \) if for the element \( e \in A, F(e) \subseteq G(e) \).

**Definition 11:** [8].

Two soft sets \((G,A),(H,A)\) in \(SS(X)_A\) are said to be soft disjoint, written \((G,A) \cap (H,A) = \emptyset \) if \(G(e) \cap H(e) = \emptyset \) for all \( e \in A \).

**Definition 12:** [8].

The soft point \( e_G, e_H \) in \(X_A\) are disjoint, written \( e_G \neq e_H \) if their corresponding soft sets \((G,A)\) and \((H,A)\) are disjoint.

**Definition 13:** [8].

The union of two soft sets \((F,A)\) and \((G,B)\) over the common universe of discourse \(X\) is the soft set \((H,C)\), where, \( C = A \cup B \) For all \( e \in C \)

\[ H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in (B - A) \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases} \]

Written as \((F,A) \cup (G,B) = (H,C)\)

**Definition 14:** [8].

The intersection \((H,C)\) of two soft sets \((F,A)\) and \((G,B)\) over common universe \(X\), denoted \((F,A) \cap (G,B)\) is defined as \( C = A \cap B \) and \( H(e) = F(e) \cap G(e), \forall e \in C \)

**Definition 15:** [8].

Let \((F,E)\) be a soft set over \(X\) and \(Y\) be a non-empty subset of \(X\). Then the sub soft set of \((F,E)\) over \(Y\) denoted by \((Y,F,E)\), is defined as follow \( Y_{F(a)} = Y \cap F(a), \forall a \in E \) in other words \((Y,F,E) = Y \cap (F,E)\).

**Definition 16:** [9].

Let \( \tau \) be the collection of soft sets over \(X\), then \( \tau \) is said to be a soft topology on \(X\), if
1. \( \emptyset, X \) belong to \( \tau \)
2. The union of any number of soft sets in \( \tau \) belongs to \( \tau \)
3. The intersection of any two soft sets in \( \tau \) belong to \( \tau \)

The triplet \((X,\tau,E)\) is called a soft topological space.

**Definition 17:** [9].

Let \((X,F,E)\) be a soft topological space over \(X\), then the member of \( \tau \) are said to be soft open sets in \(X\).

**Definition 18:** [9].

Let \((X,F,E)\) be a soft topological space over \(X\). A soft set \((F,A)\) over \(X\) is said to be a soft closed set in \(X\) if its relative complement \((F,A)^C\) belong to \(\tau\).

**Definition 19:** [16].

A soft set \((A,E)\) in a soft topological space \((X,\tau,E)\) will be termed soft semi open (written S.S.O) if and only if there exists a soft open set \((O,E)\) such that \((O,E) \subseteq (A,E) \subseteq Cl(O,E)\).

**Proposition 1.** Let \((X,\tau,E)\) be a soft topological space over \(X\). If \((X,\tau,E)\) is soft semi \(T_3\)-space, then for all \( x \in X, x_E = (x,E) \) is semi-closed soft set.

**Proposition 2.** Let \((Y,\tau_Y,E)\) be a soft sub space of a soft topological space \((X,\tau,E)\) and \((F,E) \in SS(X)\) then
1. If \((F,E)\) is soft semi open set in \(Y\) and \(Y \in \tau\), then \((F,E) \in \tau\)
2. \((F,E)\) is soft semi open soft set in \(Y\) if and only if \((F,E) = Y \cap (G,E)\) for some \((G,E) \in \tau\).
3. \((F,E)\) is soft semi closed soft set in \(Y\) if and only if \((F,E) = Y \cap (H,E)\) for some \((H,E) \in \tau\) soft semi closed.

**3. SOFT SEMI SEPARATION AXIOMS OF SOFT QUAD TOPOLOGICAL SPACES**

In this section we introduced soft Semi separation axioms in soft quad topological space with respect to ordinary points and discussed some results with respect to these points in detail.

**Definition 20:**

Let \((X,\tau_1,E), (X,\tau_2,E), (X,\tau_3,E)\) and \((X,\tau_4,E)\) be four different soft topologies on \(X\). Then \((X,\tau_1,\tau_2,\tau_3,\tau_4,E)\) is called a soft quad topological space. The soft four topologies \((X,\tau_1,E),\)
(X, τ₂, E), (X, τ₃, E) and (X, τ₄, E) are independently satisfying the axioms of soft topology. The members of τ₁ are called τ₁ soft open set. And complement of τ₁ Soft open set is called τ₁ soft closed set. Similarly, the members of τ₂ are called τ₂ soft open sets and the complement of τ₂ soft open sets are called τ₂ soft closed set. The members of τ₃ are called τ₃ soft open set. And complement of τ₃ Soft open set is called τ₃ soft closed set and the members of τ₄ are called τ₄ soft open set. And complement of τ₄ Soft open set is called τ₄ soft closed set.

Definition 21:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft quad topological space over X and Y be a non-empty subset of X. Then τ₁Y' = {(Y, E); (F, E) ∈ τ₁}, τ₂Y' = {(Y, E); (G, E) ∈ τ₂}, τ₃Y' = {(H, E); (H, E) ∈ τ₃} and τ₄Y' = {(I, E); (I, E) ∈ τ₄} are said to be the relative topological on Y. Then (Y, τ₁Y, τ₂Y, τ₃Y, τ₄Y, E) is called relative soft quad topological space of (X, τ₁, τ₂, τ₃, τ₄, E). Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft quad topological space over X, where (X, τ₁, E), (X, τ₂, E), (X, τ₃, E) and (X, τ₄, E) be four different soft topologies on X. Then a sub set (F, E) is said to be quad-open (in short hand q-open) if (F, E) ⊆ τ₁ ∪ τ₂ ∪ τ₃ ∪ τ₄ and its complement is said to be soft q-closed.

3.1 SOFT SEMI SEPARATION AXIOMS OF SOFT QUAD TOPOLOGICAL SPACES WITH RESPECT TO ORDINARY POINTS.

In this section we introduced soft semi separation axioms in soft quad topological space with respect to ordinary points and discussed some attractive results with respect to these points in detail.

Definition 22:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft quad topological space over X and x, y ∈ X such that x \neq y
If we can find soft q-open sets (F, E) and (G, E) such that x ∈ (F, E) and y \notin (F, E) or y ∈ (G, E) and x \notin (G, E) then (X, τ₁, τ₂, τ₃, τ₄, E) is called soft qT₀ space.

Definition 23:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft quad topological space over X and x, y ∈ X such that x \neq y
If we can find two soft q-open sets (F, E) and (G, E) such that x ∈ (F, E) and y \notin (F, E) or y ∈ (G, E) and y \notin (G, E) then (X, τ₁, τ₂, τ₃, τ₄, E) is called soft qT₁ space.

Definition 24:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft quad topological space over X and x, y ∈ X such that x \neq y
If we can find two q-open soft sets such that x ∈ (F, E) and y ∈ (G, E) moreover (F, E) ∩ (G, E) = φ Then (X, τ₁, τ₂, τ₃, τ₄, E) is called a soft qT₂ space.

Definition 25:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft topological space (G, E) be q-closed soft set in X and x ∈ X such that x \notin (G, E). If there occurs soft q-open sets (F₁, E) and (F₂, E) such that x ∈ (F₁, E), (G, E) ⊆ (F₂, E) and (F₁, E) ∩ (F₂, E) = φ. Then (X, τ₁, τ₂, τ₃, τ₄, E) is called soft q-regular spaces. A soft q-regular q₁ Space is called soft qT₀ space.

Then (X, τ₁, τ₂, τ₃, τ₄, E) is called a soft q-regular spaces. A soft q-regular q₁ Space is called soft qT₂ space.

Definition 26:
(X, τ₁, τ₂, τ₃, τ₄, E) be a soft quad topological space (F₁, E), (G, E) be closed soft sets in X such that (F, E) ∩ (G, E) = φ. If there exists soft q-open sets (F₁, E) and (F₂, E) such that (F₁, E) ∩ (G, E) ⊆ (F₂, E) and (F₁, E) ∩ (F₂, E) = φ. Then (X, τ₁, τ₂, τ₃, τ₄, E) is called a q-soft normal space. A soft q-normal q₁ Space is called soft qT₂ space.

Definition 27:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft Topological space over X and e₁, e₂, e₃, e₄ ∈ X such that e₁ \neq e₂ if there can happen at least one soft q-open set (F₁, A) or (F₂, A) such that e₁ ∈ (F₁, A), e₂ \notin (F₁, A) or e₃ ∈ (F₂, A), e₃ \notin (F₂, A) then (X, τ₁, τ₂, τ₃, τ₄, E) is called a soft qT₀ space.

Definition 28:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft Topological spaces over X and e₁, e₂, e₃, e₄ ∈ X such that e₁ \neq e₂ if there can happen soft q-open sets (F₁, A) and (F₂, A) such that e₁ ∈ (F₁, A), e₂ \notin (F₁, A) and e₃ ∈ (F₂, A), e₃ \notin (F₂, A) then (X, τ₁, τ₂, τ₃, τ₄, E) is called soft qT₂ space.

Definition 29:
Let (X, τ₁, τ₂, τ₃, τ₄, E) be a soft Topological space over X and e₁, e₂, e₃, e₄ ∈ X such that e₁ \neq e₂ if there can happen soft q-open sets (F₁, A) and (F₂, A) such that e₁ ∈ (F₁, A), and e₂ ∈ (F₂, A)
\[(F_1, A) \cap (F_2, A) = \phi \_A.\text{ Then } (X, \tau_1, \tau_2, \tau_3, \tau_4, E) \text{ is called soft } qT_2 \text{ space.}

**Definition 30:**

Let \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) be a soft topological space \((G, E)\) be q-closed soft set in \(X\) and \(e \in X\) such that \(e \notin (G, E)\). If there occurs soft q-open sets \((F_1, E)\) and \((F_2, E)\) such that \(e \notin (F_1, E), (G, E) \subseteq (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\). Then \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is called soft q-

regular spaces. A soft q-

regular \(qT_2\) Space is called soft \(qT_2\) space.

**Definition 31:**

In a soft quad topological space \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\)

1) \(\tau_1 \cup \tau_2\) is said to be soft semi \(T_0\) space with respect to \(\tau_3 \cup \tau_4\) if for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and a to \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\) such that \(x \in (F, E)\) and \(y \notin (G, E)\) or \(y \in (G, E)\) and \(x \notin (G, E)\). Similarly, \(\tau_3 \cup \tau_4\) is said to be soft semi \(T_0\) space with respect to \(\tau_1 \cup \tau_2\) if for each pair of distinct points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_3 \cup \tau_4\) soft semi open set \((F, E)\) and a to \(\tau_1 \cup \tau_2\) soft semi open set \((G, E)\) such that \(x \in (F, E)\) and \(y \notin (G, E)\) and \(y \in (G, E)\) and \(x \notin (G, E)\). Soft quad topological spaces \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is said to be pair wise soft semi \(T_0\) space if \(\tau_1 \cup \tau_2\) is soft semi \(T_0\) space with respect to \(\tau_3 \cup \tau_4\) and to \(\tau_3 \cup \tau_4\) is soft semi \(T_0\) space with respect to \(\tau_1 \cup \tau_2\).

2) \(\tau_1 \cup \tau_2\) is said to be soft semi \(T_1\) space with respect to \(\tau_3 \cup \tau_4\) if for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and a to \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\) such that \(x \in (F, E)\) and \(y \notin (G, E)\) and \((F, E) \cap (G, E) = \phi\). Similarly, \(\tau_3 \cup \tau_4\) is said to be soft semi \(T_1\) space with respect to \(\tau_1 \cup \tau_2\) if for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_3 \cup \tau_4\) soft semi open set \((F, E)\) and a to \(\tau_1 \cup \tau_2\) soft semi open set \((G, E)\) such that \(x \in (F, E)\) and \(y \notin (G, E)\) and \((F, E) \cap (G, E) = \phi\). Soft quad topological spaces \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is said to be pair wise soft semi \(T_1\) space if \(\tau_1 \cup \tau_2\) is soft semi \(T_1\) space with respect to \(\tau_3 \cup \tau_4\) and to \(\tau_3 \cup \tau_4\) is soft semi \(T_1\) space with respect to \(\tau_1 \cup \tau_2\).

3) \(\tau_1 \cup \tau_2\) is said to be soft semi \(T_2\) space with respect to \(\tau_1 \cup \tau_2\) if for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and a to \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\) such that \(x \in (F, E)\) and \(y \notin (G, E)\) and \((F, E) \cap (G, E) = \phi\). Similarly, \(\tau_3 \cup \tau_4\) is said to be soft semi \(T_2\) space with respect to \(\tau_1 \cup \tau_2\) if for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and a to \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\) such that \(x \in (F, E)\) and \(y \notin (G, E)\) and \((G, E) \cap (F, E) = \phi\). Soft quad topological spaces \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is said to be pair wise soft semi \(T_2\) space if \(\tau_1 \cup \tau_2\) is soft semi \(T_2\) space with respect to \(\tau_3 \cup \tau_4\) and \(\tau_3 \cup \tau_4\) is soft semi \(T_2\) space with respect to \(\tau_1 \cup \tau_2\).

**Definition 32:**

In a soft quad topological space \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\)

1) \(\tau_1 \cup \tau_2\) is said to be soft semi \(qT_3\) space with respect to \(\tau_3 \cup \tau_4\) if \(\tau_1 \cup \tau_2\) is soft semi \(T_1\) space with respect to \(\tau_3 \cup \tau_4\) and for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\) such that \(x \notin (F, E)\), \(\tau_1 \cup \tau_2\) soft semi open set \((F_1, E)\) and \(\tau_3 \cup \tau_4\) soft semi open set \((F_2, E)\) such that \(x \in (F_1, E), (G, E) \subseteq (F_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\). Similarly, \(\tau_3 \cup \tau_4\) is said to be soft semi \(T_3\) space with respect to \(\tau_1 \cup \tau_2\) if \(\tau_3 \cup \tau_4\) is soft semi \(T_3\) space with respect to \(\tau_1 \cup \tau_2\) and for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_3 \cup \tau_4\) soft semi closed set \((G, E)\) such that \(x \notin (G, E)\), \(\tau_1 \cup \tau_2\) soft semi closed set \((F_1, E)\) and \(\tau_3 \cup \tau_4\) soft semi closed set \((F_2, E)\) such that \(\phi \in (F_1, E)\) and \((F_2, E)\) and \((G, E) \cap (F_2, E) = \phi\). Also there exists \(\tau_1 \cup \tau_2\) soft semi closed set \((F_3, E)\) is soft \(\tau_1 \cup \tau_2\) semi open set, \((G_1, E)\) is soft \(\tau_3 \cup \tau_4\) semi open set such that \((G_1, E) \subseteq (F_1, E), (F_2, E) \subseteq (G_2, E)\) and \((F_1, E) \cap (F_2, E) = \phi\). Similarly, \(\tau_3 \cup \tau_4\) is said to be soft semi \(T_3\) space with respect to \(\tau_1 \cup \tau_2\) if \(\tau_3 \cup \tau_4\) is soft semi \(T_3\) space with respect to \(\tau_1 \cup \tau_2\) and for each pair of points \(x, y \in X\) such that \(x \neq y\) there exists \(\tau_1 \cup \tau_2\) soft semi closed set \((F_1, E)\) and \(\tau_3 \cup \tau_4\) soft semi closed set \((F_2, E)\) such that \((F_1, E) \cap (F_2, E) = \phi\). Also there exist \(\tau_1 \cup \tau_2\) soft semi closed set \((F_3, E)\) and \(\tau_3 \cup \tau_4\) soft semi closed set \((G_1, E)\) such that \((F_3, E) \subseteq (G_1, E)\) and \((F_3, E) \cap (G_1, E) = \phi\). Thus, \((X, \tau_1, \tau_2, E)\)
said to be pair wise soft semi $T_4$ space if $\tau_1 \cup \tau_2$ is soft semi $T_4$ space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi $T_4$ space with respect to $\tau_1 \cup \tau_2$.

**Proposition 3.** Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over $X$. Then, if $(X, \tau_1, \tau_2, E)$ and $(X, \tau_3, \tau_4, E)$ are soft semi $T_3$ space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft semi $T_2$ space.

**Proof:** Suppose $(X, \tau_1, \tau_2, E)$ is a soft semi $T_3$ space with respect to $(X, \tau_3, \tau_4, E)$ then according to definition for $x, y \in X$ which distinct, by using Proposition 1, $(Y, E)$ is soft semi closed set in $\tau_1 \cup \tau_4$ and $x \notin (Y, E)$ there exists a $\tau_1 \cup \tau_2$ soft semi open set $(F, E)$ and a $\tau_3 \cup \tau_4$ soft semi open set $(G, E)$ such that $x \in (F, E)$, $y \in (Y, E) \subseteq (G, E)$ and $(F, E) \cap (F_2, E) = \phi$. Hence $\tau_1 \cup \tau_2$ is soft semi $T_2$ space with respect to $\tau_3 \cup \tau_4$. Similarly, if $(X, \tau_3, \tau_4, E)$ is a soft semi $T_3$ space with respect to $(X, \tau_1, \tau_2, E)$ then according to definition for $x, y \in X, x \neq y$, by using Theorem 2, $(E, x)$ is semi closed set in $\tau_1 \cup \tau_2$ and $y \notin (E, x)$ there exists a $\tau_3 \cup \tau_4$ soft semi open set $(F, E)$ and a $\tau_1 \cup \tau_2$ soft semi open set $(G, E)$ such that $y \in (F, E)$ and $x \in (G, E)$ and for each point $x \in X$ and each $(X, \tau_1, \tau_2, E)$ semi closed set $(F_1, E)$ such that $x \notin (F_1, E)$ there exists a $(X, \tau_1, \tau_2, E)$ soft semi open set $(F_1, E)$ and $(X, \tau_3, \tau_4, E)$ soft semi open set $(F_2, E)$ such that $x \in (F_1, E)$, $(G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_3, \tau_4, E)$ is a pair wise soft semi $T_2$ space. This implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft semi $T_2$ space.

**Proposition 4.** Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over $X$. if $(X, \tau_1, \tau_2, E)$ and $(X, \tau_3, \tau_4, E)$ are soft semi $T_3$ space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft semi $T_3$ space.

**Proof:** Suppose $(X, \tau_1, \tau_2, E)$ is a soft semi $T_3$ space with respect to $(X, \tau_3, \tau_4, E)$ then according to definition for $x, y \in X, x \neq y$ there exists a $(X, \tau_1, \tau_2, E)$ soft semi open set $(F, E)$ and a $(X, \tau_3, \tau_4, E)$ soft semi open set $(G, E)$ such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each $(X, \tau_1, \tau_2, E)$ semi closed set $(F_1, E)$ such that $x \notin (F_1, E)$ there exists a $(X, \tau_1, \tau_2, E)$ semi open set $(F_1, E)$ and $(X, \tau_3, \tau_4, E)$ soft semi open set $(F_2, E)$ such that $x \in (F_1, E)$, $(G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly, to $(X, \tau_3, \tau_4, E)$ is a soft semi $T_3$ space with respect to $(X, \tau_1, \tau_2, E)$. So according to definition for $x, y \in X, x \neq y$ there exists a $(X, \tau_3, \tau_4, E)$ soft semi open set $(F, E)$ and $(X, \tau_1, \tau_2, E)$ soft semi open set $(G, E)$ such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each $(X, \tau_3, \tau_4, E)$ semi closed set $(F_1, E)$ such that $x \notin (F_1, E)$ there exists a $(X, \tau_3, \tau_4, E)$ soft semi open set $(F_1, E)$ and $(X, \tau_1, \tau_2, E)$ soft semi open set $(F_2, E)$ such that $x \in (F_1, E)$, $(G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi $T_3$ space.

**Proposition 5.** If $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over $X$. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ and $(X, \tau_3, \tau_4, E)$ are soft semi $T_3$ space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi $T_3$ space.

**Proof:** Suppose $(X, \tau_1, \tau_2, E)$ is soft semi $T_3$ space with respect to $(X, \tau_3, \tau_4, E)$ so accordingly to definition for $x, y \in X, x \neq y$ there exist a $(X, \tau_1, \tau_2, E)$ soft semi open set $(F, E)$ and $a(X, \tau_3, \tau_4, E)$ soft semi open set $(G, E)$ such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ each $(X, \tau_1, \tau_2, E)$ soft semi closed set $(F_1, E)$ and a $(X, \tau_3, \tau_4, E)$ soft semi closed set $(F_2, E)$ such that $(F_1, E) \cap (F_2, E) = \phi$. There exist $(F_3, E)$ and $(G_1, E)$ such that $(F_3, E)$ is soft $(X, \tau_3, \tau_4, E)$ soft semi closed set $(F_1, E)$, $(G_1, E)$ soft semi open set $(F_1, E)$ and $(F_3, E) \cap (F_2, E) = \phi$. Similarly, $(X, \tau_3, \tau_4, E)$ is soft semi $T_3$ space with respect to $\tau_1$ so accordingly to definition for $x, y \in X, x \neq y$ there exists a $(X, \tau_3, \tau_4, E)$ soft semi open set $(F, E)$ and a $(X, \tau_1, \tau_2, E)$ soft semi open set $(G, E)$ such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each $(X, \tau_3, \tau_4, E)$ soft semi closed set $(F_1, E)$ and $(X, \tau_1, \tau_2, E)$ soft semi closed set $(F_2, E)$ such that $(F_1, E) \cap (F_2, E) = \phi$. There exist soft semi open sets $(F_3, E)$ and $(G_1, E)$ such that $(F_3, E)$ is soft $(X, \tau_3, \tau_4, E)$ semi open set $(F_1, E)$, $(G_1, E)$ soft semi open set $(F_1, E)$ and $(F_3, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi $T_3$ space.

**Proposition 6.** Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over $X$ and $Y$ be a non-empty subset of $X$. if $(X, \tau_Y, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi $T_3$ space. Then $(Y, \tau_Y, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi $T_3$ space.

**Proof:** First we prove that $(X, \tau_Y, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft semi $T_3$ space. Let $x, y \in X, x \neq y$ if $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft space then this implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft space. So there exists $(X, \tau_1, \tau_2, E)$ soft semi open $(F, E)$ and $(X, \tau_3, \tau_4, E)$ soft semi open set $(G, E)$ such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ now $x \in Y$ and $x \notin (G, E)$. Hence $x \in Y$ and $(F, E) = (Y, E)$ then $y \notin Y$ for some $\alpha \in E$. This means that $\alpha \in E$ then $y \notin Y$ for some $\alpha \in E$. 

557
Therefore, \( y \not\in Y \cap (F, E) = (Y_f, E) \). Now \( y \in Y \) and \( y \in (G, E) \). Hence \( y \in Y \cap (G, E) = (G_f, E) \) where \((G, E) \in (X, \tau_3, \tau_4, E)\). Consider \( x \not\in (G, E) \) this means that \( \alpha \in E \) then \( x \not\in Y \cap G(\alpha) \) for some \( a \in E \). Therefore \( \not\exists Y \cap (G, E) = (G_f, E) \) thus \((Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)\) is pair wise soft semi \( T_1 \) space.

Now we prove that \((Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)\) is pair wise soft semi \( T_3 \) space then \((Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)\) is pair wise soft semi regular space.

Let \( y \in Y \) and \( (G, E) \) be a soft semi closed set in \( Y \) such that \( y \not\in (G, E) \) where \((G, E) \in (X, \tau_1, \tau_2, \tau_3, \tau_4, E) \) then \((G, E) = (Y, E) \cap (F, E) \) for some soft semi closed set \( \text{int}(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E) \). Hence \( y \not\in (Y, E) \cap (F, E) \) but \( y \in (Y, E) \), so \( y \not\in (F, E) \) since \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is soft semi \( T_3 \) space. 

\((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is soft semi regular space so there exists \((X, \tau_1, \tau_2, E)\) soft semi open set \((F_1, E)\) and \((X, \tau_3, \tau_4, E)\) soft semi open set \((F_2, E)\) such that \(y \in (F_1, E), (G, E) \subseteq (F_1, E)\)  

\((F_1, E) \cap (F_2, E) = \phi \)

Take \((G_1, E) = (Y, E) \cap (F_1, E)\) then \((G_1, E), (G_2, E)\) are soft semi open set in \( Y \) such that \(y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)\) \((G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi \)

Therefore \( \tau_{1Y} \cup \tau_{2Y} \) is soft semi regular space with respect \( \tau_{3Y} \cup \tau_{4Y} \). Similarly, let \( y \in Y \) and \( (G, E) \) be a soft semi closed sub set in \( Y \) such that \( y \not\in (G, E) \) where \((G, E) \in (X, \tau_3, \tau_4, E) \) then \((G, E) = (Y, E) \cap (F, E) \) where \((F, E) \) is some soft semi closed set \( \text{int}(X, \tau_{3Y}, \tau_{4Y}, E) \). \( y \not\in (Y, E) \cap (F, E) \) but \( y \in (Y, E) \) since \((X, \tau_3, \tau_2, E)\) is soft semi regular space so there exists \((X, \tau_3, \tau_4, E)\) soft semi open set \((F_1, E)\) and \((X, \tau_1, \tau_2, E)\) soft semi open set \((F_2, E)\). Therefore \( y \in (F_1, E), (G, E) \subseteq (F_2, E)\) \((F_1, E) \cap (F_2, E) = \phi \)  

Take \((G_1, E) = (Y, E) \cap (F_1, E)\) \((G_1, E) = (Y, E) \cap (F_2, E)\)

Then \((G_1, E)\) and \((G_2, E)\) are soft semi open set in \( Y \) such that \(y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)\) \((G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi \)  

Therefore \( \tau_{3Y} \cup \tau_{4Y} \) is soft semi regular space with respect \( \tau_{1Y} \cup \tau_{2Y} \) is pair wise soft semi \( T_3 \) space.

**Proposition 7.** Let \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) be a soft quad topological space over \( X \) and \( Y \) be a soft semi closed sub space of \( X \). If \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is pair wise soft semi \( T_4 \) space then \((Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)\) is pair wise soft semi \( T_4 \) space.

**Proof:** Since \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is pair wise soft semi \( T_4 \) space so this implies that \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is pair wise soft semi \( T_1 \) space as proved above.

We prove \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is pair wise soft semi normal space.

Let \((G_1, E), (G_2, E)\) be soft semi closed sets in \( Y \) such that \((G_1, E) \cap (G_2, E) = \phi \)

Then \((G_1, E) = (Y, E) \cap (F_1, E)\) And \((G_2, E) = (Y, E) \cap (F_2, E)\)

For some soft semi closed sets such that \((F_1, E)\) is soft semi closed set in \( \tau_1 \cup \tau_2 \) soft semi closed set \((F_2, E)\) in \( \tau_3 \cup \tau_4 \).

And \((F_1, E) \cap (F_2, E) = \phi \) From Proposition 2. Since, \( Y \) is soft semi closed sub set of \( X \) then \((G_1, E), (G_2, E)\) are soft semi closed sets in \( X \) such that \((G_1, E) \cap (G_2, E) = \phi \)

Since \((X, \tau_1, \tau_2, \tau_3, \tau_4, E)\) is pair wise soft semi normal space. So there exists soft semi open sets \((H_1, E)\) and \((H_2, E)\) such that \((H_1, E)\) is soft semi open set in \( \tau_1 \cup \tau_2 \) and \((H_2, E)\) is soft semi open set in \( \tau_3 \cup \tau_4 \) such that \((G_1, E) \subseteq (H_1, E)\) \((G_2, E) \subseteq (H_2, E)\)

\((H_1, E) \cap (H_2, E) = \phi \)

Since \((G_1, E), (G_2, E) \subseteq (Y, E)\)
Then \((G_1, E) \subseteq (Y, E) \cap (H_1, E)\)
\((G_2, E) \subseteq (Y, E) \cap (H_2, E)\)
And \([\{Y, E\} \cap (H_1, E)] \cap [\{Y, E\} \cap (H_2, E)] = \emptyset\)
Where \((Y, E) \cap (H_1, E)\) and \((Y, E) \cap (H_2, E)\) are soft semi open sets in \(Y\) there fore \(\tau_{1Y} \cup \tau_{2Y}\) is soft semi normal space with respect to \(\tau_{3Y} \cup \tau_{4Y}\). Similarly, let \((G_1, E), (G_2, E)\) be soft semi closed sub set in \(Y\) such that
\((G_1, E) \cap (G_2, E) = \emptyset\)
Then \((G_1, E) = (Y, E) \cap (F_1, E)\)
And \((G_2, E) = (Y, E) \cap (F_2, E)\)
For some soft semi closed sets such that \((F_1, E)\) is soft semi closed set in \(\tau_3 \cup \tau_4\) and \((F_2, E)\) soft semi closed set in \(\tau_1 \cup \tau_2\) and
\((F_1, E)(F_2, E) = \emptyset\)
From Proposition 2. Since, \(Y\) is soft semi closed sub set in \(X\) then \((G_1, E), (G_2, E)\) are soft semi closed sets in \(X\) such that
\((G_1, E) \cap (G_2, E) = \phi\)
Since \((X, \tau_1, \tau_2, \tau_1, \tau_2, E)\) is pair wise soft semi normal space so there exists soft semi open sets \((H_1, E)\) and \((H_2, E)\)
Such that \((H_1, E)\) is soft semi open set is \(\tau_3 \cup \tau_4\) and \((H_2, E)\) is soft semi open set in \(\tau_1 \cup \tau_2\) such that
\((G_1, E) \subseteq (H_1, E)\)
\((G_2, E) \subseteq (H_2, E)\)
\((H_1, E) \cap (H_2, E) = \phi\)
Since \((G_1, E), (G_2, E) \subseteq (Y, E)\)
Then \((G_1, E) \subseteq (Y, E) \cap (H_1, E)\)
\((G_2, E) \subseteq (Y, E) \cap (H_2, E)\)
And \([\{Y, E\} \cap (H_1, E)] \cap [\{Y, E\} \cap (H_2, E)] = \emptyset\)
Where \((Y, E) \cap (H_1, E)\) and \((Y, E) \cap (H_2, E)\) are soft open sets in \(Y\) there fore \(\tau_{3Y} \cup \tau_{4Y}\) is soft semi normal space with respect to \(\tau_{1Y} \cup \tau_{2Y}\)
\(\Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)\) is pair wise soft semi \(T_4\) space.

3.2 SOFT SEMI SEPARATION AXIOMS IN SOFT QUAD TOPOLOGICAL SPACES WITH RESPECT TO SOFT POINTS.

In this section, we introduced soft topological structures known as semi separation axioms in soft quad topology with respect to soft points. With the applications of these soft semi separation axioms different result are brought under examination.

**Definition 3:**

In a soft quad topological space \((X, \tau_1, \tau_2, \tau_1, \tau_2, E)\)
1) \(\tau_1 \cup \tau_2\) is said to be soft semi \(T_0\) space with respect to \(\tau_3 \cup \tau_4\) if for each pair of distinct points \(e_G, e_H \in X_A\) there happens \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and a \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\) such that \(e_G \in (F, E)\) and \(e_H \notin (G, E)\). Similarly, \(\tau_3 \cup \tau_4\) is said to be soft semi \(T_0\) space with respect to \(\tau_1 \cup \tau_2\) if for each pair of distinct points \(e_G, e_H \in X_A\) there happens \(\tau_3 \cup \tau_4\) soft semi open set \((F, E)\) and a \(\tau_1 \cup \tau_2\) semi soft open set \((G, E)\) such that \(e_G \in (F, E)\) and \(e_H \notin (G, E)\) or \(e_H \in (G, E)\) and \(e_G \notin (G, E)\). Soft quad topological spaces \((X, \tau_1, \tau_2, \tau_1, \tau_2, E)\) is said to be pair wise soft semi \(T_0\) space if \(\tau_1 \cup \tau_2\) is soft semi \(T_0\) space with respect to \(\tau_3 \cup \tau_4\) and \(\tau_3 \cup \tau_4\) is soft semi \(T_0\) spaces with respect to \(\tau_1 \cup \tau_2\).

2) \(\tau_1 \cup \tau_2\) is said to be soft semi \(T_1\) space with respect to \(\tau_3 \cup \tau_4\) if for each pair of distinct points \(e_G, e_H \in X_A\) there happens a \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\) such that \(e_G \in (F, E)\) and \(e_H \notin (G, E)\) and \(e_H \in (G, E)\) and \(e_G \notin (G, E)\). Similarly, \(\tau_3 \cup \tau_4\) is said to be soft semi \(T_1\) space with respect to \(\tau_1 \cup \tau_2\) if for each pair of distinct points \(e_G, e_H \in X_A\) there exist a \(\tau_3 \cup \tau_4\) soft semi open set \((F, E)\) and a \(\tau_1 \cup \tau_2\) Soft semi open set \((G, E)\) such that \(e_G \in (F, E)\) and \(e_H \notin (G, E)\) and \(e_H \in (G, E)\) and \(e_G \notin (G, E)\). Soft quad topological space \((X, \tau_1, \tau_2, \tau_1, \tau_2, E)\) is said to be pair wise soft semi \(T_1\) space if \(\tau_1 \cup \tau_2\) is soft semi \(T_1\) space with respect to \(\tau_3 \cup \tau_4\) and \(\tau_3 \cup \tau_4\) is soft semi \(T_1\) spaces with respect to \(\tau_1 \cup \tau_2\).

3) \(\tau_1 \cup \tau_2\) is said to be soft semi \(T_2\) space with respect to \(\tau_3 \cup \tau_4\), if for each pair of distinct points \(e_G, e_H \in X_A\) there happens a \(\tau_1 \cup \tau_2\) soft semi open set \((F, E)\) and a \(\tau_3 \cup \tau_4\) soft semi open set \((G, E)\)
such that $e_g \in (F, E)$ and $e_h \not\in (G, E)$ and $e_g \in (G, E)$ and $e_g \not\in (G, E) \cap (F, E) = \phi$.
Similarly, $\tau_3 \cup \tau_4$ is aid to be soft semi $T_2$ space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_g, e_h \in X_A$, there exists a $\tau_1 \cup \tau_2$ soft semi open set $(F, E)$ and $e_g \in (G, E)$ and $e_g \not\in (G, E) \cap (F, E) = \phi$. The soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft semi $T_2$ space if $\tau_1 \cup \tau_2$ is soft semi $T_2$ space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft semi $T_2$ space with respect to $\tau_1 \cup \tau_2$.

**Definition 34:**

In a soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$

1) $\tau_1 \cup \tau_2$ is said to be soft semi $T_3$ space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft semi $T_1$ space with respect to $\tau_3 \cup \tau_4$ and for each pair of distinct points $e_g, e_h \in X_A$, there exists a $\tau_1 \cup \tau_2$ soft semi open set $(F, E)$ such that $e_g \in (F, E), (G, E) \subseteq (F, E)$ and $(F, E) \cap (F, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi $T_3$ space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft semi $T_1$ space with respect to $\tau_1 \cup \tau_2$ and for each pair of distinct points $e_g, e_h \in X_A$, there exists a $\tau_3 \cup \tau_4$ soft semi open set $(F, E)$ such that $e_g \in (F, E), (G, E) \subseteq (F, E)$ and $(F, E) \cap (F, E) = \phi$.

2) $\tau_1 \cup \tau_2$ is said to be soft semi $T_4$ space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft semi $T_1$ space with respect to $\tau_3 \cup \tau_4$, there exists a $\tau_1 \cup \tau_2$ soft semi closed set $(F, E)$ and $\tau_3 \cup \tau_4$ soft semi closed set $(F, E)$ such that $(F, E) \cap (F, E) = \phi$. Also, there exists $(F, E)$ and $(G, E)$ such that $(F, E)$ is soft $\tau_1 \cup \tau_2$ semi open set, $(G, E)$ is soft $\tau_3 \cup \tau_4$ semi open set such that $(F, E) \subseteq (F, E), (F, E) \subseteq (G, E)$ and $(F, E) \cap (G, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft semi $T_4$ space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft semi $T_1$ space with respect to $\tau_1 \cup \tau_2$ and for each pair of distinct points $e_g, e_h \in X_A$, there exists a $\tau_3 \cup \tau_4$ soft semi closed set $(F, E)$ such that $e_g \in (F, E), (G, E) \subseteq (F, E)$ and $(F, E) \cap (F, E) = \phi$. Similarly, $\tau_1 \cup \tau_2$ is said to be soft semi $T_4$ space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft semi $T_1$ space with respect to $\tau_3 \cup \tau_4$ and for each pair of distinct points $e_g, e_h \in X_A$, there exists a $\tau_1 \cup \tau_2$ soft semi closed set $(F, E)$ such that $e_g \in (F, E), (G, E) \subseteq (F, E)$ and $(F, E) \cap (F, E) = \phi$.

**Proposition 8.** Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space over $X$. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft semi $T_3$ space, then for all $e_g \in X_E$, $e_g \in (e_g, E)$ is soft semi-closed set.

**Proof:** We want to prove that $e_g$ is semi closed soft set, which is sufficient to prove that $e_g \subseteq \subseteq (e_g, E)$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft semi $T_3$ space, then there exists soft semi sets $(F, E)$ such that $e_g (F, E) \subseteq (F, E)$ and $e_g \not\in (F, E) \cap (F, E) = \phi$. It follows that, $e_g \subseteq (F, E) \subseteq (F, E)$. When we want to prove that $e_g \subseteq \subseteq (e_g, E)$. Let $e_g \subseteq (e_g, E)$, then $(F, E) \subseteq (F, E)$. Where $(H(e)) = \subseteq (e_g, E) \subseteq (F, E)$. Thus, $e_g \subseteq (e_g, E) \subseteq (F, E)$. So, $e_g \subseteq\subseteq (e_g, E)$. This means that, $e_g \subseteq (e_g, E)$ is soft semi open set for all $e_g \in (e_g, E)$. Therefore, $e_g \subseteq (e_g, E)$ is soft semi closed set.

**Proposition 9.** Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over $X$. Then, if $(X, \tau_1, \tau_2, E)$ and $(X, \tau_3, \tau_4, E)$ are soft semi $T_3$ space, then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft semi $T_2$ space.

**Proof:** Suppose if $(X, \tau_1, \tau_2, E)$ is a soft semi $T_3$ space with respect to $(X, \tau_3, \tau_4, E)$, then according to definition for $e_g \not\in (e_g, E) \not\in (e_g, E) \subseteq (e_g, E)$ and $e_g \not\in (e_g, E) \not\in (e_g, E) \subseteq (e_g, E) \cap (e_g, E) = \phi$. Hence, $(X, \tau_1, \tau_2, E)$ is soft semi $T_2$ space with respect to $(X, \tau_3, \tau_4, E)$. Similarly, if $(X, \tau_3, \tau_4, E)$ is a soft semi $T_3$ space with respect to $(X, \tau_1, \tau_2, E)$, then according to definition for $e_g \not\in (e_g, E) \not\in (e_g, E) \subseteq (e_g, E)$ and $e_g \not\in (e_g, E) \not\in (e_g, E) \subseteq (e_g, E) \cap (e_g, E) = \phi$. Hence, $(X, \tau_3, \tau_4, E)$ is soft semi $T_2$ space with respect to $(X, \tau_1, \tau_2, E)$. Similarly, if $(X, \tau_1, \tau_2, E)$ is a soft semi $T_3$ space with respect to $(X, \tau_3, \tau_4, E)$, then according to definition for $e_g \not\in (e_g, E) \not\in (e_g, E) \subseteq (e_g, E)$ and $e_g \not\in (e_g, E) \not\in (e_g, E) \subseteq (e_g, E) \cap (e_g, E) = \phi$. Hence, $(X, \tau_3, \tau_4, E)$ is soft semi $T_2$ space with respect to $(X, \tau_1, \tau_2, E)$.
such that $e_H \in (F, E)$, $e_G \in (x, E) \subseteq (G, E)$ and $(F_2, E) \cap (F_2, E) = \emptyset$. Hence, $(X, (\tau_3, \tau_4, E))$ is a soft semi $T_2$ space. Thus $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is a pair wise soft semi $T_2$ space.

**Proposition 10.** Let $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ be a soft quad topological space over $X$. If $(X, (\tau_1, \tau_2, E))$ and $(X, (\tau_3, \tau_4, E))$ are soft semi $T_3$ space then $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is a pair wise soft semi $T_3$ space.

**Proof:** Suppose $(X, (\tau_1, \tau_2, E))$ is a soft semi $T_3$ space with respect to $(X, (\tau_3, \tau_4, E))$ then according to definition for $e_G, e_H \in X_A$, $e_G \notin e_H$ there happens $\tau_1 \cup \tau_2$ soft semi open set $(F_1, E)$ and a $\tau_3 \cup \tau_4$ soft semi open set $(G, E)$ such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each point $e_G \in X_A$ and each $\tau_1 \cup \tau_2$ semi closed set $(G_1, E)$ such that $e_G \notin (G_1, E)$ there happens a $\tau_1 \cup \tau_2$ soft semi open set $(F_1, E)$ and $\tau_3 \cup \tau_4$ soft semi open set $(F_2, E)$ such that $e_G \notin (F_1, E)$, $(G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \emptyset$. Similarly $(X, (\tau_3, \tau_4, E))$ is a soft semi $T_3$ space with respect to $(X, (\tau_1, \tau_2, E))$. So according to definition for $e_G, e_H \in X_A$, $e_G \notin e_H$ there exists a $\tau_3 \cup \tau_4$ soft semi open set $(F_1, E)$ and $\tau_1 \cup \tau_2$ soft semi open set $(G, E)$ such that $e_H \notin (F, E)$ and $e_H \notin (G, E)$ and for each point $e_G \in X_A$ and each $\tau_3 \cup \tau_4$ semi closed set $(G_1, E)$ such that $e_G \notin (G_1, E)$ there exists $\tau_3 \cup \tau_4$ soft semi open set $(F_1, E)$ and $\tau_1 \cup \tau_2$ soft semi open set $(G, E)$ such that $e_H \notin (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \emptyset$. Hence $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_3$ space.

**Proposition 11.** If $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ be a soft quad topological space over $X$, $(X, (\tau_1, \tau_2, E))$ and $(X, (\tau_3, \tau_4, E))$ are soft semi $T_4$ space then $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_4$ space.

**Proof:** Suppose $(X, (\tau_1, \tau_2, E))$ is soft semi $T_4$ space with respect to $(X, (\tau_3, \tau_4, E))$. So according to definition for $e_G, e_H \in X_A$, $e_G \notin e_H$ there happens a $\tau_1 \cup \tau_2$ soft semi open set $(F_1, E)$ and $\tau_3 \cup \tau_4$ soft semi open set $(G, E)$ such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each point $e_G \in X_A$ and each soft semi closed set $(G_1, E)$ such that $e_G \notin (G_1, E)$ there happens a $\tau_1 \cup \tau_2$ soft semi open set $(F_1, E)$ and $\tau_3 \cup \tau_4$ soft semi open set $(F_2, E)$ such that $(F_1, E) \cap (F_2, E) = \emptyset$. There occurs $(F_3, E)$ and $(G_1, E)$ such that $(F_3, E)$ is soft $\tau_3 \cup \tau_4$ semi open set $(G_1, E)$ is soft $\tau_1 \cup \tau_2$ semi open set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E)$ and $(F_3, E) \cap (G_1, E) = \emptyset$. Similarly $\tau_3 \cup \tau_4$ is soft semi $T_4$ space with respect to $(X, (\tau_1, \tau_2, E))$. So according to definition for $e_G, e_H \in X_A$, $e_G \notin e_H$ there happens a $\tau_1 \cup \tau_2$ soft semi open set $(F_1, E)$ and for some soft semi closed set $(G, E)$ there occurs $(F_3, E)$ and $(G_1, E)$ such that $(F_3, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \emptyset$. Hence $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_4$ space.

**Proposition 12.** Let $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ be a soft quad topological space over $X$ and $Y$ be a non-empty subset of $X$. If $(Y, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_3$ space. Then $(Y, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_3$ space.

**Proof.** First we prove that $(Y, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_1$ space. Let $e_G, e_H \in X_A$, $e_G \notin e_H$ if $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft space then this implies that $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $\tau_1 \cup \tau_2$ space. So there exists $\tau_1 \cup \tau_2$ soft semi open set $(G, E)$ such that $e_G \in (F, E)$ and $e_H \in (F, E)$ or $e_H \notin (G, E)$ and $e_G \notin (G, E)$ and for each $e_G \in X_A$ and each soft semi closed set $(G_1, E)$ such that $e_G \notin (G_1, E)$ there happens a $\tau_1 \cup \tau_2$ soft semi open set $(F_1, E)$ and $\tau_3 \cup \tau_4$ soft semi closed set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \emptyset$. Hence $e_G \notin Y \cap (F, E) = (Y, E)$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. This means that $\alpha \in E$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. Therefore, $e_G \in Y \cap (F, E) = (Y, E)$. Now $e_H \in Y \cap F(\alpha) = (G, E)$ where $(G, E) \subseteq (\tau_3, \tau_4)$. Consider $x \notin (G, E)$, this means that $\alpha \in E$ and $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. Therefore, $e_G \in Y \cap (G, E) = (Y, G)$ thus $(Y, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_1$ space.

Now, we prove that $(Y, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is pair wise soft semi $T_1$ space. Let $e_G \in Y$ and $(G, E)$ be soft semi closed set in $Y$ such that $e_G \notin (F, E)$ where $(G, E) \subseteq (\tau_1, \tau_2)$ then $(G, E) \subseteq (Y, E) \cap (F, E)$ for some soft semi closed set in $\tau_1 \cup \tau_2$. Hence $e_G \notin (Y, E) \cap (F, E)$ but $e_G \notin (Y, E)$, so $e_G \notin (F, E)$ since $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is soft semi $T_3$ space $(X, (\tau_1, \tau_2, \tau_3, \tau_4, E))$ is soft semi regular space so there happens $\tau_1 \cup \tau_2$ soft semi open set $(F_1, E)$ and $\tau_3 \cup \tau_4$ soft semi open set $(F_2, E)$ such that $e_G \notin (F_1, E), (G, E) \subseteq (F_2, E)$ $(F_1, E) \cap (F_2, E) = \emptyset$. Take $(G_1, E) = (Y, E) \cap (F_2, E)$ then $(G_1, E), (G_2, E)$ are soft semi open sets in $Y$ such that
Therefore, $(\tau_{1Y}, \tau_{2Y})$ is soft semi regular space with respect to $(\tau_{3Y}, \tau_{4Y})$. Similarly, Let $eH \in Y$ and $(G,E)$ be a soft semi closed sub set in Y such that $eH \notin (G,E)$. Where $(G,E) \in \tau_3 \cup \tau_4$ then $(G,E) = (Y,E) \cap (F,E)$ where $(F,E)$ is some soft semi closed set in $\tau_3 \cup \tau_4$. $eH \notin (Y,E) \cap (F,E)$ but $eH \in (Y,E)$ and $eH \notin (F,E)$ since $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is soft semi regular space so there happens $\tau_3 \cup \tau_4$ soft semi open set $(F_1, E)$ and $\tau_1 \cup \tau_2$ soft semi open set $(F_2, E)$. Such that $eH \in (F_1, E), (G,E) \subseteq (F_2, E)$, $(G_1, E) = (Y,E) \cap (F_1, E)$, $(G_2, E) = (Y,E) \cap (F_2, E)$, $(G_1, E) \subseteq (F_1, E)$, $(G_2, E) \subseteq (F_2, E)$, $(F_1, E) \cap (F_2, E) = \phi$. Take

Then $(G_1, E) = (Y,E) \cap (F_1, E)$, $(G_2, E) = (Y,E) \cap (F_2, E)$, $(G_1, E) \subseteq (F_1, E)$, $(G_2, E) \subseteq (F_2, E)$, $(F_1, E) \cap (F_2, E) = \phi$. Therefore $(\tau_{3Y}, \tau_{4Y})$ is soft semi regular space.

4. Conclusion

A soft set with single specific topological structure is unable to shoulder up the responsibility to construct the whole theory. So to make the theory healthy, some additional structures on soft set has to be introduced. It makes, it more springy to develop the soft topological spaces with its infinite applications. In this regard we introduce strong topological structure known as soft quad topological structure in this paper.

Topology is the supreme branch of pure mathematics which deals with mathematical structures. Freshly, many scholars have studied the soft set theory which is coined by Molodtsov [1] and carefully applied to many difficulties which contain uncertainties in our social life. Shabir and Naz [3] familiarized and deeply studied the origin of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we constantly study the behavior of soft semi separation axioms in soft quad topological spaces with respect to soft points as well as ordinary points of a soft topological space. We introduce soft semi $qT_0$ structure, soft semi $qT_1$ structure, soft semi $qT_2$ structure, Soft semi $qT_3$ and soft semi $qT_4$ structure with respect to soft and ordinary points. In future we will plant these structures in different results. More over defined soft semi $T_0$ structure w.r.t. soft semi $T_1$ structure and versa, soft semi $T_1$ structure w.r.t soft semi $T_2$ structure and versa and soft semi $T_3$ space w.r.t soft semi $T_4$ and vice versa with respect to ordinary and soft points in soft quad topological spaces and studied their activities in different results with respect to ordinary and soft points. We also planted these axioms to different results. These soft semi separation axioms in quad structure would be valuable for the development of the theory of soft topology to solve complicated problems, comprising doubts in economics, engineering, medical etc. We also attractively discussed some soft transmissible properties with respect to ordinary as well as soft points. We expect that these results in this article will do help the researchers for strengthening the toolbox of soft topological structures. In the forthcoming spread the idea of soft $a$- open, and soft $b^{**}$ open sets in soft quad topological structure with respect to ordinary and soft points.

References